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The Gift of
Rev. Thomas Hill, D.D.,
of Cambridge,
(H. C. 1843,)
7 Oct. 1868.

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NATURAL SERIES.

THE ANALYSIS OF WRITTEN ARITHMETIC.

BOOK SECOND,

DESIGNED FOR

PUBLIC AND PRIVATE SCHOOLS,

CONTAINING

MENTAL, SLATE, AND BLACKBOARD EXERCISES;

BY

S. A. FELTER, A. M.,

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AND POLYTECHNIC INSTITUTE, AUTHOR OF PRIMARY ARITHMETIC, ETC.

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PREFACE.

In the preparation of this work the author has labored to secure, 1st, *discipline of mind* by requiring a *formula* for each step of the analysis; 2nd, *rapidity, accuracy, and complete familiarity with commercial transactions*, by giving a large number of abstract and practical examples and problems; 3rd, *thoroughness*, by a systematic daily review.

Upon the introduction of Mental Arithmetic into our schools, the solution of problems by rule gave place, in some degree, to analysis, which has, however, been applied but little to the more advanced subjects of Commercial Arithmetic. A complete system of analysis constitutes an important, and it is hoped, a very advantageous feature of this work.

In most published text books on arithmetic more attention has been given to the development of the science and the properties of numbers, and the logical arrangement of subjects than to their application to business transactions. Most of the examples are prepared with particular reference to the subject which they are to illustrate, much time and space are devoted to arithmetical puzzles, and the commercial examples often contain most improbable conditions. While the arrangement of subjects is *logical*, it is not *natural*, and does not follow the order of simplicity, thereby involving the pupil in diffiulties before his mind is sufficiently developed to master them. The subject of Proportion, or the Rule of Three, has received undue prominence; for all commercial questions can be more easily solved by Analysis. Reduction of Currencies, Duodecimals, Permutation, and the Progressions are superfluous in a commercial arithmetic, and should be discussed in a higher arithmetic only. In this work an effort has been made to avoid these and similar faults.

The work is divided into two parts; the first containing a complete theoretical review of the subjects contained in the First Book, with important additions of principles and contractions in the fundamental rules and denominate numbers. Exercises in the composition of examples may be carried to any desirable extent. The second part commences with the Properties of Numbers, and Fractions, and is, really, the continuation of Book First. No pupil should begin it without being thoroughly acquainted with the subjects treated in that volume; for a neglect of this precaution will greatly embarrass both pupil and teacher.

The author wishes to acknowledge his indebtedness to those teachers who have shown their interest in this series by kindly offering valuable suggestions, as well as to R. S. Delisser, Esq., for his method of averaging accounts by interest, which he has kindly given permission to use.

COLLEGIATE AND POLYTECHNIC INSTITUTE,
BROOKLYN, N. Y., May, 1864.

INTRODUCTION.

In presenting this work to his fellow teachers, the author deems it proper to exhibit its peculiarities more fully than he can do in a preface for the general reader.

Analysis.—This work is intended to continue the subject of commercial arithmetic as left in Book First; and, also, to make it in some measure complete for those who wish a theoretical review of the subjects treated in that volume. All the tables, principles, analyses, formulas, &c., given in the First Book are, therefore, reproduced in the second, with model examples for composition, by which the key to the formation of problems as well as their true analysis is clearly exhibited. (See Anal., page 27.)

Every complex, concrete, or commercial problem, can be separated into elementary questions, and each question involves one of six *arithmetical formulas*. Using the symbols x and y, we have:

I. If some oranges cost x cts., and some apples cost y cts., both will cost the sum of these quantities, which are $x+y$ cts.

II. If the oranges cost x cts., and the apples cost y cts., the oranges will cost as many more cents than the apples as the difference of these quantities, which is $x-y$ cts.

III. If one orange cost x cts., y oranges will cost y times x cts., which is $x \times y$ cts.

IV. If x cts. are equally divided among y boys, each boy will receive *one y* part of x cts., which is $x \div y$ cts.

V. If one orange costs y cts., as many oranges can be bought for x cts. as y cts. are contained times in x cts., which are $x \div y$ times.

VI. If a boy have x cts. and he give away y cts., he will give away the $\frac{y}{x}$ part of his money.

Substituting 18 as the value of x, and 9 as the value of y, we have the numerical values of each respectively, as follows: 27 cts., 9 cts., 162 cts., 2 cts., 2 oranges, and $\frac{9}{18}$ or $\frac{1}{2}$ of his money. By substituting other numerical values and denominations a large number of examples may be formed, and, by combination, the ingenious teacher may find pleasant, and the pupil, profitable employment.

While undue prominence should not be given to this method, it is important that in the analysis of problems the pupil be continually required to point out the more important of these elementary questions, and the formulas involved in them. By this means, he will become self-reliant, unaccustomed to flee for refuge in every difficulty to some RULE; but will depend upon the exercise of his reason for a solution.

ARRANGEMENT.—It has been objected that, in a logical order of subjects, fractions should precede denominate numbers, and U. S. Currency should be placed with decimals, where it logically belongs. The author would say, in answer, that since no scale can be added or subtracted without reduction, logic would require reduction to be taught before addition, and the complete decimal scale before fractions or denominate numbers. He would not follow out this absurdity but counsel a *natural* instead of a *logical* order. All will agree that a child comprehends the simple before the complex. What, then, is the simplest subject in arithmetic? Certainly it is the integral portion of the decimal scale. The reduction of this is so simple that it is omitted, and the fundamental rules are taken next. None will dispute that the integral portion of the denominate scale is more simple than the fractional scale, and hence should precede it. An objection is sometimes raised that fractions are found in denominate numbers; but, certainly, those few fractions can be more easily explained than the whole subject of fractions, and the objection is, therefore, not valid. This arrangement of subjects has the further advantage, that, after the scholar has passed through the fundamental rules, he is made to apply his knowledge to business transactions in the currency which he is daily using, and is thus made to feel that arithmetic really means something in life. He then gets a thorough knowledge of the application of the tables of denominate numbers with especial reference to their use in purchases and sales. The pupil is better fitted to leave school with a thorough knowledge of these subjects, and the discipline which the acquisition will give him, than with a knowledge of the simple operations of fractions and decimals combined; for the latter he will soon forget, while the former will constitute a capital at compound interest.

Omissions and Additions.—A number of subjects which are usually introduced into arithmetics are omitted in this treatise for the purpose of giving more space to those which are more practical, and many additions are made to the applications of fractions and per centage.

Review.—The method of review in this series is such that a pupil entering at almost any stage of progress will be required to review all the subjects over which the class have passed, before finishing the book, thus avoiding the necessity of turning the class back on account of a few of its members.

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ARITHMETIC.

BOOK SECOND.

SECTION I.

DEFINITIONS.

LESSON I.*

1. A **Unit** is a single thing; as, 1, 1 apple, 1 cow, 1 peach.

ILLUSTRATION.*—*Def.* 1.—A unit is a single thing.

1 cow, 1 peach represent each a single thing, and, therefore, are units.

2. Number is an expression that tells how many; as, 1, 3, 5, 19, 4 horses, 3 oxen, 2 cows.

3. An **Abstract Number** is one used without reference to any object; as, 5, 18, 13, 25.

4. A **Concrete,** or **Denominate Number,** is one used with reference to some object or collection of objects; as, 3 pecks, 1 pound, 9 melons, 11 dollars.

ILLUSTRATION.—A concrete, or denominate number, is one used with reference to some object, or collection of objects.

1 peck, 9 horses, 11 melons, are each used either with reference to some object or collection of objects, and are, therefore, concrete, or denominate numbers.

---

*NOTE.—The teacher should require the pupil to illustrate each of the following definitions and axioms.

5. **Arithmetic** treats of quantity represented by numbers.

6. **Quantity** is that which can be measured.

7. A **Problem** in Arithmetic is a question with conditions, proposed for solution.

8. A **Simple Problem** is a problem containing but one elementary question.

9. A **Complex Problem** is a problem involving two or more elementary questions.

10. An **Analytical Step** is the solution of one of the elementary questions composing a complex problem.

11. An **Analysis*** is the method of solving a complex problem by resolving it into elementary questions.

12. A **Rule** is a concise direction for solving problems, and may be deduced from an analysis.

13. A **Sign** in Arithmetic is a symbol employed to show the relations of numbers, and to denote operations to be performed upon them.

14. An **Axiom** is a self-evident truth.

15. The principal, or fundamental rules in Arithmetic, are Notation and Numeration, Addition, Subtraction, Multiplication, and Division.

16. AXIOMS.

(*a.*) Things can be added to like things only.

---

*Note.—The term Analysis is, also, somewhat loosely applied to the solution of an Elementary Question by means of an Arithmetical formula, thus:—

Elementary Question.—If 1 pound of sugar costs 11 cents, what will 6 pounds cost?

Arithmetical Formula.—If 1 pound of sugar costs 11 cents, 6 pounds will cost 6 times 11 cents, which are 66 cents.

(*b.*) Things can be subtracted from like things only.

(*c.*) Things can be compared with like things only.

(*d.*) If the same or equal numbers be *added* to equal numbers, the *sums* will be equal.

(*e.*) If the same or equal numbers be *subtracted* from equal numbers, the *remainders* will be equal.

(*f.*) If equal numbers be *multiplied* by the same or equal numbers, the *products* will be equal.

(*g.*) If equal numbers be *divided* by the same or equal numbers, the *quotients* will be equal.

(*h.*) If the same number be both *added to*, and *subtracted from* another number, the *value* of the latter number will not be changed.

(*i.*) If a number be both *multiplied* and *divided* by the same number, its *value* will not be changed.

(*j.*) If two numbers be equally *increased* or *diminished*, their *difference* will not be changed.

(*k.*) Numbers equal to the *same* number are equal to each other.

(*l.*) A number is *greater* than any of its parts.

(*m.*) A number is equal to the *sum* of all its parts.

(*n.*) If both *divisor* and *dividend* be *multiplied* by the same number, the quotient will not be changed.

(*o.*) If both *divisor* and *dividend* be *divided* by the same number, the quotient will not be changed.

QUESTIONS.—What is a unit? (1.)* What is a simple problem? (8.) What is an analytical step? (10.) What is a number? (2.) What is an abstract number? (3.) Of what does Arithmetic treat? (5.)

---

*NOTE FOR THE TEACHER.—The numbers at the end of the questions refer to the paragraphs containing the answers.

N. B.—The Review questions appended to each lesson should by no means be omitted.

What is a quantity? (6.) What is a problem? (7.) What is an analysis? (11.) What is a rule? (12.) What is a sign? (13.) What is an axiom? (14.) Which are the fundamental rules? (15.) Illustrate the 1st definition after the model. (1.* See note.) Illustrate the 4th definition. (See note.) Illustrate the 8th definition. Illustrate each of the first fourteen definitions on the blackboard.

SECTION II.

NOTATION AND NUMERATION.

LESSON I.

NOTATION.

17. Notation is a method of expressing numbers by words, letters, or figures. There are two methods in common use, the Roman and the Arabic.

18. The Roman, used in numbering the chapters, sections, and other divisions of books, expresses numbers by letters, thus:

I	one.	C	one hundred.
V	five.	D	five hundred.
X	ten.	M	one thousand.
L	fifty.	$\bar{M}$	one million.

(*a.*) All other numbers are expressed by the use of these letters repeated or combined, as:

I	one	IX	nine	LXXX	eighty	DCC	seven hundred
II	two	X	ten	XC	ninety	DCCC	eight hundred
III	three	XX	twenty	C	one hundred	DCCCC	nine hundred
IV	four	XXX	thirty	CC	two hundred	M	one thousand
V	five	XL	forty	CCC	three hundred	MD	fifteen hundr'd
VI	six	L	fifty	CCCC	four hundred	MM	two thousand
VII	seven	LX	sixty	D	five hundred	$\bar{X}$	ten thousand
VIII	eight	LXX	seventy	DC	six hundred	$\bar{M}$	one million

(*b.*) By repeating a letter the value of that letter is multiplied as many times as the letter is repeated; as, XXX, *thirty*, CC, *two hundred.*

(*c.*) If a letter of *less value* be written *before* a letter of *greater value*, the value of the less is subtracted from that of the greater; as, X, *ten*, IX, *nine;* L, *fifty*, XL, *forty.*

(*d.*) If a letter of *less value* be written *after* a letter of *greater value*, the value of the *less* is *added* to that of the greater; as, V, *five*, VI, *six;* M, *one thousand*, MD, *fifteen hundred.*

(*e.*) A dash (—) placed over a letter or combination of letters increases the value a thousand times; as, X, *ten*, $\overline{X}$, *ten thousand;* XII, *twelve*, $\overline{XII}$, *twelve thousand.*

QUESTIONS.—What is the axiom for addition? (16., *a.*) What is the axiom for subtraction? (16., *b.*) What is the axiom for comparison? (16., *c.*) If you add the same or equal quantities to equal quantities, how will the sums be affected? (16., *d.*) Illustrate each of the axioms on the blackboard. (16.) What is notation? (17.) What of the Roman notation? (18.) How many letters are used in the Roman notation? (18.) Name them. What number is represented by each? (18.) How are other numbers expressed? (18., *a.*) What effect does repeating a letter have? (18., *b.*) What effect has a letter of less value when placed before one of greater value? (18., *c.*) What effect has a letter of less value when placed after one of greater value? (18., *d.*) What effect has a dash placed over a letter or combination of letters? (18., *e.*)

LESSON II.

19. The Arabic notation is generally used in arithmetical computations. It consists of ten characters, or figures, nine of which have each a simple and a local value. They are:—

1, *one;* 3, *three;* 5, *five;* 7, *seven;* 9, *nine;* 2, *two;* 4, *four;* 6, *six;* 8, *eight;* 0, *naught or cipher.*

20. The *simple value* of a figure is the one expressed by it when standing alone or in the units' place of a number.

21. The *local value* of a figure is that which depends upon the place which the figure occupies in a number; thus, in 37 the figure 3 denotes *thirty;* and in 904, 9 denotes *nine hundred.*

22. The *naught* or *cipher* (0) has no value, but is used to show that a denomination is wanting; thus, in the number 6074, the hundreds being wanting, the place is occupied by a cipher.

QUESTIONS.—Of what does the Arabic notation consist? (19.) What is the simple value of a figure? (20.) What is the local value of a figure? (21.) What is the use of the cipher? (22.) Show on the blackboard what is meant by simple and local values. What is the difference between the Roman and the Arabic notation? (18.) (19.)

LESSON III.

NUMERATION.

23. Numeration is the art of naming in their regular order the places occupied by numbers.

24. Reading numbers is the art of expressing their written value orally.

NUMERATION OF A PERIOD.

Period.

Hundreds.	Tens.	Units.
0	0	0

25. NUMERATION TABLE.

7th Period.*			6th Period.			5th Period.			4th Period.			3rd Period.			2nd Period.			1st Period.		
Quintillions.			Quadrillions.			Trillions.			Billions.			Millions.			Thousands.			Units.		
Hundreds.	Tens.	Units.	Hundreds.	Teus.	Units.	Hundreds.	Tens.	Units.	Hundreds.	Tens.	Units.	Hundreds.	Tens.	Units.	Hundreds.	Tens.	Units.	Hundreds.	Tens.	Units.
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

26. Every *vacant place* in each period, with the exception of the left hand places of the left hand period, must be filled with a cipher.

27. Every *vacant period* in each number, with the exception of the left hand period, must be filled with ciphers.

NOTE.—(*a*) In writing numbers periods are usually separated by commas.

---

*From quintillions the table may be extended to sextillions, septillions, octillions, nonillions, decillions, undecillions, duodecillions, &c.

LESSON IV.

EXERCISES IN NOTATION AND NUMERATION.

Write

Three hundred and eighty-two;
Seven hundred and forty-nine;
Eight hundred and sixty-seven;
One thousand three hundred and ninety-seven;
Fifteen thousand eight hundred and seventy-three;
Eleven thousand and eight.

MODEL OPERATION.

382
749
867
1,397
15,873
11,008

28. Rule for Notation.—*Begin at the left hand, and write the figures belonging to the highest period.*

Write the hundreds, tens, and units, of each successive period in their proper order, observing to write a cipher in each vacant place.

Note for the Teacher.—For exercises in notation and numeration the pupil is referred to the First Book of Arithmetical Analysis.

LESSON V.

29. ANALYSIS OF NUMBERS.

MODEL OPERATION.

It is required to analyze the number 3,917,893.

(*a.*) For convenience, every number is divided into *periods* of three *places* each, counting from the right.

(*b.*) The 1st *period* of every number is called *Units*;

the 2d, *Thousands;* the 3d, *Millions*; the 4th, *Billions;* the 5th, *Trillions;* the 6th, *Quadrillions;* and the 7th, *Quintillions*, &c.

(*c.*) The *first* place of every period is called *units;* the *second*, *tens;* the *third*, *hundreds.*

(*d.*) The value of the places increases in a tenfold ratio from the right to the left; that is, tens' place has ten times the value of units' place; hundreds' place has ten times the value of tens' place; thousands' place has ten times the value of hundreds' place, &c.

(*e.*) In the units' period of the given number are written 3 units, 9 tens, 8 hundreds; in the thousands' period are written 7 units, 1 ten, 9 hundreds; in the millions' period are written 3 units.

(*f.*) In the units' period of the given number, 9 tens are equal to 90 units; 8 hundreds are equal to 80 tens, or 800 units; in the thousands' period, 7 units of thousands are equal to 70 hundreds, or 700 tens, or 7000 units; 1 ten of thousands is equal to ten units of thousands, or 100 hundreds, or 1000 tens, or 10000 units; 9 hundreds of thousands are equal to 90 tens of thousands, or 900 units of thousands, or 9000 hundreds, or 90,000 tens, &c.

After the model, analyze the following numbers:—

1. One million three hundred and eighty-five thousand six hundred seventy-four.
2. 19 millions 713 thousand 486.
3. 186 millions 347 thousand 391.
4. 986 trillions 421 million 223 thousand, 86.
5. Five trillions 876 millions 876 thousand.
6. 189 quadrillions 317 thousands 346.
7. 597 trillions 896 millions 348 thousand.
8. 876 millions 384 thousand.

9. 968 millions 117 thousand 18.
10. 678 trillions 347 millions 396 thousand.
11. 876 billions 546 millions 321 thousand.
12. 347 billions 390 millions 416 units.
13. 974 quadrillions 14 trillions 318.
14. 764 millions 39 thousand 340.
15. 97 thousand, four hundred and ninety-five.

QUESTIONS.—What is numeration? (23) What is the reading of numbers? (24) Repeat the numeration table. (25) What is to be done with vacant places? (26) What is to be done with vacant periods? (27) How are periods usually separated? (27 *a*) What is the rule for notation? (28) Give the analysis of the number 3,917,-893. (29)

SECTION III.

ADDITION.

LESSON I.

30. The **Sum** of two or more numbers is a number which contains as many units as all the numbers taken together.

31. **Addition** is the process of finding the sum of two or more numbers.

32. The SIGN OF ADDITION is a horizontal cross, thus, (+), and is called *plus*, which signifies more. When placed between two numbers, it denotes that they are to be added together; as, 4+6 are equal to 10; to be read 4 plus 6 are equal to ten.

33. The SIGN OF EQUALITY is two, short, parallel, horizontal lines; thus, (=), and when placed between two numbers or quantities, it denotes that they are equal; as 8+6=14; to be read, 8 plus 6 equals 14.

34. The DOLLAR SIGN is an S crossed by two perpendicular parallel lines; thus, ($), and when prefixed to numbers, denotes that they represent dollars; as, $86; to be read, 86 dollars.

35. Order of solving problems.

(*a.*) State the problem.

(*b.*) Give necessary definitions.

(*c.*) Analyze.

(*d.*) Give conclusion.

(*e.*) Deduce a rule.

LESSON II.

36. ANALYSIS OF ADDITION.

(*a.*) Statement.—It is required to find the sum of the following numbers: 3896; 4896; 3894; and 437.

(*b.*) Definition.—The sum of two or more numbers is a number which contains as many units as all the numbers taken together.

MODEL OPERATION.

```
 3896
 4896
 3894
  437
 ----
13123 sum.
```

(*c.*) Analysis.—For convenience, the numbers are written so that units shall stand in the column of units, tens in the column of tens, and hundreds in the column of hundreds, &c.

2. For convenience, commence by adding the units' column, thus: 7, 11, 17, 23; 23 units are equal to 3 units and 2 tens: write the units under the column of units, and add the tens to the column of tens: thus,

3. 2, 5, 14, 23, 32; 32 tens are equal to 2 tens and three hundreds: write the tens under the column of tens, and add the hundreds to the column of hundreds: thus,

4. 3, 7, 15, 23, 31; 31 hundreds are equal to one hundred and three thousands: write the hundred under the column of hundreds, and add the thousands to the column of thousands: thus,

5. 3, 6, 10, 13; 13 thousands are equal to 3 thousands and one ten thousand: write the 3 thousands under the column of thousands, and the 1 ten thousand in the place of ten thousands.

(*d.*) CONCLUSION.—Therefore, the sum of 3896, 4896, 3894,and 437 is thirteen thousand one hundred and twenty-three.

From the preceding we derive the following

(*e.*) RULE.—*Write the numbers so that units shall stand in the column of units, tens in the column of tens, &c.*

Beginning with units, add each column separately, and if the sum be less than ten, write it under the column added.

If the sum of any column be ten or more, write the units figure only, and add the others with the next column.

Write the entire sum of the last column.

(*f.*) PROOF.—*Addition may be proved by adding the columns downward in the same manner, and if the result is the same, it is supposed to be correct.*

QUESTIONS.—What is the sum of two or more numbers? (30.) What is the sign of addition? (32.) What is the sign of equality? (33.) What is the dollar sign? (34.) Give the order of solving problems. (35.) Give analysis of addition. (36.) Give Rule for addition. (36., *e.*)

LESSON III.

37. BLACKBOARD EXERCISES.

1. What is the sum of each of the following columns?

4	6	9	7	0	1	3	8	2	5
3	3	3	3	3	3	3	3	3	3
—	—	—	—	—	—	—	—	—	—

Note.—The Teacher should copy the foregoing and similar exercises upon the blackboard, and pointing to each of the columns successively, require a pupil, or the class in concert, to name the sum of each column instantaneously, as it is pointed to; thus, pointing to the right hand column, the pupil should say "Eight;" not, "Three and five are eight" Pointing to the next column, the pupil should say at once, "Five;" not, "Three and two are five."

These exercises are important in enabling the pupil to acquire a facility in combining numbers rapidly and accurately, and should be continued until this is accomplished.

2.	2	0	4	6	9	5	7	1	3	8
	4	4	4	4	4	4	4	4	4	4
3.	3	8	1	7	5	9	6	4	0	2
	5	5	5	5	5	5	5	5	5	5
4.	7	1	8	3	2	0	4	6	9	5
	6	6	6	6	6	6	6	6	6	6
5.	5	7	1	8	3	2	0	4	6	9
	7	7	7	7	7	7	7	7	7	7
6.	1	7	5	9	8	3	2	0	4	6
	8	8	8	8	8	8	8	8	8	8
7.	8	3	2	0	4	6	1	7	5	9
	9	9	9	9	9	9	9	9	9	9
8.	5	6	8	9	4	7	6	9	8	5
	9	9	9	9	9	9	9	9	9	9
9.		2	7	3	8	0	4	6	1	4
		3	7	2	9	8	5	4	9	6
10.		2	8	4	6	2	1	5	8	7
		5	3	9	8	7	6	4	3	2

11.	3	5	7	9	2	6	4	2	1
	5	7	9	2	3	5	4	8	7
	—	—	—	—	—	—	—	—	—

12.	1	9	7	8	6	3	4	2	5
	8	3	5	4	9	7	2	8	8
	—	—	—	—	—	—	—	—	—

BLACKBOARD EXERCISES IN COLUMNS.

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)	(18)
&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.
2	2	2	3	3	3	1	4	4	4	4	1	5	5	5	5	5	4
2	2	1	3	3	3	2	4	4	4	4	2	5	5	5	5	5	3
2	2	2	3	3	3	3	4	4	4	4	3	5	5	5	5	5	4
2	2	1	3	3	3	2	4	4	4	4	4	5	5	5	5	5	5
2	2	1	3	3	3	3	4	4	4	4	1	5	5	5	5	5	4
2	2	2	3	3	3	1	4	4	4	4	3	5	5	5	5	5	3
2	2	2	3	3	3	2	4	4	4	4	2	5	5	5	5	5	5
2	2	2	3	3	3	3	4	4	4	4	4	5	5	5	5	5	4
2	2	1	3	3	3	3	4	4	4	4	1	5	5	5	5	5	1
2	2	2	3	3	3	2	4	4	4	4	3	5	5	5	5	5	4
2	2	2	3	3	3	1	4	4	4	4	2	5	5	5	5	5	5
2	1	1	3	1	2	3	4	1	2	3	4	5	1	2	3	4	3
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

(19)	(20)	(21)	(22)	(23)	(24)	(25)	(26)	(27)	(28)	(29)	(30)	(31)	(32)	(33)	(34)	(35)	(36)
&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.
6	6	6	6	6	6	1	7	7	7	7	7	7	7	1	8	8	8
6	6	6	6	6	6	5	7	7	7	7	7	7	7	7	8	8	8
6	6	6	6	6	6	4	7	7	7	7	7	7	7	2	8	8	8
6	6	6	6	6	6	3	7	7	7	7	7	7	7	7	8	8	8
6	6	6	6	6	6	4	7	7	7	7	7	7	7	5	8	8	8
6	6	6	6	6	6	6	7	7	7	7	7	7	7	7	8	8	8
6	6	6	6	6	6	3	7	7	7	7	7	7	7	2	8	8	8
6	6	6	6	6	6	3	7	7	7	7	7	7	7	3	8	8	8
6	6	6	6	6	6	2	7	7	7	7	7	7	7	0	8	8	8
6	6	6	6	6	6	6	7	7	7	7	7	7	7	2	8	8	8
6	6	6	6	6	6	1	7	7	7	7	7	7	7	7	8	8	8
6	1	2	3	4	5	2	7	1	2	3	4	5	6	2	8	1	2
—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—	—

(37)	(38)	(39)	(40)	(41)	(42)	(43)	(44)	(45)	(46)	(47)	(48)	(49)	(50)	(51)	(52)
&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.	&c.
8	8	8	8	8	4	9	9	9	9	9	9	9	9	9	3
8	8	8	8	8	8	9	9	9	9	9	9	9	9	9	5
8	8	8	8	8	3	9	9	9	9	9	9	9	9	9	8
8	8	8	8	8	7	9	9	9	9	9	9	9	9	9	7
8	8	8	8	8	3	9	9	9	9	9	9	9	9	9	6
3	4	5	6	7	4	9	1	2	3	4	5	6	7	8	9

Note.—The above blackboard exercises in adding columns at sight, should be practiced as follows: After copying the exercise on the blackboard, let a pupil or the class in concert commence at the bottom and add, repeating only the result of each addition, thus: (Exercise 1st.) 2, 4, 6, 8, 10, 12, 14, &c., to fifty or more, as the teacher may judge best. When the 1st exercise is THOROUGHLY mastered, commence with the second in the same manner, thus MASTERING each exercise successively until all are accomplished. It must not be understood that the exercises in this lesson are to be completed before the pupil proceeds; but they should, however, form a part of each recitation during his entire arithmetical course, if not finished sooner.

LESSON IV.

PRACTICAL EXAMPLES CONTAINING BUT ONE ELEMENTARY QUESTION.

If a merchant sells at one time 91834 yards of calico, at another 34186 yards of silk, and at another 46741 yards of army cloth, how many yards does he sell in all?

MODEL OPERATION.

91834 yards sold the 1st time.
34186 " " 2d "
46741 " " 3d "

172761 yards sold in all.

Analysis.—Arithmetical Formula.—If a merchant sells 91834 yards of calico at one time, 34186 yards of silk at another, and 46741 yards of army cloth at another, he sells in all the sum of these quantities, which is 172761 yards.

Conclusion.—Therefore, if a merchant sells 91834 yards at one time, 34186 yards at another, and 46741 yards at another, he sells in all 91834 yards.

EXERCISE.

N. B.—The teacher should require each pupil to compose and analyze ten examples similar to the model example, each containing but one elementary question.

LESSON V.

EXAMPLES CONTAINING TWO OR MORE ELEMENTARY QUESTIONS.

A farmer sold 325 bushels of wheat to one man, and to another 437 more than to the first; how many bushels did he sell in all?

MODEL OPERATION.

(*a*) 325 bushels sold to the 1st man.
437 " " 2d " more than to the 1st.
762 bushels sold to the 2d man.

(*b*) 325 bushels sold to the 1st man.
762 " " 2d "
1087 bushels sold in all.

ANALYSIS.

Elementary Question.—(*a*) If a farmer sold 325 bushels of wheat to one man, and to another 437 more than to the first, how many bushels did he sell to the second man?

Arithmetical Formula.—If a farmer sold 325 bushels of wheat to one man, and to another 437 more than to the first, he must have sold to the second man the sum of these quantities, which is 762 bushels.

Elementary Question.—(*b*) If a farmer sold 325 bushels of wheat to one man, and 762 bushels to a second, how many bushels did he sell to both?

Arithmetical Formula.—If a farmer sold 325 bushels of wheat to one man, and 762 bushels to a second, he must have sold to both the sum of these quantities, which is 1087 bushels.

Conclusion.—Therefore, if a farmer sold 325 bushels of wheat to one man, and to another 437 more than to the first, he sold to both 1087 bushels.

EXERCISE.

Require the pupil to compose and analyze ten examples similar to the model example, each containing two or more elementary questions.

TO THE TEACHER.—In the analysis of all complex problems the pupil should be required to state each *Elementary Question* contained in the complex problem, until he can do so with accuracy and promptness. The pupil should also be required to give the *Arithmetical Formula* for each question. It would be well, at first, to require the scholar to *write* the *elementary questions*, and to read them to the class, to be criticised.

Should the teacher think that this method of solving problems too *tedious* and *formal*, let us *earnestly* counsel *patience*, *thoroughness*, and *perseverance*, and from *actual experience we assure him* that the *final results* will be amply gratifying. (See Introduction.)

SECTION IV.

SUBTRACTION.

LESSON I.

38. The **Difference** between two numbers is such a number as added to the less will make it equal the greater.

39. **Subtraction** is the process of finding the difference between two numbers.

40. The **Minuend** is the number subtracted from.

41. The **Subtrahend** is the number subtracted.

42. The **Remainder** is the number left after subtraction.

43. The SIGN of SUBTRACTION, is a short horizontal line, thus: (—), and is called *minus*, signifying less. When it is placed between two numbers it denotes that the one on the right is to be taken from the one on the left; thus, for 6—4=2, to be read, 6 minus 4 equals 2.

LESSON II.

44. ANALYSIS OF SUBTRACTION.

Required the difference between 3897 and 2938.

MODEL OPERATION.

		PROOF.	
3897	minuend.	2938	subtrahend.
2938	subtrahend.	959	difference.
959	difference.	3897	a sum equal to the minuend.

(*a.*) For convenience write the *subtrahend* under the *minuend,* so that units may stand under units, tens under tens, &c.

(*b.*) For convenience commence at the units' column to subtract, thus:—

(*c.*) As 8 units can not be taken from 7 units, take 1 ten, which equals 10 units, from the 9 tens; 10 units and 7 units equal 17 units; 8 units from 17 units leave 9 units, to be written in the place of units.

(*d.*) 3 tens from 8 tens leave 5 tens, to be written in the place of tens.

(*e.*) As 9 hundreds can not be taken from 8 hundreds, take 1 thousand, which equals 10 hundreds, from the 3 thousands; 10 hundreds added to 8 hundreds equal 18 hundreds; 9 hundreds from 18 hundreds leave 9 hundreds, to be written in the place of hundreds.

(*f.*) 2 thousands from two thousands leave no thousands.

(*g.*) Therefore, the difference between 3897 and 2938 is 959.

(*h.*) RULE.—*Write the subtrahend under the minuend so that units may stand under units, tens under tens, &c.*

Begin at the right hand, and subtract each figure of the subtrahend from the figure above it in the minuend, and write the result underneath.

If any figure of the subtrahend is greater than the corresponding figure of the minuend, add ten to the figure of the minuend before subtracting, and write the result underneath. In this case, before subtracting the next left hand figure in the subtrahend, consider the corresponding figure in the minuend diminished by ONE.

(*i.*) PROOF.—*Add the difference to the subtrahend, and if the sum equals the minuend, the work is correct.*

Questions.—What is the difference of two numbers? (38.) What is Subtraction? (39.) What is the minuend? (40.) What is the subtrahend? (41.) What is the remainder? (42.) What is the sign of Subtraction? (43.) Repeat the analysis of Subtraction. (44.) Give the rule. (44., *h.*) Give the proof. (44., *i.*)

45. BLACKBOARD EXERCISES.

LESSON III.

What is the difference of the numbers in each of the following columns?

1.	2	4	7	1	9	5	8	3	6	0
	2	2	2	2	2	2	2	2	2	2

Note.—Let the teacher write the preceding, and the similar exercises which follow, upon the blackboard, and pointing to each, require a pupil or the class in concert to name the difference in each as the teacher points to it.

This should be repeated until the pupil has thoroughly mastered these exercises.

2.	8	4	7	6	9	5	8	3	9	7
	3	3	3	3	3	3	3	3	3	3
3.	9	4	7	6	9	5	8	5	7	5
	4	4	4	4	4	4	4	4	4	4
4.	9	8	7	6	9	5	8	7	8	9
	5	5	5	5	5	5	5	5	5	5
5.	7	9	8	9	8	9	9	8	6	5
	6	6	6	7	7	8	9	3	2	2
6.	10	12	15	13	14	19	17	18	16	11
	3	3	3	3	3	3	3	3	3	3

7.	10	12	15	13	14	19	17	18	16	11
	4	4	4	4	4	4	4	4	4	4
8.	10	12	15	13	14	19	17	18	16	11
	5	5	5	5	5	5	5	5	5	5
9.	10	12	15	13	14	19	17	18	16	11
	6	6	6	6	6	6	6	6	6	6
10.	10	12	14	13	14	19	17	18	16	11
	7	7	7	7	7	7	7	7	7	7
11.	10	12	15	13	14	18	17	19	16	11
	8	8	8	8	8	8	8	8	8	8
12.	10	12	15	13	14	19	17	18	16	11
	9	9	9	9	9	9	9	9	9	9

LESSON IV.

PRACTICAL EXAMPLES CONTAINING BUT ONE *ELEMENTARY QUESTION.*

A boy had 347 chestnuts, and gave 284 to his sister; how many had he remaining?

MODEL OPERATION.

347 No. of chestnuts the boy had.
284 “ “ he gave away.
———
63 No. of chestnuts he had left.

ANALYSIS.

Arithmetical Formula.—If a boy had 347 chestnuts and gave away 284, he must have had left the difference between 347 and 284, which is 63 chestnuts.

CONCLUSION.—Therefore, if a boy had 347 chestnuts, and gave away 284, he had 63 remaining.

EXERCISE.

Require the pupil to compose and analyze ten examples similar to the model example.

LESSON V.

PRACTICAL EXAMPLES COMBINING ADDITION AND SUBTRACTION, AND CONTAINING TWO OR MORE ANALYTICAL STEPS.

A farmer sold a span of horses for $138, a cow for $38, some butter for $13, and some straw for $15; he purchased 10 yards of broadclothl for $40, a stove for $35, some milk-pans for $13; how much money had he left?

MODEL OPERATION.

(*a.*)
$138 received for horses.
38 " cow.
13 " butter.
15 " straw.
———
$204 received for all.

(*b.*)
$40 paid for broadcloth.
35 " stove.
13 " milk-pans.
——
$88 paid for all.

(*c.*)
$204 amount received.
88 " paid.
———
$116 amount left.

ANALYSIS.

ANALYTICAL STEP. (*a.*) Find the amount of money received.

Elementary Question.—If a farmer sold a span of horses for $138, a cow for $38, some butter for $13, and some straw for $15, how much did he receive?

Arithmetical Formula.—If a farmer sold a span of horses for $138, a cow for $38, some butter for $13, and some straw for $15, he must have received for all the sum of these amounts, which is $204.

ANALYTICAL STEP.—(*b.*) Find the amount of the money expended.

Elementary Question.—If a farmer purchased 10 yards of broadcloth for $40, a stove for $35, and some milk-pans for $13, how much did he expend for all?

If a farmer purchased broadcloth for $40, a stove for $35, and some milk-pans for $13, he must have expended for all the sum of these amounts, which is $88.

ANALYTICAL STEP.—(*c*.) Find the amount of money left.

Elementary Question.—If a farmer received $204, and spent $88, how much had he left?

Arithmetical Formula.—If a farmer received $204, and spent $88, he must have had left the difference between these amounts, which is $116.

CONCLUSION.—Therefore, if a farmer sold a span of horses for $138, a cow for $38, some butter, &c.

NOTE.—The teacher should require the pupil either to state each *elementary question* distinctly, or, which would be far better, require it written.

EXERCISE.

Require the pupil to compose and analyze ten examples consisting of at least three analytical steps.

SECTION V.

MULTIPLICATION.

LESSON I.

46. **Multiplication** is the repetition of one number as many times as there are units in another, and is, therefore, a short method of addition.

47. The **Multiplicand** is the number to be repeated or multiplied.

48. The **Multiplier** is the number by which we multiply, and denotes the number of repetitions to be made.

49. The **Product** is the number produced by multiplication.

50. The multiplicand and multiplier are often called **Factors.**

51. The SIGN OF MULTIPLICATION is an oblique cross

(×), and when placed between numbers, it denotes that they are to be multiplied together; as, $4\times8=32$; to be read, 4 multiplied by 8 equals 32.

52. MULTIPLICATION TABLE.

Once 0 is	0	2 times 0 are	0	3 times 0 are	0
Once 1 is	1	2 times 1 are	2	3 times 1 are	3
Once 2 is	2	2 times 2 are	4	3 times 2 are	6
Once 3 is	3	2 times 3 are	6	3 times 3 are	9
Once 4 is	4	2 times 4 are	8	3 times 4 are	12
Once 5 is	5	2 times 5 are	10	3 times 5 are	15
Once 6 is	6	2 times 6 are	12	3 times 6 are	18
Once 7 is	7	2 times 7 are	14	3 times 7 are	21
Once 8 is	8	2 times 8 are	16	3 times 8 are	24
Once 9 is	9	2 times 9 are	18	3 times 9 are	27
Once 10 is	10	2 times 10 are	20	3 times 10 are	30
Once 11 is	11	2 times 11 are	22	3 times 11 are	33
Once 12 is	12	2 times 12 are	24	3 times 12 are	36
4 times 0 are	0	5 times 0 are	0	6 times 0 are	0
4 times 1 are	4	5 times 1 are	5	6 times 1 are	6
4 times 2 are	8	5 times 2 are	10	6 times 2 are	12
4 times 3 are	12	5 times 3 are	15	6 times 3 are	18
4 times 4 are	16	5 times 4 are	20	6 times 4 are	24
4 times 5 are	20	5 times 5 are	25	6 times 5 are	30
4 times 6 are	24	5 times 6 are	30	6 times 6 are	36
4 times 7 are	28	5 times 7 are	35	6 times 7 are	42
4 times 8 are	32	5 times 8 are	40	6 times 8 are	48
4 times 9 are	36	5 times 9 are	45	6 times 9 are	54
4 times 10 are	40	5 times 10 are	50	6 times 10 are	60
4 times 11 are	44	5 times 11 are	55	6 times 11 are	66
4 times 12 are	48	5 times 12 are	60	6 times 12 are	72
7 times 0 are	0	8 times 0 are	0	9 times 0 are	0
7 times 1 are	7	8 times 1 are	8	9 times 1 are	9
7 times 2 are	14	8 times 2 are	16	9 times 2 are	18
7 times 3 are	21	8 times 3 are	24	9 times 3 are	27
7 times 4 are	28	8 times 4 are	32	9 times 4 are	36
7 times 5 are	35	8 times 5 are	40	9 times 5 are	45
7 times 6 are	42	8 times 6 are	48	9 times 6 are	54
7 times 7 are	49	8 times 7 are	56	9 times 7 are	63
7 times 8 are	56	8 times 8 are	64	9 times 8 are	72
7 times 9 are	63	8 times 9 are	72	9 times 9 are	81
7 times 10 are	70	8 times 10 are	80	9 times 10 are	90
7 times 11 are	77	8 times 11 are	88	9 times 11 are	99
7 times 12 are	84	8 times 12 are	96	9 times 12 are	108

10 times 0 are 0	11 times 0 are 0	12 times 0 are 0
10 times 1 are 10	11 times 1 are 11	12 times 1 are 12
10 times 2 are 20	11 times 2 are 22	12 times 2 are 24
10 times 3 are 30	11 times 3 are 33	12 times 3 are 36
10 times 4 are 40	11 times 4 are 44	12 times 4 are 48
10 times 5 are 50	11 times 5 are 55	12 times 5 are 60
10 times 6 are 60	11 times 6 are 66	12 times 6 are 72
10 times 7 are 70	11 times 7 are 77	12 times 7 are 84
10 times 8 are 80	11 times 8 are 88	12 times 8 are 96
10 times 9 are 90	11 times 9 are 99	12 times 9 are 108
10 times 10 are 100	11 times 10 are 110	12 times 10 are 120
10 times 11 are 110	11 times 11 are 121	12 times 11 are 132
10 times 12 are 120	11 times 12 are 132	12 times 12 are 144

LESSON II.

53. ANALYSIS OF MULTIPLICATION.

Required to multiply 9,873 by 24.

MODEL OPERATION.

$$\left.\begin{array}{r}9873\text{ multiplicand}\\24\text{ multiplier}\end{array}\right\}\text{factors.}$$

39492 product of 4 times multiplicand.
19746 " 20 "

236,952 " 24 "

(*a.*) For convenience write the multiplier under the multiplicand, so that units may stand under units, tens under tens, &c.

(*b.*) For convenience commence with the right hand or units' figure to multiply.

(*c.*) 4 times 3 units are 12 units; 12 units are equal to 1 ten and 2 units; write the units in units' place, and add the 1 ten to the product of tens.

(*d.*) 4 times 7 tens are 28 tens, which with the 1 ten added are 29 tens; 29 tens are equal to 2 hundreds and 9 tens; write the 9 tens in tens' place, and add the 2 hundreds to the product of hundreds.

(*e.*) 4 times 8 hundreds are 32 hundreds, which with the 2 hundreds added are 34 hundreds; 34 hundreds are equal to 3 thousands and 4 hundreds; write the 4 hundreds in the place of hundreds, and add the 3 thousands to the product of thousands.

(*f.*) 4 times 9 thousands are 36 thousands, which with the 3 thousands added are 39 thousands; 39 thousands are equal to 3 tens of thousands and 9 thousands; write the 9 thousands in the place of thousands, and the 3 tens of thousands in the place of tens of thousands.

*(*g.*) Next multiply by the 2 tens of the multiplier. 2 tens are equal to 20 units, then 20 times 3 units are 60 units; 60 units are equal to 6 tens and 0 units; the 0 units need not be written, but write the 6 tens in the place of tens.

(*h.*) 20 times 7 tens are 140 tens; 140 tens are equal to 1 thousand, 4 hundreds, and 0 tens; write the 4 hundreds in the place of hundreds and add the 1 thousand to the thousands of the product.

(*i.*) 20 times 8 hundreds are 160 hundreds; 160 hundreds are equal to 1 ten of thousands, 6 thousands, and 0 hundreds; 6 thousand with the 1 thousand added are 7 thousand, which write in the place of thousands, and add the 1 ten of thousands to the tens of thousands of the product.

(*j.*) 20 times 9 thousands are 180 thousands; 180 thousands are equal to 1 hundred of thousands, 8 tens of thousands, and 0 thousands; 8 tens of thousands with 1 ten of thousands added, &c.

(*k.*) Next proceed to add the partial products 39,492 and 197,460, which give the total product of 236,952.

---

*NOTE.—Any number of figures in the multiplier may be analysed after this method by changing all the different orders to units.

(*l.*) Therefore, 9,873 multiplied by 24 gives a product of 236,952.

(*m.*) RULE.—*Write the multiplier under the multiplicand so that units may stand under units, tens under tens, &c.*

Beginning with the unit figure, multiply each figure of the multiplicand by each figure of the multiplier, successively, observing to write the first figure of each partial product directly under its multiplier, writing down and carrying as in addition.

Add the partial products, and their sum will be the product sought.

(*n.*) PROOF.—I. *Interchange the multiplier with the multiplicand, and multiply as before; if the two products are the same, the work is supposed to be correct.*

* II. *Divide the product by the multiplier, and if the quotient equals the multiplicand, the work is correct.*

FIRST METHOD OF PROOF.

MODEL OPERATION.

(*a.*)

```
   8467 multiplicand.
    371 multiplier.
   ----
   8467
  59269
 25401
-------
3141257 product.
```

(*b.*)

PROOF.—(See 53 *n.*) I.

```
    371 multiplier.
   8467 multiplicand.
   ----
   2597
  2226
 1484
2968
-------
3141257 product equal to product (a.)
```

QUESTIONS.—What is Multiplication? (46.) What is the multiplicand? (47.) What is the multiplier? (48.) What is the product? (49.) Which terms are called factors? (50.) What is the sign of multiplication? (51.) Repeat the analysis of multiplication. (53.) Repeat the rule. (53., *m.*) Give both methods of proof. (53., *n.*) What is the difference between addition and subtraction? (31.,) (39.) What are the terms of subtraction? (38.,) (39.,) (40.,) (41.)

---

* NOTE.—This method of proof presupposes a knowledge of division; but, as the pupil should already have a knowledge of the fundamental operations, and as it is the method usually adopted in business, we think it best to insert it here.

LESSON III.

PRACTICAL EXAMPLES CONTAINING BUT ONE ANALYTICAL STEP.

What will 1983 gallons of wine cost at 97 cts. a gallon?

MODEL OPERATION.*

```
  1983 gallons.
    97 cents.
  ----
 13881
17847
------
192,351 cents.
```

ANALYSIS.—*Arithmetical Formula.*—If 1 gallon of wine cost 97 cents, 1,983 gallons will cost 1,983 times 97 cents, which are 192,351 cents.

Therefore, if 1 gallon of wine cost 97 cents, 1,983 gallons will cost 192,351 cents.

EXERCISE.

Require the pupil to compose and analyze ten examples similar to the model.

LESSON IV.

PRACTICAL EXAMPLES

COMBINING ADDITION, SUBTRACTION, AND MULTIPLICATION, AND WHICH CONTAIN TWO OR MORE ANALYTICAL STEPS.

I purchased of a farmer 138 bushels of potatoes at 46 cents a bushel, and 37 bushels of corn at 96 cents a bushel;

---

* NOTE.—Throughout the work the name of each denominate number is appended to it for the purpose of aiding the memory, the *analysis* in multiplication and division determining *which* number ought to be used as *abstract*.

I sold the farmer in return 63 yards of muslin at 13 cents per yard, and 41 yards of calico at 17 cents; how much money must I pay to balance the account?

MODEL OPERATION.

(*a.*)
138 bushels potatoes.
46 cents.
828
552
6,348 cts., cost of potatoes.

(*b.*)
37 bushels corn.
96 cents.
222
333
3,552 cts., cost of corn.

(*c.*)
6348 cents, cost of potatoes.
3552 " " corn.
9900 cents, cost of both.

(*d.*)
63 yards muslin.
13 cents.
189
63
819 cents, received for muslin.

(*e.*)
41 yds. calico.
17 cents.
287
41
697 cts., rec'd for calico.

(*f.*)
819 cents, received for muslin.
697 " " calico.
1,516 cents, received for both.

(*g.*)
9900 cents, cost of articles purchased.
1516 " paid with articles in return.
7384 cents, balance due to farmer.

*ANALYSIS.

Analytical Step. (*a.*) Find the cost of the potatoes.

---

*Note to the Teacher.—The pupil should be required to give the *full forms* until he can do so promptly and accurately.

Elementary Question.—If I purchase 1 bushel of potatoes for 46 cents, what will 138 bushels cost?

Arithmetical Formula.—If 1 bushel of potatoes cost 46 cents, 138 bushels, &c.

ANALYTICAL STEP.—(*b.*) Find the cost of the corn.

Elementary Question.—If I purchase 1 bushel of corn, &c.

Arithmetical Formula.—If 1 bushel of corn costs, &c.

ANALYTICAL STEP.—(*c.*) Find the cost of both the corn and the potatoes.

Elementary Question.—If the potatoes cost 6348 cents, and the corn 3552 cents, what will both cost?

Arithmetical Formula.—If the potatoes, &c.

ANALYTICAL STEP.—(*d.*) Find the amount received for the muslin.

Elementary Question.—If one yard of muslin costs, &c.

Arithmetical Formula.—If one yard of muslin costs, &c.

ANALYTICAL STEP.—(*e.*) Find the amount received for the calico.

Elementary Question.—If one yard of calico costs, &c.

Arithmetical Formula.—If one yard of calico costs, &c.

ANALYTICAL STEP.—(*f.*) Find the amount received for both muslin and calico.

Elementary Question.—If the muslin costs, &c.

Arithmetical Formula.—If, &c.

ANALYTICAL STEP.—(*g.*) Find the balance due.

Elementary Question.—If, &c.

Arithmetical Formula.—If, &c.

CONCLUSION.—If I purchase of a farmer 138 bushels of potatoes, and &c.

EXERCISE.

Require the pupil to compose and analyze ten examples similar to the model.

SECTION VI.

DIVISION

LESSON I.

54. Division is the process of finding how many times one number is contained in another, or of separating one number into as many parts as there are units in another.

55. The **Dividend** is the number to be *divided.*

56. The **Divisor** is the number by which the *dividend* is divided.

57. The **Quotient** is the number of times the *divisor* is contained in the *dividend.*

58. There are two methods of *division,* called *Long Division* and *Short Division.*

59. Short Division is the method generally used when the *divisor* does not exceed *twelve.*

60. In **Long Division** each step of the process is written, and it is the method generally used when the *divisor* is greater than *twelve.*

61. A **Remainder** in Division is that part of the *dividend* which remains undivided.

62. The SIGN of DIVISION is a short horizontal line, with a point above and another below it (÷), and when placed between numbers, it indicates that the number before it is to be divided by the number after it; thus, $20 \div 4 = 5$, to be read, 20 divided by 4 is equal to 5.

(*a.*) Division is also expressed by writing the dividend *above,* and the divisor *below,* a short horizontal line; thus, $\frac{12}{3} = 4$, shows that 12 divided by 3 equals 4. This is sometimes called the *fractional sign.*

63. DIVISION TABLE.

1 in	1	1 time	2 in	2	1 time	3 in	3	1 time
1 in	2	2 times	2 in	4	2 times	3 in	6	2 times
1 in	3	3 times	2 in	6	3 times	3 in	9	3 times
1 in	4	4 times	2 in	8	4 times	3 in	12	4 times
1 in	5	5 times	2 in	10	5 times	3 in	15	5 times
1 in	6	6 times	2 in	12	6 times	3 in	18	6 times
1 in	7	7 times	2 in	14	7 times	3 in	21	7 times
1 in	8	8 times	2 in	16	8 times	3 in	24	8 times
1 in	9	9 times	2 in	18	9 times	3 in	27	9 times
4 in	4	1 time	5 in	5	1 time	6 in	6	1 time
4 in	8	2 times	5 in	10	2 times	6 in	12	2 times
4 in	12	3 times	5 in	15	3 times	6 in	18	3 times
4 in	16	4 times	5 in	20	4 times	6 in	24	4 times
4 in	20	5 times	5 in	25	5 times	6 in	30	5 times
4 in	24	6 times	5 in	30	6 times	6 in	36	6 times
4 in	28	7 times	5 in	35	7 times	6 in	42	7 times
4 in	32	8 times	5 in	40	8 times	6 in	48	8 times
4 in	36	9 times	5 in	45	9 times	6 in	54	9 times
7 in	7	1 time	8 in	8	1 time	9 in	9	1 time
7 in	14	2 times	8 in	16	2 times	9 in	18	2 times
7 in	21	3 times	8 in	24	3 times	9 in	27	3 times
7 in	28	4 times	8 in	32	4 times	9 in	36	4 times
7 in	35	5 times	8 in	40	5 times	9 in	45	5 times
7 in	42	6 times	8 in	48	6 times	9 in	54	6 times
7 in	49	7 times	8 in	56	7 times	9 in	63	7 times
7 in	56	8 times	8 in	64	8 times	9 in	72	8 times
7 in	63	9 times	8 in	72	9 times	9 in	81	9 times
10 in	10	1 time	11 in	11	1 time	12 in	12	1 time
10 in	20	2 times	11 in	22	2 times	12 in	24	2 times
10 in	30	3 times	11 in	33	3 times	12 in	36	3 times
10 in	40	4 times	11 in	44	4 times	12 in	48	4 times
10 in	50	5 times	11 in	55	5 times	12 in	60	5 times
10 in	60	6 times	11 in	66	6 times	12 in	72	6 times
10 in	70	7 times	11 in	77	7 times	12 in	84	7 times
10 in	80	8 times	11 in	88	8 times	12 in	96	8 times
10 in	90	9 times	11 in	99	9 times	12 in	108	9 times

LESSON II.

64. ANALYSIS OF SHORT DIVISION.

Divide 70301 by 4.

MODEL OPERATION.

Divisor. 4)70301 Dividend.

$17575\frac{1}{4}$ Quotient.

PROOF.

$17575\frac{1}{4}$ Quotient.

4 Divisor.

70301 = Dividend.

(*a.*) *ANALYSIS.—For convenience write the divisor at the left of the dividend, and commence by dividing the first left hand figure.

(*b.*) *One fourth* of 7 tens of thousands is 1 ten of thousands, with a remainder of 3 tens of thousands equal to 30 thousand; write the 1 ten of thousands in the place of tens of thousands, and add the 30 thousands to the thousands.

(*c.*) *One fourth* of 30 thousands is 7 thousands, with a remainder of two thousands which is equal to 20 hundreds; write the 7 thousands in the place of thousands, and add the 20 hundreds to the hundreds; 20 hundreds and 3 hundreds are 23 hundreds.

(*d.*) *One fourth* of 23 hundreds is 5 hundreds, with a remainder of 3 hundreds equal to 30 tens; write the 5 hundreds in hundreds' place, and add the 30 tens to the tens.

(*e.*) *One fourth* of 30 tens is 7 tens, with a remainder of 2 tens equal to 20 units; write the 7 tens in the place of tens, and add the 20 units to the units; 20 units and 1 unit are 21 units.

(*f.*) *One fourth* of 21 units is 5 units, with a remainder of 1 unit, which divided into 4 parts which gives 1 fourth of a unit, written $\frac{1}{4}$; write the 5 units in the place of units, and annex the $\frac{1}{4}$ unit.

---

*In this analysis we prefer the *fractional* form, as it is simpler, and one which the pupil readily understands, although liable to some objections. The teacher will find another form in the analysis of Long Division, (65), which he can use here if he prefers it.

(*g.*) Therefore, *one fourth* of 70301 is $17575\frac{1}{4}$.

(*h.*) RULE FOR SHORT DIVISION.—*Write the divisor at the left of the dividend.*

Begin at the left and divide each figure of the dividend by the divisor and set each quotient figure under its dividend.

If there is a remainder after any division, prefix it to the next figure of the dividend and divide as before.

If any partial dividend be less than the divisor, write a cipher for the quotient, prefix the partial dividend to the next figure of the dividend, and divide as before.

If there is a remainder after dividing the last figure of the dividend, it can be changed to a FRACTIONAL QUOTIENT, *by writing the divisor under it and annexing the result to the quotient.*

(*i.*) PROOF.—*Division may be proved by multiplying the* QUOTIENT *by the* DIVISOR *and adding to the product the remainder, if there be any. The result will be equal to the dividend, if the work is correct.*

QUESTIONS.—What is Division? (54.) What is the dividend? (55.) What is the divisor? (56.) What is the quotient? (57.) How many methods of division are there? (58.) What is Short Division? (59.) What is Long Division? (60.) What is the remainder in division? (61.) What are the signs of division? (62.) Repeat the analysis of short division. (64.) Give the rule. (64., *h.*) Give the method of proof. (64., *i.*)

LESSON III.

BLACKBOARD EXERCISES.

NOTE.—Perform the divisions in the following exercises in the same manner as in similar exercises in Addition and Subtraction. Let the teacher point, and the pupil simply give the quotients.

1. 2)2 6 8 4 12 10 18 16 14

2. 3)9 12 3 6 15 21 18 27 24

3. 4)8 4 16 20 12 24 32 28 36

4. 5)10 5 15 25 35 20 40 30 45

5. 6)24 12 6 18 42 36 48 24 30

6. 7)14 35 21 42 28 56 7 49 63

7. 8)8 24 16 40 32 72 56 48 64

8. 9)27 18 9 45 36 63 54 81 72

9. 10)30 50 20 10 40 60 80 70 90

10. 12)12 96 108 36 24 48 72 60 84

Give the quotients and the remainders of the following examples; thus, 2 and 1 remainder, &c.

11. 4)9 13 12 27 34 29 6 38 22

12. 5)12 8 19 29 36 21 41 33 37

13. 6)27 15 18 21 45 39 52 25 34

14. 7)17 39 28 43 34 62 59 50 65

15.	8)28	20	46	37	79	61	54	65	72	
16.	9)20	30	54	49	39	70	60	84	77	
17.	10)41	63	72	12	84	45	67	83	82	91
18.	11)56	37	46	83	72	94	63	78	34	44
19.	12)83	72	46	37	83	46	49	13	78	99

Note.—Let the teacher dwell on these and similar exercises, until the pupil can answer them rapidly.

LESSON IV.

LONG DIVISION.

Divide 26,306,241 by 483.

MODEL OPERATION.

Divisor. Dividend. Quotient.

483)26306241(54464$\frac{122}{483}$
2415

2156
1932

2242
1932

3104
2898

2061
1932

129 Rem.÷483 Divis.

65. ANALYSIS.

(*a.*) For convenience write the divisor at the left, and the quotient at the right of the dividend.

(*b.*) 483 is contained in 2630 tens of thousands, 5 tens of thousands times, with a remainder of 215 tens of thousands, equal to 2150 thousands; write the 5 tens of thousands in the place of tens of thousands in the quotient, and add the 2150 thousands to the 6 thousands of the dividend, which makes 2156 thousands.

(*c.*) 483 is contained in 2156 thousand 4 thousand times, with a remainder of 224 thousand equal to 2240 hundreds; write the 4 thousands in the place of thousands in the quotient, and add 2240 hundreds to the 2 hundreds of the dividend, which makes 2242 hundreds.

(*d.*) 483 is contained in 2242 hundreds 4 hundred times, with a remainder of 310 hundreds equal to 3100 tens; write the 4 hundred in hundreds' place in the quotient, and add the 3100 tens to the 4 tens of the dividend, which makes 3104 tens.

(*e.*) 483 is contained in 3104 tens 6 tens times, with a remainder of 206 tens equal to 2060 units; write the 6 tens in the tens' place in the quotient, and add the 2060 units to the 1 unit of the dividend, which makes 2061 units.

(*f.*) 483 is contained in 2061 units 4 units times, with a remainder of 129 units; write the 4 units in units' place in the quotient, and divide the remainder of 129 units into 483 parts, giving $\frac{129}{483}$ of a unit, which annex to the quotient.

(*g.*) Therefore 26,306,241 divided by 483 gives a quotient of $54{,}464\frac{129}{483}$.

*Note.—It is, perhaps, more convenient to place the divisor at the right of the dividend over the quotient.

(*h.*) RULE FOR LONG DIVISION.—*Write the divisor at the left of the dividend as in short division.*

Find how many times the divisor is contained in the LEAST *number of the left hand figures of the dividend that will contain it, and place the quotient at the right of the dividend.*

Multiply the divisor by this quotient figure, subtract the product from this partial dividend, and to the remainder annex the next figure of the dividend.

Divide as before, until all the figures of the dividend have been annexed and divided.

If any partial dividend will not contain the divisor, place a cipher in the quotient, annex the next figure of the dividend, and divide as before.

If there be a remainder after dividing all the figures of the dividend, it may be changed to a FRACTIONAL QUOTIENT *by writing the* DIVISOR *underneath it.*

(*i.*) PROOF.—*The proof of long division is the same as that of short division.*

NOTES.—1. The first step in Long Division is to find the quotient figure of the highest denomination.

2. Multiply the divisor by that figure and write the product under the first partial dividend, then *observe* that if the *product* is *greater* than the *partial dividend*, the *quotient figure* is TOO LARGE.

3. Subtract that product from the first partial dividend, and *observe*, that if the REMAINDER is *equal to*, or GREATER than the DIVISOR, the *quotient figure* is TOO SMALL

4. Change the remainder to the next lower denomination, and add to it the corresponding denomination in the dividend.

LESSON V.

PRACTICAL EXAMPLES INVOLVING THE FUNDAMENTAL RULES.

Mr. Jones died leaving an estate worth $3,746, to be divided equally between 3 daughters and 2 sons after his wife had taken out her share amounting to $1,479; what was the share of each child?

MODEL OPERATION.

	\$3746 worth of estate.	(*a.*)	(*b.*)
	1479 right of his wife.		3 childr'n, daughters.
	——		2 " sons.
(*c.*)	5)2267 am't to be divided.		—
	——		5 children.
	\$453$\frac{2}{5}$ each child received.		

*ANALYSIS.

Analytical Step.—(*a.*) Find the amount to be divided after the wife has taken her share from the estate.

(*b.*) Find the number of children among whom the remainder is to be divided.

(*c.*) Find the amount which each child received.

EXERCISE.

Require the pupil to compose and analyze ten examples similar to the model.

Questions.—Repeat the analysis of Long Division. (65.) Repeat the rule. (65., *h.*) Give the proof. (65., *i.*) What is addition? (31.) Illustrate the use of the sign of Addition. (32.) Subtraction. (43.) Multiplication. (51.) Division. (62.) What is the use of the cipher? (22.) What is the difference between the Roman and the Arabic notation? (18.,) (19.) What is the difference between the simple and the local value of a figure? (20.,) (21.) What is the difference between a simple and a complex problem? (8.,) (9.) What is the difference between a concrete and an abstract number? (4.,) (3.)

LESSON VI.

THE FUNDAMENTAL RULES—PRINCIPLES AND CONTRACTIONS.

66. PRINCIPLES.

(*a.*) The *greater* of two numbers is equal to the *less* added to their *difference.*

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* Note for the Teacher.—The teacher should in addition to the analytical steps require the *Elementary Questions* and *Arithmetical Formulas*, until the pupil can give them with promptness and accuracy, when they may be omitted.

MODEL ILLUSTRATION.

Let 4186 be the greater number, and
" 3712 " less "
———
474 " difference.

Then $\overset{\text{greater.}}{4186}=\overset{\text{less.}}{3712}+\overset{\text{their difference.}}{474}$ [Repeat prin. (*a*.)]

NOTE.—Let each principle be illustrated after the same manner on the blackboard.

(*b*.) The *smaller* of two numbers is equal to the *remainder* obtained by subtracting their *difference* from the greater.

(*c*.) If the *multiplier* be a unit, the *product* will be equal to the multiplicand.

(*d*.) The *multiplicand* remaining the same, if the multiplier be increased or diminished any number of times, the product will be increased or diminished in the same ratio.

(*e*.) The *multiplier* remaining the same, if the *multiplicand* be increased or diminished any number of times, the product will be increased or diminished in the same ratio.

(*f*.) When the *divisor* is a unit, the quotient will be equal to the dividend.

(*g*.) When the *divisor* is equal to the dividend, the quotient will be a unit.

(*h*.) The *dividend* remaining the same, if the divisor is *increased* any number of times, the quotient will be *decreased* in the same ratio.

(*i*.) The *dividend* remaining the same, if the divisor is *decreased* any number of times, the quotient will be *increased* in the same ratio.

(*j*.) The divisor remaining the same, if the dividend is *increased* any number of times, the quotient will be *increased* in the same ratio.

(*k.*) The divisor remaining the same, if the dividend is *decreased* any number of times, the quotient will be *decreased* in the same ratio.

(*l.*) The product of the divisor and quotient *plus* the remainder, equals the dividend.

(*m.*) The product divided by the multiplier is equal to the multiplicand.

(*n.*) From the *sum* of two numbers subtract their difference, and divide the remainder by 2; and the quotient will be the smaller number: to the smaller number add their difference, and the sum will be the larger number.

LESSON VII.

PROBLEMS FOUNDED ON THE PRECEDING PRINCIPLES.

NOTE FOR THE TEACHER.—The teacher should require the pupil to repeat the principle involved in each of these problems.

1. The *less* of two numbers is 398; their *difference* is 698; what is the greater? See prin. (*a.*)

2. The greater of two numbers is 7863; their difference is 6713; what is the smaller? See prin. (*b.*)

3. The multiplicand is 3679; the multiplier is 327; the product is 1203033; if the multiplier be increased 7 times, what will be the product?

4. The multiplier is 12; the product is 148,140; if the multiplier be increased 9 times, what will be the product?

5. The multiplier is 96; the product is 49164; if the multiplier be diminished 4 times, what will be the product?

6. The multiplier is 16; the multiplicand is 41862; if the multiplier be diminished 8 times, what will be the product?

7. The multiplier is 24; the multiplicand is 691,344; if the multiplicand be diminished 4 times, what will be the product?

8. The multiplier is 36; the product is 91884; if the multiplicand is diminished 3 times, what will be the product?

9. The divisor is 9; the quotient is 6847; if the divisor be diminished 3 times, what will be the quotient?

10. The divisor is 28; the quotient is 418,374; if the divisor be increased 8 times, what will be the quotient?

11. The quotient is 9186; what will be the quotient if the divisor is increased 8 times? 13 times? If decreased 9 times? 17 times?

12. The divisor is 34; the quotient is 7641; the remainder is 11; what is the dividend?

13. The divisor is 78; the quotient is 3746; what is the dividend?

14. The product of two factors is 39746; the multiplier is 374; what is the multiplicand?

15. The *sum* of two numbers is 3974; their difference is 1389; what are the numbers?

16. The *sum* of two numbers is 49683; their difference is 3841; what are the numbers?

LESSON VIII.

67. CONTRACTED OPERATIONS IN THE FUNDAMENTAL RULES.

(*a*.) To multiply by 10, 100, 1000, &c.

Annexing a cipher to any number changes the local value of the unit figure to tens, and of tens to hundreds, &c.; and therefore increases the number *ten* times; thus, $\overset{\text{tens.}}{5}\overset{\text{units.}}{3}$ by annexing a cipher, thus: $\overset{\text{hundreds.}}{5}\overset{\text{tens.}}{3}\overset{\text{units.}}{0}$, the number is increased *ten* times.

According to the same principle *two* ciphers increases the value one hundred times, *three* ciphers one thousand times, &c.

(*b.*) To multiply by 25.

RULE.—*Annex two ciphers to the multiplicand and divide the result by* 4.

ILLUSTRATION.—If we annex two ciphers to the multiplicand we multiply it by 100 (67 *a*); but as 100 is 4 times larger than 25, the product is 4 times too large (66 *d*); therefore *one fourth* of this product is the product required.

(*c.*) To multiply by $12\frac{1}{2}$.

RULE.—*Annex two ciphers to the multiplicand and divide the result by* 8.

NOTE.—Let the pupil be required to illustrate each of these rules after the model given.

(*d.*) To multiply by $33\frac{1}{3}$.

RULE.—*Annex two ciphers to the multiplicand and divide the result by* 3.

(*e.*) To multiply by 125.

RULE.—*Annex three ciphers to the multiplicand and divide the result by* 8.

(*f.*) To divide by 10, 100, 1000, &c.

By removing a figure from the right of any number the local value of ten's figure is changed to units, and hundreds to tens, &c., therefore the number is decreased *ten* times, thus: $\overset{\text{hundreds.}}{6}\overset{\text{tens.}}{3}\overset{\text{units.}}{7}$, by removing the right hand figure, 7, thus: $\overset{\text{tens.}}{6}\overset{\text{units.}}{3}$, the number is decreased *ten* times with a remainder of 7.

According to the same principle removing two figures decreases the value a hundred times, three figures a thousand times, &c.

(*g*.) To divide by 25.

Rule.—*Multiply the number by* 4, *and divide the product by* 100.

Illustration.—As 100 is 4 times larger than 25, if we divide any number by 100, the quotient will be 4 times too small, (see 66, *h*); therefore, 4 times this quotient will be the quotient required. It is most convenient first to multiply the number by the 4, and then divide the product by 100.

(*h*.) To divide by 12½.

Rule.—*Multiply the number by* 8, *and divide the product by* 100.

(*i*.) To divide by 33⅓.

Rule.—*Multiply the number by* 3, *and divide the product by* 100.

(*j*.) To divide by 125.

Rule.—*Multiply the number by* 8, *and divide the product by* 1000.

SECTION VII.

LESSON I.

UNITED STATES MONEY.

68. The Currency of the United States is a decimal currency, and is sometimes called Federal Money.

(*a*.) TABLE OF DENOMINATIONS.

10 mills	make	1 cent.	ct.
10 cents	"	1 dime, ...	d.
10 dimes	"	1 dollar.	$.
10 dollars	"	1 eagle.	E.

(*b.*) NUMERATION TABLE.

dollars.		cents.		mills.
eagles.	dollars.	dimes.	cents.	mills.
0	0 .	0	0	0

(*c.*) TABLE OF ALIQUOT PARTS.

$\frac{1}{4}$ of a dollar = 25 cents.
$\frac{1}{2}$ of a dollar = 50 cents.
$\frac{3}{4}$ of a dollar = 75 cents.
$\frac{1}{2}$ of a cent = 5 mills.

69. (*a.*) COINS.—The **Gold coins** are the *double-eagle, eagle, half-eagle, quarter-eagle, three-dollar piece,* and *dollar.*

(*b.*) The **Silver coins** are the *half-dollar, quarter-dollar, dime, half-dime,* and *three-cent piece.*

(*c.*) The cent is a **Nickel coin.**

70. The *decimal point,* (.), is used to separate dollars from cents and mills; thus, $46.873 equals 46 dollars 87 cents and 3 mills.

NOTES.—1. Since the places in Federal Money increase in value from the right to the left in a ten-fold ratio like simple numbers, it may be added, subtracted, multiplied, and divided, according to the same rules.

2. In business transactions, eagles are read as tens of dollars, and dimes as tens of cents; thus, 4 eagles 9 dollars 5 dimes 6 cents and 3 mills, are read, 49 dollars 56 cents and 3 mills.

3. The fifty-dollar piece, the half-dollar and quarter-dollar pieces are not legal coin.

4. The copper cent and half-cent are no longer coined. The mill is not a coin.

5. The dollar sign ($) is by some supposed to be a contraction of U. S.

LESSON II.

71. REDUCTION.

1. How many cents in 75 dollars?

MODEL OPERATION.

$75
100 Cents.

7500 Cents.

(*a.*) **Analysis.**—Since there are 100 cents in one dollar, in 75 dollars there are 75 times 100 cents, which are 7500 cents.

(*b.*) Therefore, there are 7500 cents in 75 dollars.

(*c.*) **Rules.**—*To change dollars to cents, multiply by* 100; *that is, annex* TWO *ciphers.*

(*d.*) *To change dollars to mills, annex* THREE *ciphers.*

(*e.*) *To change cents to mills, annex* ONE *cipher.*

EXAMPLES FOR PRACTICE.

2. Reduce $24 to cents.
3. Reduce $36 to cents.
4. Reduce $14 to mills.
5. Reduce $102 to mills.
6. Reduce $100 to cents.
7. Change 66 cents to mills.
8. Change 7 cents to mills.
9. Change $5.05 to mills.

Note.—To change dollars and cents to cents, or dollars, cents and mills to mills, remove the decimal point and the dollar sign.

10. Change $4.36 to cents.
11. Change $3.73 to cents.
12. Change $6.374 to mills.
13. Change $487.411 to mills.

14. How many dollars in 3746 cents?

MODEL OPERATION.

Cents, 1‘00)37‘46 cents.

$37.46

Analysis.—Since in 100 cents there is 1 dollar, in 3746 cents there are as many dollars as 100 cents is contained times in 3746 cents, which are 37 times, with a remainder of 46 cents.

Therefore, in 3746 cents there are $37.46.

(*f.*) Rules.—*To change cents to dollars, divide by* 100; *that is, point off* TWO *figures from the right.*

(*g.*) *To change mills to dollars, point off* THREE *figures.*

(*h.*) *To change mills to cents, point off* ONE *figure.*

LESSON III.

72. ADDITION.

MODEL OPERATION.

1. Add

37	dollars	37	cents	8	mills	=	$37.378
9	"	14	"	1	"	=	9.141
108	"	00	"	0	"	=	108.000
59	"	00	"	3	"	=	59.003
1	"	1	"	1	"	=	1.011
							$214.533 Sum total.

(*a.*) Analysis.—For convenience write the numbers so that mills shall stand in the column of mills, cents in the column of cents, and dollars in the column of dollars.

For convenience, begin at the right hand column to add, carrying as in simple numbers, and placing the decimal point between dollars and cents.

2. Add

Fourteen cents,	=$ 0.14
One hundred and ninety-eight dollars,	=198.00
One dollar and one mill,	= 1.001
Three mills,	= 0.003
Sixteen dollars three cents and two mills,	= 16.032
	Sum total.

73. SUBTRACTION

LESSON IV.

Subtract $189.437 from $200.

MODEL OPERATION.

$200.000 minuend.
189.437 subtrahend.

$10.563 remainder.

(*a.*) ANALYSIS.—The analysis and proof are the same as in simple subtraction. Always remember to place the decimal point between dollars and cents.

74. MULTIPLICATION.

LESSON V.

If 1 bushel of corn costs $1.252, how much will 29 bushels cost?

MODEL OPERATION.

$1.252 cost of 1 bushel.
29 number of bushels.

11.268
25.04

$36.308 cost of 29 bushels.

(*a.*) *ANALYSIS.—$1.252 are equal to one thousand two hundred and fifty-two mills.

Since one bushel of corn costs 1252 mills, 29 bushels will cost 29 times 1252 mills, which are 36308 mills, equal to $36.308.

Therefore, if 1 bushel of corn cost $1.252, 29 bushels will cost $36.308.

---

*The reduction to mills or cents, as the example may require, may be omitted after the pupil thoroughly understands the process.

75. DIVISION.

LESSON VI.

Divide $3786. by 7.

MODEL OPERATION.

Divisor, 7)$3786.000 dividend.

$540.857+ quotient.

PROOF.

$540.857+ quotient.
7 divisor.

$3785.999, equal to the dividend lacking 1 mill.

(*a*.) ANALYSIS.—$3786 are equal to 3,786,000 mills.

One seventh of 3,786,000 mills is 540,857 mills, equal to $540.857+.

Therefore, $3786 divided by 7 equals $540.857+.

NOTE.—The division of United States money may be proved as simple division.

1. If 9 pounds of sugar cost $1.12½, what will one pound cost?

MODEL OPERATION.

No. of pounds 9)$1.125 cost of 9 pounds.

$0.125 cost of 1 pound.

ANALYSIS.—$1.125 are equal to 1125 mills.

If 9 pounds cost 1125 mills, 1 pound will cost 1 ninth of 1125 mills, which is 125 mills, equal to $0.125.

Therefore, if 9 pounds of sugar cost $1.12½, one pound will cost $0.125.

NOTE.—Should there be any remainder in dividing *mills*, place the sign, (+) plus after the answer; as, $9.034+. In proving examples having such remainders, slight inaccuracies will occur, unless the remainders are added as in the proof of simple division.

2. If 27 pounds of cheese cost $12.13, what will 1 pound cost?

MODEL OPERATION.

No. of pounds, 27)$12.130 cost of 27 pounds.

$0.449+ cost of 1 pound.

3. If I pay $4.50 a ton for coal, how many tons can I buy for $67.50?

MODEL OPERATION. (*a.*)

Cost per ton, $4.50)$67.50 am't with which to purchase coal.

15 No. of tons which I can purchase.

MODEL OPERATION. (*b.*)

```
                   Am't to spend.
Cost per ton $4.50)$67.50(15 No. tons purchased.
                    450
                    ----
                    2250
                    2250
                    ----
```

ANALYSIS.—In $67.50 there are 6750 cents; in $4.50 there are 450 cents.

If 1 ton of coal costs 450 cents, I can buy as many tons for 6750 cents, as 450 cents are contained times in 6750 cents, which are 15 times.

Therefore, if I pay $4.50 for a ton of coal, I can buy 15 tons for $67.50.

NOTE.—The dividend and divisor must always be changed to the lowest denomination found in either; thus, if the divisor contain only dollars, and the dividend contain cents or mills, the divisor must be changed to cents or mills.

QUESTIONS.—Illustrate the effect of annexing a cipher to a number. (67. *a.*) Explain on the blackboard the rules for the following contractions:—To multiply by 25. (67. *b.*) By 12½. (67. *c.*) By 33⅓. (67. *d.*) By 125. (67. *e.*) To divide by 10, 100, &c. (67. *f.*) By 25. (67. *g.*) By 12½. (67. *h.*) By 33⅓. (67. *i.*) By 125. (67. *j.*) What is United States Money? (68.) Which are the Gold coins? (69.) (*a.*) Silver? (69. *b.*) Nickel? (69. *c.*) Where should the decimal point be placed? (70.) Give the analysis for the reduction of dollars to cents. (71. *a.*) Of cents to mills. Give the rules. 71. (*c.*) (*d.*) (*e.*)

LESSON VII.

BILLS OF PARCELS.

76. A Bill of Parcels is a paper given by merchants, containing a statement of goods sold and their prices; or a paper given by a creditor to a debtor to show the items of an account.

1. NEW YORK, April 5, 1863.
MR. SAM'L PRATT,
Bo't of J. TRUESDELL,
16 pounds of tea, @ 85 cts.,
27 pounds of coffee, @ 15½ cts.,
15 yards of linen, @ 66 cts.,

$

Received Payment,
J. TRUESDELL.

2. NEW YORK, May 17, 1863.
F. CURTISS & Co.,
Bo't of J. H. JOHNSTON & Co.
2 doz. plain gold rings, @ $36.
½ " gold rings, opal settings, @ $180.
5 gold watches, @ $75.
7 gold chains, @ $18.
4 sets teaspoons, @ $6.

$

Received Payment,
J. H. JOHNSTON & Co.,
* per ROBINSON.

---

* When a clerk, or agent, receipts a bill, he writes the signature of the firm, and attaches his own as factor, or agent.

SECTION VIII.

COMPOUND DENOMINATE NUMBERS.

LESSON I.

77. A **Simple** number is either abstract or denominate, and of but one denomination; as, 48678, 59 dollars, 337 mills.

78. A **Compound Denominate** number is a denominate number of two or more different denominations; as, 3 furlongs, 19 rods, 4 yards; 15 days, 6 hours, 37 minutes.

79. The **Reduction** of Compound Denominate numbers is the process of changing a number of one denomination to that of another without altering its value.

80. **Reduction Descending** is the process of changing a number from a greater to a less denomination; as, pounds to farthings.

81. **Reduction Ascending** is the process of changing a number from a less to a greater denomination; as, farthings to pounds.

LESSON II.

ENGLISH MONEY.

82. (*a.*) English, or Sterling money, is the currency of Great Britain.

(*b.*) TABLE.

4 farthings (far. or qr.)	make	1 penny,	marked d.
12 pence	"	1 shilling,	" s.
20 shillings	"	1 pound, or sovereign,	£. sov.
21 shillings	"	1 guinea,	marked guin.

(*c.*) COINS.—The *gold* coins are the *Sovereign*, (1£.), and the *Half-Sovereign*, (10s.)

(*d.*) The *silver* coins are the *Crown*, (5s.), the *Half-crown*, (2s. 6d.), the *shilling* (12d.), and the *sixpenny piece*, (6d.)

(*e.*) The *copper* coins are the *penny*, *half-penny*, and *farthing*. Farthings are generally written as fractions of a penny: thus, 1 far. = $\frac{1}{4}$d.; 2 far. = $\frac{2}{4}$, or $\frac{1}{2}$d.; 3 far. = $\frac{3}{4}$d.

NOTE.—Canada currency is decimal, and the denominations are the same as Federal Money.

LESSON III.

WEIGHTS.

83. Weight is the measure of the quantity of matter a body contains, determined by some standard.

TROY WEIGHT.

84. Troy Weight is used in weighing gold, silver, and jewels; in philosophical experiments; and in determining the specific gravity of bodies, &c.

(*a.*) TABLE.

24 grains (gr.)	make	1 pennyweight,	marked	pwt.
20 pennyweights	"	1 ounce,	"	oz.
12 ounces	"	1 pound,	"	lb.
4 grains	"	1 carat,	"	k.

NOTE.—A carat is a weight of about 3 grains, and is used to weigh diamonds and other precious stones. The term *carat* is also used to denote the fineness of gold. Pure gold is said to be 24 carats fine. Gold 18 carats fine contains 18 parts of pure gold and 6 parts of alloy; gold 14 carats fine contains 14 parts pure gold and 10 parts alloy, &c. The alloy is usually silver and copper.

LESSON IV.

AVOIRDUPOIS WEIGHT.

85. Avoirdupois weight is used for all ordinary purposes.

(*a.*) TABLE.

16 drams (dr.)	make	1 ounce,	marked	oz.
16 ounces	"	1 pound,	"	lb.

25 pounds	make	1 quarter,	marked	qr.
4 quarters	"	1 hundred weight,	"	cwt.
20 hundred weight	"	1 ton,	"	T.

LESSON V.

86. TABLE OF THE GROSS TON.

28 pounds (lbs.)	make	1 quarter,	marked	qr.
4 quarters	"	1 hundred weight,	"	cwt.
20 hundred weight	"	1 gross ton,	"	G. T.

(*a.*) NOTE.—The long, or gross, ton, hundred weight, and quarter, were formerly in common use; but they are now seldom used except in estimating goods at the U. S. Custom House, and in freighting and wholesaling coal from the Pennsylvania coal mines.

87. TABLE OF MISCELLANEOUS WEIGHTS.

56 lbs.	make	1 firkin	of	butter.
196 "	"	1 barrel	of	flour.
200 "	"	1	"	beef, pork, or fish.
280 "	"	1	"	salt.
32 "	"	1 bushel	of	oats.
48 "	"	1	"	barley.
56 "	"	1	"	corn or rye.
60 "	"	1	"	wheat.

LESSON VI.

APOTHECARIES' WEIGHT.

88. Apothecaries' Weight is used by apothecaries and physicians in preparing prescriptions; but medicines are bought and sold by avoirdupois weight.

(*a.*) TABLE.

20 grains (gr.)	make	1 scruple.	marked	sc. or ℈.
3 scruples	"	1 dram.	"	dr. or ʒ.
8 drams	"	1 ounce.	"	oz. or ℥.
12 ounces	"	1 pound.	"	lb. or ℔.

(*b.*) TABLE OF THE COMPARISON OF WEIGHTS.

[*]1 pound Troy = 5760 grs.
1 " Apothecaries' = 5760 grs.
1 " Avoirdupois = 7000 grs.
144 " " = 175 lbs. Troy.

LESSON VII.

MEASURES OF EXTENSION.

89. Extension has the dimensions of length, breadth, and thickness.

(*a.*) A **Line** has the dimension of length.

(*b.*) A **Surface** has the dimensions of length and breadth.

(*c.*) A **Solid** has the dimensions of length, breadth, and thickness.

LONG MEASURE.[†]

90. Long Measure is used for measuring distances.

(*a.*) TABLE.

12	inches (in.)	make	1 foot,	marked	ft.
3	feet	"	1 yard,	"	yd.
$5\frac{1}{2}$	yards	"	1 rod, perch, or pole,	"	rd.
40	rods	"	1 furlong,	"	fur.
8	furlongs	"	1 mile,	"	mi.
$69\frac{1}{4}$	statute miles	"	1 degree, {(On the Equator)}	"	deg. or °
360	degrees	"	1 circle of the earth,	"	cir.

---

[*] The *pound*, *ounce*, and *grain*, of Troy and Apothecaries' weight, are equal, although the intermediate denominations differ.

[†] The yard used by merchants is divided into halves, quarters, eighths, and sixteenths. Ells are not used in the United States.

(*b.*) TABLE OF MISCELLANEOUS LONG MEASURE.

4 inches	make	1 hand.	Used in measuring the height of horses.
9 inches	"	1 span.	
3 feet	"	1 pace, or step.	
6 feet	"	1 fathom.	Used in measuring depths at sea.
3 geographic miles	"	1 league.	
60 " "	"	1 degree.	Of latitude. Of longitude on the equator.

(*c.*) NOTES.—1. A knot is a nautical, or geographical mile. Thus, the phrase, "thirteen knots an hour," denotes thirteen geographical miles per hour

2. 1 English mile equals 5280 feet, and 1 nautical, or geographical mile, equals 6086 feet.

LESSON VIII.

GUNTER'S CHAIN.

91. Gunter's Chain is used by land surveyors. It is 4 rods or 66 feet long, and contains 100 links.

(*a.*) TABLE.

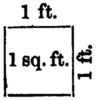
25 links (l.)	make	1 rod,	marked rd.
4 rods	"	1 chain,	" ch.
80 chains	"	1 mile,	" m.

NOTE.—A link is 7.92 inches in length.

LESSON IX.

SQUARE MEASURE.

92. A **Square** is a figure having four equal sides and four equal angles; a square foot is a square, 1 foot in length, and 1 foot in breadth.



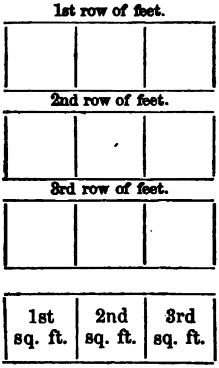
Required, the number of square feet in one square yard.

ILLUSTRATION.
1 yard=3 feet.

1 sq. ft.	1 sq. ft.	1 sq. ft.
1 sq. ft.	1 sq. ft.	1 sq. ft.
1 sq. ft.	1 sq. ft.	1 sq. ft.

1 yard=3 feet.

(*a.*) A **Square yard** is a square 1 yard, or three feet, in length, and 1 yard, or three feet, in breadth; therefore, a *square yard* will consist of 3 *rows* of feet, with 3 feet in each row.



(*b.*) ANALYSIS.—Since in one row there are 3 square feet, in 3 rows there are 3 times 3 square feet, which are 9 square feet. Therefore, in 1 square yard there are 9 square feet. Hence to find the area of any parallelogram, is deduced the following

RULE.—*Multiply the* LENGTH *by the* BREADTH, *and the product will be the square contents.*

(*c.*) There is a difference in meaning between such expressions as *five feet square* and *five square feet.* The expression *five feet square* signifies a square five feet long and five feet wide, containing twenty-five square feet. So *six feet square* signifies a square containing *thirty-six* square feet, &c.

(*d.*) *Square measure* is used in computing the superficial contents of land, boards, plastering, &c.

(*d.*) The work of Artificers is estimated as follows:

1. By the square foot, as in glazing, stone-cutting, &c.
2. By the square yard, as in papering, painting, ceiling, paving, &c.
3. By the 100 square feet, as in flooring, slating, roofing, *brick-laying*, &c.

4. By the thousand, as in laying brick and shingles, &c.

(*e.*) NOTES.—1. The painting of irregular surfaces, as mouldings, cornices, pillars, &c., is estimated by finding the superficial contents of the mouldings, &c.

2. When brick-laying is estimated by the square yard or 100 square feet, the work is understood to be 1½ bricks or 12 inches thick.

(*f.*) TABLE.

144	square inches (sq. in.)	make	1 square foot,	marked	sq. ft.
9	square feet	"	1 square yard,	"	sq. yd.
30¼	square yards	"	1 square rod,	"	sq. rd.
40	square rods	"	1 rood,	"	R.
4	roods	"	1 acre,	"	A.
640	acres	"	1 squ're mile, or section,	"	sq. m., sec.

LESSON X.

SURVEYORS' SQUARE MEASURE.

93. Surveyors' measure is used in computing the area, or contents, of portions of land.

(*a.*) TABLE.

625	square links (sq. l.)	make	1 sq. rod,	marked	sq. rd.
16	sq. rods	"	1 sq. chain,	"	sq. ch.
10	sq. chains	"	1 acre,	"	A.
640	acres		1 sq. mile,	"	sq. m.
36	sq. miles (6 miles square)	"	1 township,	"	Tp.

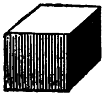
(*b*) NOTES.—1. A square mile is sometimes called a *section*.

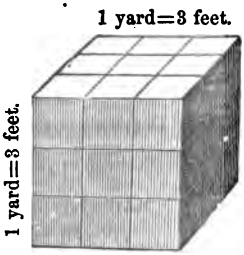
2 Civil engineers commonly use an engineer's chain which consists of 100 links, each 1 foot long

LESSON XI.

CUBIC MEASURE.

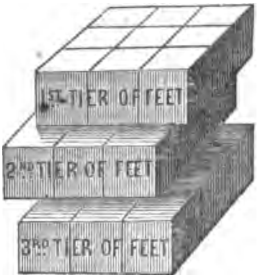
94. A Cube is a solid having six equal square sides, or faces. A cubic foot is a body, whose dimensions are, length, breadth, and thickness, each 1 foot.

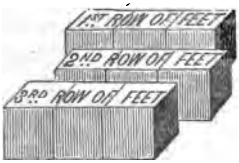


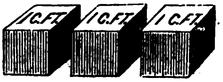

1 yard=3 feet.
1 yard=3 feet.

Required the number of cubic feet in one cubic yard.

(*a.*) A *cubic yard* is a solid, one yard, or three feet, in length, 1 yard, or three feet, in width, and 1 yard, or three feet, in thickness; therefore, a cubic yard will consist of three tiers of feet, each tier consisting of three rows, and each row consisting of 3 cubic feet.






1 C.FT.
1 C.FT.
1 C.FT.

(*b.*) ANALYSIS.—Since there are 3 *rows* of cubic feet in 1 *tier* of feet, in 3 *tiers* there are 3 times 3 *rows*, which are 9 *rows*.

Since in one row there are 3 *cubic feet*, in 9 *rows* there are 9 times 3 cubic feet, which are 27 cubic feet.

Therefore, in one cubic yard there are 27 cubic feet. Hence, to find the solid contents of a body, is deduced the following:

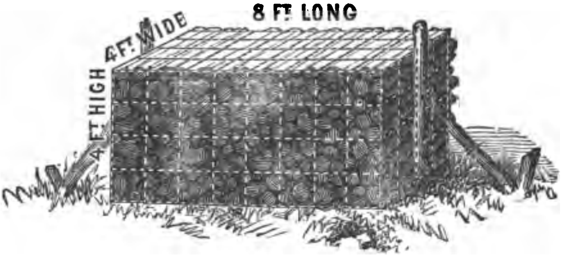
RULE.—*Multiply the* LENGTH *by the* BREADTH, *and this product by the* THICKNESS.

(*c.*) *Cubic measure* is used in estimating the contents of solids; as wood, stone, capacity of cisterns, &c.

(*d.*) 1. A load of earth contains a cubic yard.

2. Light freight is estimated by the cubic foot, and heavy freight by weight.

3. A pile of wood 8 feet long, 4 feet wide, and 4 feet high, contains 1 *cord;* and 1 foot in length of such a pile is called a *cord foot.*



4. A perch of stone or of masonry contains 24¾ cubic feet, and is 16½ feet long, 1½ feet wide, and 1 foot high.

5. The dimensions of a brick are, usually, length 8 inches, breadth 4 inches, and thickness 2 inches.

6. Joiners, painters, and masons, make no allowance for windows, doors, &c. Masons make no allowance for the corners of the walls of houses or cellars.

(*e.*) TABLE.

1728	cubic inches (cu. in.)	make	1 cubic foot,	marked	cu. ft.
27	cubic feet	"	1 cubic yard,	"	cu. yd.
40 50	cubic ft. of round timber, or cubic feet of hewn "	"	1 ton or load,	"	T.
16	cubic feet	"	1 cord foot,	"	cd. ft.
8 128	cord feet, or cubic feet,	"	1 cord of wood,	"	Cd.
24¾	cubic feet	"	1 perch of stone, or masonry,	"	Pch.

LESSON XII.

MEASURES OF CAPACITY.

95. Measures of Capacity are cubic measures, and consist of measures of liquids, and measures of dry substances.

LIQUID, OR WINE MEASURE.

(*a.*) **Liquid measure** is used in measuring liquids; as, molasses, water, vinegar, &c.

(*b.*) TABLE.

4	gills (gi.)	make	1 pint,	marked	pt.
2	pints	"	1 quart,	"	qt.
4	quarts	"	1 gallon,	"	gal.
31½	gallons	"	1 barrel,	"	bbl.
2	barrels, or 63 gallons	"	1 hogshead,	"	hhd.

NOTE.—The barrel and hogshead are often used in estimating the capacity of cisterns, reservoirs, &c.

(*c.*) The following table is sometimes used.

36 gallons	make	1 barrel of beer.
54 gallons, or 1½ barrels,	"	1 hogshead of beer.
42 gallons	"	1 tierce.
2 hogsheads, or 120 gallons,	"	1 pipe, or butt.
2 pipes	"	1 tun.

(*d.*) NOTES.—1. The hogshead, tierce, pipe, and tun, are names of casks, and are usually gauged and have the number of gallons they contain marked on them

2. Ale or beer measure, formerly used in measuring ale and milk, is now seldom used

3. The English *quarter* consists of 8 bu. of 70 lbs. each, and is ¼ of the gross ton.

LESSON XIII.

DRY MEASURE.

96. Dry measure is used in measuring vegetables and articles not fluid; as grain, fruit, salt, ashes, &c.

(*a.*) TABLE.

2 pints (pt.)	make 1 quart,	marked	qt.
8 quarts	" 1 peck,	"	pk.
4 pecks	" 1 bushel,	"	bu.

(*c.*) TABLE OF THE COMPARISON MEASURES OF CAPACITY.

1 gallon, or 4 qts.,	Wine measure,	contains	231	cubic inches.
½ pk, or 4 qts.,	Dry measure,	"	268⅘	"
1 gallon, or 4 qts.,	Beer measure,	"	282	"
1 bushel,	Dry measure,	"	2150⅖	"

A cylindrical box, 18½ inches in diameter, and 8 inches deep, contains 1 bushel.

LESSON XIV.

MEASURE OF TIME.

97. Time is the measure of duration.

(*a.*) TABLE.

60 seconds (sec.)	make 1 minute,	marked	min.
60 minutes	" 1 hour,	"	h.
24 hours	" 1 day,	"	da.
7 days	" 1 week,	"	wk.
4 weeks 2 days, or 30 days,	" 1 month,	"	mo.
365 days 52 weeks, 1 day 12 calendar months	" 1 year,	"	yr.
100 years	" 1 century,	"	C.

(*b.*) The calendar year is divided as follows:—

Season.	Names of month.	No. of days.	Abbreviations.
Winter,	1. January,	31.	Jan.
	2. February,	28 or 29.	Feb.
Spring,	3. March,	31.	Mar.
	4. April,	30.	Apr.
	5. May,	31.	—
Summer,	6. June,	30.	Jun.
	7. July,	31.	—
	8. August,	31.	Aug.
Autumn,	9. September,	30.	Sept.
	10. October,	31.	Oct.
	11. November,	30.	Nov.
Winter,	12. December,	31.	Dec.
		365 or 366.	

(*c.*) NOTES.—1. The exact length of a solar year is 365 days 5 h. 48 min. 46 sec.; but, for convenience, it is reckoned 11 min. 14 sec. more than this, or 365 da. 6 h. = 365¼ days. This ¼ day in four years makes one day, which every fourth year, (called bissextile, or leap year,) is added to the shortest month, giving it 29 days. The numbers denoting leap years are exactly divisible by 4; as, 1856, 1860, 1864.

2. The pupil should carefully commit to memory the names of the months in their order, and the number of days in each. He should associate the name of each month with the number of days it contains, independently, rather than rely on the common rhyme to assist the memory. Many persons through their whole lives are obliged to repeat, "Thirty days hath September, &c.," whenever they wish to recall the number of days in any particular month.

3. In most business transactions 30 days are called a month, and 52 weeks a year.

4. The centuries are numbered from the commencement of the Christian era; the months from the commencement of the year; the days from the commencement of the month; and the hours from the commencement of the day (12 o'clock, midnight.) Thus, 9 o'clock A. M., May 23, 1860, is the end of the 9th hour of the 23rd day of the 5th month of the 60th year of the 19th century.

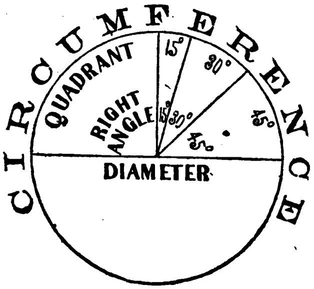
LESSON XV.

CIRCULAR MEASURE.

98. This measure is used in measuring arcs of circles.

(*a.*) TABLE.

60 seconds (″) make 1 minute, marked ′.
60 minutes " 1 degree, " °.
90 degrees " 1 quadrant, " quad.
4 quadrants " 1 circumference, " cir.



(*b*) NOTES.—1. The longest distance across a circle is called the *diameter*. The distance around it is called the *circumference*. Any part of the circumference is called an *arc*.

2. If any circumference, whether large or small, be divided into 360 equal arcs, each arc is called a degree. The degree is divided into 60 minutes, and the minute into 60 seconds. The length of a degree, minute, or second, depends on the size of the circle. If the size of the circle is increased or decreased, the length of the degree, minute, or second, is also increased or decreased.

3. The greatest circumference of the earth's surface is about 24,930 miles; 1° of that circumference is one-360th of 24,930 miles, which is 69¼ miles.

4 A geographical or nautical mile is equal to 1′ of the earth's greatest circumference, which is found to be a little more than 1 statute mile and 49 rods.

5 Latitude is measured north or south from the equator on any meridian, and is expressed in degrees, minutes, and seconds; thus, 43° 17′ 31″ north lat. denotes a position 43° 17′ 31″ north from the equator.

6 Longitude is measured east or west from any given meridian, Washington being generally used in the United States, Greenwich in England, and Paris in France; thus, 43° 17′ 13″ east long. from Greenwich denotes a position 43° 17′ 13″ east from the meridian which passes through Greenwich.

7. The linear extent of a degree of longitude depends upon the latitude, which grows less as the latitude increases; thus at latitude 10° its extent is 359640 feet; at lat. 40° it is 280106; and at lat. 80° it is only 63612 feet.

8 Since the sun apparently goes around the earth once in 24 hours, in 1 hour it must pass over *one* 24th of 360°, which is 15°; hence, if, in the sun's apparent motion, 15° of long require 1 hour, or 60 minutes of time, *one* deg. will require *one* 15th of 60 minutes, which is 4 minutes.

Since 1° or 60′ long. requires 4 min., or 240 sec. of time, 1′ long. will require *one* 60th of 240 sec., which is 4 sec.; therefore, 15° long require 1 hour of time; 1° long. requires 4 minutes of time; 1′ long requires 4 seconds of time.

ANGULAR MEASURE.

(*c*.) This measure is used for measuring difference of direction.

(*d*.) TABLE.

60 seconds (″)	make	1 minute,	marked ′.
60 minutes	"	1 degree,	" °.
90 degrees	"	1 right angle,	" r. a.

LESSON XVI.

99. MISCELLANEOUS TABLES.

(*a*.) PARTICULARS.

12 units, or things,	make	1 dozen.
12 dozen	"	1 gross.
12 gross	"	1 great gross.
20 units	"	1 score.

(*b*.) PAPER.

24 sheets	make	1 quire.
20 quires	"	1 ream.
2 reams	"	1 bundle.
5 bundles	"	1 bale.

(*c.*) BOOKS.

A sheet folded in 2 leaves is called a folio.
" " 4 " " a quarto, or 4to.
" " 8 " " an octavo, or 8vo.
" " 12 " " a 12mo.
" " 16 " " a 16mo.
" " 18 " " an 18mo.
" " 24 " " a 24mo.
" " 32 " " a 32mo.

(*d.*) COPYING.

100 words make 1 folio, or sheet, of common law.
100 " " 1 " " chancery.

LESSON XVII.

100. REDUCTION DESCENDING.

EXAMPLES FOR PRACTICE.

1. Change 46£. 6s. 8d. to the denomination of farthings.

MODEL OPERATION.

```
          £.  s. d.
        ⎧ 46+6+8
        ⎪ 20s.
        ⎪ ——                           PROOF.
  1st.  ⎨ 920s.                 4 far.)44480far.
        ⎪   6                          ———
        ⎪ ——                     12d.)11120d.
        ⎩ 926s. ⎫                     ———
           12d. ⎪                 20s.)926s.+8d.
         ———    ⎪                      ——
         1852d. ⎪                     46£.+6s.
         926    ⎬ 2nd.
        ———     ⎪
        11112d. ⎪
            8d. ⎪
        ———     ⎪
      ⎧ 11120d. ⎭
  3rd.⎨    4 far.
      ⎪ ———
      ⎩ 44480 "
```

(*a.*) ANALYSIS.—1st. Change the pounds to shillings, and add the 6 shillings.

Since in 1 pound there are 20 shillings, in 46 pounds there are 46 times 20 shillings, which are 920 shillings, and 6 shillings added make 926 shillings.

2nd. Change shillings to pence, and add the 8 pence.

Since in 1 shilling there are 12 pence, in 926 shillings there will be 926 times 12 pence, which are 11112 pence, which with 8 pence added are 11120 pence.

3rd. Change pence to farthings.

Since in one penny there are 4 farthings, in 11120 pence there are 11120 times 4 farthings, which are 44480 farthings.

Therefore, in 46£. 6s. 8d. there are 44480 farthings.

(*b.*) RULE FOR REDUCTION DESCENDING.—*Multiply the highest denomination of the given compound number by the number of units that it takes of the next lower denomination to make one of this higher, and add to the product the given number, if any, of the lower denomination.*

Proceed in the same manner with each lower denomination, until the required result is obtained.

EXERCISE.

Require the pupil to compose, analyze, and prove, ten examples similar to the model example.

101. REDUCTION ASCENDING.

LESSON XVIII.

Change 8416784 far. to pounds.

MODEL OPERATION.

$$\begin{array}{lrl}
 & 4\text{ far.})\underline{8416784\text{ far.}} & \\
 & 12\text{d.})\underline{2104196\text{d.}} & \text{1st.} \\
\text{2nd.} & 20\text{s.})\underline{175349\text{s.}+8\text{d.}} & \\
 & 8767\pounds.+9\text{s.} & \text{3rd.}
\end{array}$$

PROOF.

```
 8767£. 9s. 8d.
    20s.
 -------
 175340s.
      9s.
 -------
 175349s.
     12d.
 -------
2104188d.
      8d.
 -------
2104196d.
    4 far.
 -------
8416784 "
```

(*a.*) ANALYSIS.—1st. Change farthings to pence.

Since in 4 farthings there is 1 penny, in 8416784 farthings there are as many pence as 4 farthings are contained times in 8416784 farthings, which are 2104196.

2nd. Change pence to shillings.

Since in 12 pence there is 1 shilling, in 2104196 pence there are as many shillings as 12 pence are contained times in 2104196 pence, which are 175349, with a remainder of 8 pence.

3rd. Change shillings to pounds.

Since in 20 shillings there is 1 pound, in 175349 shillings there are as many pounds as 20 shillings are contained times in 175349 shillings, which are 8767, with a remainder of 9 shillings.

Therefore, in 8416784 farthings there are 8767£. 9s. 8d.

(*b.*) RULE FOR REDUCTION ASCENDING.—*Divide the given denomination, by that number which it takes of this denomination to make one of the next higher.*

Proceed in the same manner to divide each quotient thus obtained, until the number is reduced to the denomination required. The last quotient, with the several remainders annexed, will be the answer.

EXERCISE.

Require the pupil to compose, analyze and prove ten examples similar to the model example.

Change 6 T. 17 cwt. 3 qrs. 19 lbs. 8 oz. 11 drs. to drams.

MODEL OPERATION.

```
        ⎧  6 T. 17 cwt. 3 qrs. 19 lbs. 8 oz. 11 drs.
        ⎪  20 cwt.
        ⎪  ——
  1st.  ⎨ 120  "
        ⎪  17  "
        ⎪  ——
        ⎩ 137  "   ⎫
           4 qrs.  ⎪
          ——       ⎪
          548  "   ⎬ 2nd.
            3  "   ⎪
          ——       ⎪
       ⎧  551  "   ⎭
       ⎪   25 lbs.
       ⎪  ————
       ⎪  2755  "
  3rd. ⎨ 1102
       ⎪ ————
       ⎪ 13775  "
       ⎪    19
       ⎪ ————
       ⎩ 13794  "  ⎫
            16 oz. ⎪
         ————      ⎪
         82764  "  ⎪
        13794   "  ⎬ 4th.
        220704  "  ⎪
             8  "  ⎪
        ————       ⎪
      ⎧ 220712  "  ⎭
      ⎪     16 drs.
      ⎪ —————
      ⎪ 1324272  "
 5th. ⎨ 220712
      ⎪ —————
      ⎪ 3531392  "
      ⎪      11  "
      ⎪ —————
      ⎩ 3531403  "
```

PROOF.

```
      ⎧ 16 drs.)3531403 drs.
 1st. ⎨            —
      ⎩   16 oz.)220712 oz.+11 drs.   ⎫
                 ————                 ⎬ 2nd.
      ⎧ 25 lbs.)13794 lbs.+8 oz.      ⎭
 3rd. ⎨         ————
      ⎩     4 qrs.)551 qrs.+19 lbs.   ⎫
                   ——                 ⎬ 4th.
      ⎧ 20 cwt.)137 cwt.+3 grs.       ⎭
 5th. ⎨         ——
      ⎩          6 T.+17 cwt.
```

ANALYTICAL STEPS.

1st. Change T. to cwt.
2nd. Change cwt. to qrs.
3rd. Change qrs. to lbs.
4th. Change lbs. to oz.
5th. Change oz. to drs.

ANALYTICAL STEPS TO PROOF.

1st. Change drs. to oz.
2nd. Change oz. to lbs.
3rd. Change lbs. to qrs.
4th. Change qrs. to cwt.
5th. Change cwt. to T.

LESSON XIX.

Note.—The pupil should give a full analysis of each step of every example, as heretofore.

Change 104 sq. m., 203 A., 2 R., 8 sq. rds., 2 sq. ft., 88 sq. in., to square inches.

MODEL OPERATION.

104 sq. m., 203 A., 2 R., 8 sq. rds., 21 sq. yds., 2 sq. ft., 88 sq. in
640 A.
4160 "
624
66560 "
203 "
66763 "
4 R.
267052 "
2 "
267054 "
40 sq. rds.
10682160 "
8 "
4)10682168
30¼ sq. yds.
2670542 "
320465040
323135582 "
21 "
323135603 "
9 sq. ft.
2908220427 "
2 "
2908220429 "
144 sq. in.
11632881716 "
11632881716
2908220429
418783741776 "
88 "
418783741864 "

PROOF.

144 sq. in.)418783741864 sq. in.
9 sq. ft.)2908220429 sq. ft.+88 sq. in.
30¼ sq. yds.)323135603 sq. yds.+2 sq. ft.
4 q. sq. yds. 4 quarter sq. yds.
121 q. sq. yds.)1292542412 " "
40 sq. rds.)10682168 sq. rds.+84 q. sq. yds.=21 sq. yds.
4 R.)267054 R.+8 sq. rds.
640 A.)66763 A.+2 R.
104 sq. mi.+203 A.

Analysis.—1st. Change square miles to acres and add the acres.

Formula.—Since in 1 square mile there are 640 square acres, in 104 square miles there are 104 times 640 square acres, which are 66560 acres, which added to 203 acres make 66763 acres.

2d. Change acres to roods and add the roods.

Form.—Since in 1 acre there are 4 roods, in 66763 acres there are 66763 times 4 roods, which are 267052 roods, which added to 2 roods make 267054 roods.

3d. Change roods to square rods and add the rods.

Form.—Since in 1 rood there are 40 square rods, in 237054 roods there are 267054 times 40 square rods, which are 10682160 square rods, which added to 8 square rods make 10682168 square rods.

4th. Change square rods to square yards and add the square yards.

Form.—Since in 1 square rod there are $30\frac{1}{4}$ square yards, in 10682168 square rods there are 10682168 times $30\frac{1}{4}$ square yards, which are 323135582 square yards, which added to 21 square yards make 323135603 square yards.

5th. Change square yards to square feet and add the square feet.

Form.—Since in 1 square yard there are 9 square feet, in 323135603 square yards there are 323135603 times 9 square feet, which are 2908220427 square feet, which added to 2 square feet make 2908220429 square feet.

6th. Change square feet to square inches and add the square inches.

Form.—Since in 1 square foot there are 144 square inches, in 2908220429 square feet there are 2908220429 times 144 square inches, which are 418783741776, which added to 88 square inches make 418783741864 square inches.

Therefore, in 104 sq. m., 203 A., 2 R., 8 sq. rds., 21 sq. yds., 2 sq. ft., 88 sq. in., there are 418783741864 sq. in.

ANALYSIS OF PROOF.—1st. Change square inches to square feet.

Formula.—Since in 144 square inches there is 1 square foot, in 418783741864 square inches there are as many square feet as 144 square inches is contained times in 418783741864 square inches, which are 2908220429, with a remainder of 88 square inches.

2d. Change square feet to square yards.

Form. Since in 9 square feet there is 1 square yard, in 2908220429 square feet there as many square yards as 9 square feet are contained times in 2908220429 square feet, which are 323135603, with a remainder of 2 square feet.

3d. As square yards can not conveniently be changed directly to square rods, change them to *quarter* square yards.

Form.—Since in 1 square yard there are 4 *quarter* square yards, in 30¼ square yards (the number of square yards in a square rod) there are 30¼ times 4 *quarter* square yards, which are 121 quarter square yards.

Form.—Since in 1 square yard there are 4 *quarter* square yards, in 323135603 square yards there are 323135603 times 4 *quarter* square yards, which is 1292542412 *quarter* square yards.

4th. Change *quarter* square yards to square rods.

Form.—Since in 121 *quarter* square yards there is 1 square rod, in 1292542412 *quarter* square yards there are as many square rods as 121 *quarter* square yards are contained times in 1292542412, which are 10682168, with a remainder of 84 *quarter* square yards, equal to 21 square yards.

5th. Change square rods to roods.

Form.—Since in 40 square rods there is 1 rood, in 10682168 square rods there are as many roods as 40 square rods are contained times in 10682168 square rods, which are 267054 roods, with a remainder of 8 square rods.

6th. Change roods to acres.

Form.—Since in 4 roods there is 1 acre, in 267054 roods there are as many acres as 4 roods are contained times in 267054, which are 66763, with a remainder of 2 roods.

7th. Change acres to miles.

Form.—Since in 640 acres there is 1 square mile, in 66763 acres there are as many square miles as 640 acres are contained times in 66763 acres, which are 104, with a remainder of 203 acres.

Therefore, in 418783741864 sq. in. there are 104 sq. m., 203 A., 2 R., 8 sq. rds., 21 sq. yds., 2 sq. ft., 88 sq. in.

EXERCISE.

Require the pupil to compose, prove, and analyze, twenty examples similar to the model examples.

LESSON XX.

102. ADDITION OF COMPOUND NUMBERS.

What is the sum of 36 bu. 2 pks. 6 qts. 1 pt., 25 bu. 1 pk. 2 pts., 18 bu. 3 pks. 7 qts. 1 pt., 9 bu. 2 qts. 1 pt.?

MODEL OPERATION.

36 bu.	2 pk.	6 qts.	1 pt.
25 "	1 "	0 "	2 "
18 "	3 "	7 "	1 "
9 "	0 "	2 "	1 "
90 bu.	0 pk.	1 qt.	1 pt.

(*a.*) ANALYSIS.—For convenience write the denominations so that pints may stand in the column of pints, quarts in the column of quarts, pecks in the column of pecks, &c.

(*b.*) For convenience begin the addition with the column of pints.

(*c.*) 1, 2, 4, 5; 5 pts. equal 2 quarts and 1 pint; write the 1 pint under the column of pints, and add the 2 quarts to the sum of the quarts.

(*d.*) 2, 9, 15, 17; 17 quarts are equal to 2 pks. and 1 quart; write the 1 quart under the column of quarts, and add the 2 pecks to the sum of the pecks.

(*e.*) 3, 4, 6, 8; 8 pecks are equal to 2 bushels and 0 pecks; write a cipher under the column of pecks, and add the 2 bushels to the sum of the bushels.

(*f.*) 9, 27, 52, 88, 90; write the 90 bushels under the column of bushels.

(*g.*) Therefore, the sum of 36 bu. 2 pks. 6 qts. 1 pt.; 25 bu. 1 pk. 2 pts.; 18 bu. 3 pks. 7 qts. 1 pt.; 9 bu. 2 qts. 1 pt.; is 90 bu. 1 qt. 1 pt.

(*h.*) RULE.—*Write the numbers so that those of the same denomination may stand in the same column.*

Add the lowest denomination as in simple numbers, carrying one for as many units as it takes of the denomination added to make one of the next higher.

(*i.*) PROOF.—*The same as in simple numbers.*

LESSON XXI.

103. SUBTRACTION OF COMPOUND NUMBERS.

From 24 lb. 6 oz. 5 pwt. 12 gr. take 14 lb. 9 oz. 10 pwt. 7 gr.

MODEL OPERATION.

24 lb.	6 oz.	5 pwt.	12 gr.
14 "	9 "	10 "	7 "
9 lb.	8 oz.	15 pwt.	5 gr.

(*a.*) ANALYSIS—For convenience write the subtrahend under the minuend, so that grains may stand under grains, pennyweights under pennyweights, ounces under ounces, &c.

(*b.*) For convenience begin with the lowest denomination to subtract.

7 gr. taken from 12 gr. leave 5 gr.; write the 5 gr. under the column of grains.

(*c.*) As 10 pwt. can not be subtracted from 5 pwt., take 1 oz. from 6 oz.; 1 oz. is equal to 20 pwt., which added to 5 pwt. make 25 pwt.; then 10 pwt. taken from 25 pwt. leave 15 pwt., which is written under the column of pwts.

(*d.*) As 9 oz. can not be taken from 5 oz., take 1 lb. from 24 lb.; 1 lb. is equal to 12 oz., which added to 5 oz. make 17 oz.; then 9 oz. taken from 17 oz. leave 8 oz., which is written under the column of ounces.

(*e.*) 14 lb. taken from 23 lb. leave 9 lb., which is written under the column of pounds.

(*f.*) Therefore, 14 lb. 9 oz. 10 pwt. 7 gr. taken from 24 lb. 6 oz. 5 pwt. 12 gr. leave 9 lb. 8 oz. 15 pwt. 5 gr.

(*g.*) RULE.—*Write the subtrahend under the minuend so that numbers of any denomination may stand under numbers of the same denomination.*

Subtract the lowest denomination of the subtrahend from the corresponding denomination in the minuend, as in simple numbers.

If the number of any denomination in the minuend is less than the number of the same denomination in the subtrahend, add to it as many units as it takes to make one of the next higher denomination, and then subtract; then consider the next higher denomination of the minuend diminished by one, and proceed in the same manner with each denomination.

(*h.*) PROOF.—*Prove as in simple subtraction.*

LESSON XXII.

104. MULTIPLICATION OF COMPOUND NUMBERS.

If one acre of land cost 14£. 5s. 8d. 2 far., what will 9 acres cost?

MODEL OPERATION.

14£.	5s.	8d.	2 far.	
			9 acres.	
128£.	11s.	4d.	2 far.	

(*a.*) Analysis.—If 1 acre costs 14£. 5s. 8d. 2 far., 9 acres will cost 9 times 14£. 5s. 8d. 2 far.

(*b.*) Analytical Steps.—1. 9 times 2 far. are 18 far., equal to 4 pence and 2 farthings; write the farthings in the place of farthings, and add the 4 pence to the product of the pence.

2. 9 times 8d. are 72d., which added to 4d. makes 76d.; 76d. equal 6s. and 4d.; write the 4d. in the place of pence, and add the 6s. to the product of the shillings.

3. 9 times 5s. are 45s., which added to 6s. make 51s.; 51s. equal 2£. and 11s.; write the 11s. in the place of shillings, and add the 2£. to the product of the pounds.

4. 9 times 14£. are 126£., which with 2£. added, make 128£., which is written in the place of pounds.

(*c.*) Therefore, if 1 acre of land cost 14£. 5s. 8d. 2 far., 9 acres will cost 128£. 11s. 4d. 2 far.

(*d.*) Rule.—*Write the multiplier under the lowest denomination of the multiplicand.*

Multiply as in simple numbers, and carry as in addition of compound numbers.

(*e.*) Proof.—*Prove as in simple numbers.*

LESSON XXIII.

105. DIVISION OF COMPOUND NUMBERS.

If 5 acres of land produce 102 bu. 3 pks. 1 qt. of wheat, how much will 1 acre produce?

MODEL OPERATION.

5 A.)102 bu. 3 pks. 1 qt. 0 pt.

20 bu. 2 pks. 1 qt. 1$\frac{3}{5}$ pt.

ANALYSIS.—(*a.*) If 5 acres of land produce 102 bu. 2 pks. 1 qt. of wheat, 1 acre will produce *one-fifth* of 102 bu. 2 pks. 1 qt.

(*b.*) ANALYTICAL STEPS.—1. $\frac{1}{5}$ of 102 bushels is 20 bushels, with a remainder of 2 bushels, equal to 8 pecks; write the 20 bushels in the place of bushels, and add the 8 pecks to the pecks.

2. 8 pecks and 3 pecks are 11 pecks, and $\frac{1}{5}$ of 11 pecks are 2 pecks, with a remainder of 1 peck, equal to 8 qts.; write the 2 pecks in the place of pecks, and add the 8 qts. to the qts.

3. 8 qts. and 1 qt. are 9 qts., and $\frac{1}{5}$ of 9 qts. is 1 qt., with a remainder of 4 qts., equal to 8 pints; write the 1 qt. in the place of qts., and add the pts. to the pts.

4. $\frac{1}{5}$ of 8 pts. is 1$\frac{3}{5}$ pts., which write in the place of pts.

(*c.*) Therefore, one-fifth of 102 bu. 3 pks. 1 qt. is 20 bu. 2 pks. 1 qt. 1$\frac{3}{5}$ pts.

(*d.*) RULE.—*Divide the highest denomination as in simple numbers, and each succeeding denomination in the same manner, if there be no remainder.*

If there be a remainder after dividing any denomination, change it to the next lower denomination; add it to the number of that denomination, if any, and divide as before, until each denomination is divided.

(*e.*) PROOF.—*Prove as in simple numbers.*

PART SECOND.

SECTION IX.

PROPERTIES OF NUMBERS, &c.

LESSON I.

DEFINITIONS.

106. An **Integer** is an entire unit, or a collection of entire units.

ILLUSTRATION.*—A collection of entire units is an integer; 3, 9, 7, 14, are collections of entire units, therefore they are integers.

units; 3, 9, 7, 14, are collections of entire units, therefore they are integers.

107. An **Odd number** is one that can not be divided by 2 without a remainder.

108. An **Even number** is one that *can* be divided by 2 without a remainder.

109. A **Prime number** is one that can be exactly divided only by itself and *one*.

110. A **Composite number** is one that *can* be exactly divided by an integer other than itself and *one*.

111. Numbers are said to be *prime* to each other, when no number will divide each of them without a remainder; as, 7, 4, 15, and 11.

112. A **Prime Factor** of a number is a *prime number* that will divide it without a remainder.

(*a*.) NOTE.—Every number is a prime factor, or is composed of prime factors.

113. A **Composite Factor** of a number is a *composite number* that will divide it without a remainder.

---

*NOTE.—Each of the following definitions should be illustrated in a similar manner by the pupil in the recitation.

NOTE.—(*a.*) Unity or 1 is not regarded as a material factor, since multiplying or dividing a number by 1 does not alter its value.

(*b.*) Although no direct process has been discovered by which prime numbers may be distinguished, yet the following facts will aid in determining whether a number is prime or not.

1. Every even number is divisible by 2.

INFERENCE.*—All prime numbers except 2 must be odd.

2. If the sum of the digits of any number is divisible by 3, the number is divisible by 3.

3. Every number whose right hand figure is a cipher or 5 is divisible by 5.

4. Every number occupying *three* or *four* places, whose right hand figures are contained in the left hand figure or figures exactly 3 times, is divisible by 7; as, 602, 3913.

5. Every number occupying *three* or *four* places, whose two right hand figures contain the left hand figure or figures exactly 5 times, is divisible by 7; as, 840, 1155.

6. Every number occupying four places in which two like figures have two ciphers between them is divisible by 7, 11, and 13; as, 2002, 3003.

QUESTIONS.—What is an integer? (106.) What is the difference between an odd and an even number? (107.) (108.) Between a prime and a composite number?. (109.) (110.) When are numbers prime to each other? (111.) What is the difference between prime and composite factors? (112.) (113.) Is a unit ever considered a factor? (113., *a.*)

LESSON II.

114. THE ANALYSIS OF COMPOSITE NUMBERS.

What are the prime factors of 60?

---

* NOTE.—Let the pupil be required to illustrate each of the following facts, and draw the inference from it.

MODEL OPERATIONS.

(1.)

$$\begin{array}{r} 2\overline{)60} \\ 2\overline{)30} \\ 3\overline{)15} \\ 5 \end{array}$$

(2.)

(*a.*) $60=2\times30.$

(*b.*) $30=2\times15.$

(*c.*) $15=3\times5.$

$$60=2\times2\times3\times5=2^2\times3\times5.$$

ANALYSIS.—(*a.*) By trial it is found that 60 is composed of two factors, 2 and 30, of which 2 is prime, and 30 composite.

(*b.*) The composite factor 30 is composed of two factors, 2 and 15, of which 2 is prime, and 15 is composite.

(*c.*) The composite factor 15 is composed of two factors, 3 and 5, both of which are prime.

Therefore, by trial, the prime factors of 60 are (2.) found to be 2, 2, 3, and 5, written 2^2, 3, 5.

(*d.*) RULE.*—*Divide the given number by the least prime number that will divide it without a remainder, and each quotient in the same manner, until the quotient is a prime number. The divisors and last quotient will be the prime factors required.*

(*e.*) PROOF.—*Multiply the prime factors together, and if the product equals the number, the work is correct.*

QUESTIONS.—What is a unit? (1.) What is a number? (2.) What is the difference between an abstract and a concrete number? (3.) (4.) Of what does Arithmetic treat? (5.) What is a quantity? (6.)

---

*NOTES.—1. As the first operation is most convenient in practice the rule is framed with reference to it, but the pupil should also be required to deduce a rule from the analysis.

2. When the same factor is contained more than once in the same number, the repetition is usually denoted by a small figure written above and to the right of the number which expresses the factor; thus, $7^3=7\times7\times7$; $2^3\times3\times5^2=2\times2\times2\times3\times5\times5$. The number thus placed above and to the right shows how often the factor is taken, and is said to denote the power; thus, $3\times3\times3\times3=3^4$, and is called the fourth power of three.

What a problem? (7.) What's the difference between a simple and a complex problem? (8.) (9.) What is an analytical step? (10.) What is an analysis? (11.) What is a rule? (12.) What is the difference between an analysis and a rule? (11.) (12.) What is an axiom? (14.) Which are the fundamental rules? (15.) Can unlike things or numbers be added? (16., *a.*) Can unlike things or numbers be subtracted? (16., *b.*) Can unlike things or numbers be compared? (16., *c.*)

LESSON III.

What are the prime factors of each of the following numbers?

1. 24
2. 36
3. 48
4. 144
5. 120
6. 136
7. 145
8. 225
9. 796
10. 576
11. 256
12. 344
13. 864
14. 945
15. 3420
16. 18500
17. 19965
18. 12496
19. 7007
20. 46096
21. 38148
22. 17199
23. 3913
24. 41834
25. 24024
26. 37076
27. 137456
28. 627510
29. 26840
30. 117936

QUESTIONS.—What is Notation? (17.) What is the difference between the Roman and Arabic notation? (18.) (19.) What is the difference between the simple and the local value of a figure? (20.) (21.) What is the use of the cipher? (22.) What letters are used in the Roman notation? (18.) What is the effect of repeating letters? (18., *b.*) What effect has a letter of less value written before one of greater value? (18., *c.*) What when written after? (18., *d.*) What effect has a dash placed over a letter or combination of letters? (18., *e.*) What is the difference between Notation and Numeration? (17.) (23.) Repeat the Numeration table by periods from right to left. (25.) From left to right. What should be done with vacant places? (26.) With vacant periods? (27.)

LESSON IV.

115. To multiply or divide by the factors of composite numbers.

It is required to multiply 346 by 24.

MODEL OPERATION.

346 Multiplicand.
4 One factor of 24.

1384 Product of 346×4.
6 The other factor of 24.

8304 Product of 6 times (4 times 346.)

Analysis.—It is required to multiply 346 by *twenty-four*. The multiplier 24 is composed of two factors, 4 and 6.

If the multiplicand is multiplied by 4 the product is 6 times too small, for 4 is 6 times smaller than 24; hence this product 1384 must be multiplied by 6, and 1384 multiplied by 6 equals 8304.

Therefore, 346 multiplied by 24 equals 8304.

Note.—Each pupil should be required to deduce a rule from the analysis, and present it in writing for the inspection of the teacher, or to be read in the class and subjected to its criticism.

LESSON V.

In each of the following examples multiply by the factors of the multiplier.

31. 687842×36
32. 4871067×24
33. 371862×108
34. 48678×72
35. 416072×63
36. 57868934×132
37. 467083486×66
38. 47086864×432
39. 5407864×384
40. 3716024×144
41. 4167468×77
42. 5670864×99
43. 3716742×864
44. 4716070×225
45. 372641×520

Questions.—How are periods usually separated? (27., *a.*) What is the sum of two or more numbers? (30.) What is Addition? (31.) Subtraction? (39.) Multiplication? (46.) Division? (54.) What is the sign of Addition? (32.) Of Subtraction? (43.) Of Multiplication? (51.) What are the signs of Division? (62.) What is the sign of Equality? (33.) The Dollar sign? (34.) What is the order of solving problems? (35.) Repeat the analysis of an example in Addition. (36.) How is Addition proved? (36., *f.*) Subtraction? (44., *i.*) Multiplication? (53., *n.*) Division? (64., *i.*) What is the difference of two numbers? (38.) What is the Minuend? (40.) What is the Subtrahend? (41.) What is the answer in Addition called? (30.) In Subtraction? (42.) In Multiplication? (49.) In Division? (57.)

LESSON VI.

It is required to divide 3746 by the factors of 210, and to find the true remainder.

Illustration.—What number of baskets is required to contain 3746 cherries, if each basket holds 210 cherries; and how many cherries will remain?

MODEL OPERATIONS.

(*a.*)

$$
\begin{array}{ll}
 & \text{Dividend} \\
\text{1st.}\left\{\begin{array}{l} 5)\underline{3746}\ \text{cherries.} \\ \ \ 749\ \text{bunches}+1\ \text{cherry.} \end{array}\right. & \\
 & \left.\begin{array}{l} 6)\underline{749}\ \text{bunches}+1\ \text{cherry.} \\ 7)124\ \text{piles}+5\ \text{bunches.} \end{array}\right\}\text{2d.} \\
\text{3d.}\left\{\begin{array}{l} 7)\underline{124}\ \text{piles}+5\ \text{bunches.} \\ \ \ 17\ \text{baskets}+5\ \text{piles.} \end{array}\right. & \\
\text{Divisor.} & \text{Factors.} \\
210= & 5\times6\times7
\end{array}
$$

(*b.*)

$$
\begin{array}{lll}
 & \begin{array}{lllll} \text{Piles.} & & \text{Bunches} & & \text{Cherry.} \\ 5 & + & 5 & + & 1 \end{array} & \\
\text{4th.}\left\{\begin{array}{ll} \underline{6} & \text{bunches.} \\ 30 & \text{"} \\ \underline{5} & \text{"} \\ 35 & \text{"} \end{array}\right. & & \\
 & \left.\begin{array}{lll} \underline{5} & \text{cherries.} & \\ 175 & \text{"} & \\ \underline{1} & \text{"} & \\ & \text{"} & \\ 176 & \text{"} & \text{no. remaining.} \end{array}\right\}\text{5th.} &
\end{array}
$$

Analysis.—(*a.*) Find the number of baskets.

(*b.*) Find the number of cherries remaining.

ANALYTICAL STEPS.—1. Divide 8746 cherries into bunches containing 5 cherries each.

2. Divide 749 cherries into piles containing 6 bunches each.

3. Find the number of baskets required to hold 124 piles, each basket containing 7 piles.

4. Find the number of bunches in 5 piles, and add the bunches remaining.

5. Find the number of cherries in 35 bunches, and add the cherries remaining.

CONCLUSION.—Therefore, 17 baskets are required, and 176 cherries remain.

NOTE.—Let the pupil be required to write a rule for dividing by the factors of a composite number, and finding the true remainder.

LESSON VII.

46. Divide 3786412 by the factors of 294, and find the true remainder.

47. Divide 418643 by 945 in the same manner, and find the true remainder.

48. Divide 4160703 by 344 in the same manner.

49. Divide 37064 by 18500 in the same manner.

50. Divide 460783 by 864 in the same manner.

51. Divide 3870645 by 17199 in the same manner.

52. Divide 460783 by 945 in the same manner.

53. Divide 370642 by 256 in the same manner.

54. Divide 607121 by 796 in the same manner.

55. Divide 4009001 by 344 in the same manner.

QUESTIONS.—What is the multiplicand? (47.) What is the multiplier? (48.) What is the product? (49.) Which terms are called factors? (50.) What is Division? (54.) What is the dividend? (55.) The divisor? (56.) The quotient? (57.) What is the difference between Long Division and Short Division? (60.) (59.) What is the difference between the remainder in Division and the remainder in Subtraction? (61.) (42.) To what is the greater of two numbers equal? (66., *a.*) The smaller of two numbers? (66., *b.*)

LESSON VIII.

ABBREVIATED ARITHMETICAL EXPRESSIONS, INCLUDING CANCELLATION.

116. A **Parenthesis,** ( ) or **Vinculum,** —— denotes that the parts enclosed are taken as one number; thus, $(8+2)\times 2$, or $\overline{8+2}\times 2$, signifies that the sum of 8 and 2 is to be multiplied by 2.

$(4-3)+(6+7)\times 2$ signifies that the difference between 4 and 3 is to be added to the sum of 6 and 7 multiplied by 2.

$\left(\frac{8+4}{3}+\frac{13-1}{6}\right)\times 5\div\left(\frac{6\times 4\times 3}{12}-1\right)$ signifies that the sum of 8 and 4 divided by 3, is to be added to the difference between 13 and 1 divided by 6; that this result is to be multiplied by 5, and the product divided by one less than the quotient of the product of 6 multiplied by 4, multiplied by 3, divided by 12.

What is the value of each of the following expressions?

56. $437\div 8-17+74$
57. $643\times 7+8-174$
58. $893-67\times 4+374$
59. $374\times 8\div 9+17$
60. $386\times 4\div 6-19$
61. $\frac{467}{2}+14-27$
62. $(4+6+4)-\frac{3\times 4+2}{7}$
63. $\frac{6\times 7\times 8\times 4}{8\times 2\times 3}-\frac{8\times 7+4}{5}$
64. $\frac{46\times 8}{8}-41+9\times 2$
65. $\frac{43\times 2}{9}+\frac{5-4\times 8}{2}$
66. $(3\times 4\times 6\times 8)\div 4\times 3\times 2\times 6$
67. $24\div 6\times 3\div(16\times 2-8)$
68. $8\div 4\times 3+25\times 3-11$
69. $(4\times 6)\div(3-1\times 5)-3$
70. $5-2\times(24\div 4)-(3+2)$
71. $\frac{8}{2}+\frac{6}{3}+\frac{6+2}{4}-8$
72. $\frac{5}{2}+6+\frac{(37-1)}{\frac{12}{3}}$
73. $\frac{\frac{24}{3}}{27}+\frac{63}{9}+48$
74. $(1\div 9)+\frac{63}{9}-4$
75. $\frac{47}{63}+184-\frac{96}{8}$

QUESTIONS.—If the multiplier is a unit, will the product be greater, or less, than the multiplicand? (66., *c.*) What effect will multiplying or dividing the multiplier by any number have upon the product? (66., *d.*) What effect will increasing or diminishing the multiplicand in any ratio have upon the product? (66., *e.*) If the divisor is a unit will the quotient be greater, or less, than the dividend? (66., *f.*) If the divisor equals the dividend, what will be the quotient? (66., *g.*)

LESSON IX.

117. Cancellation is the method of shortening arithmetical operations, by rejecting all factors, common to both divisor and dividend.

(*a.*) Divide 19965 by 3630.

(*b.*) Divide the product of 22, 9, 12 and 5, by the product of 3, 11, 6, and 4.

MODEL OPERATIONS.

(*a.*)

$$\begin{array}{r}\text{Dividend}\\ \text{Divisor}\end{array}\frac{19965=\cancel{3}\times\cancel{5}\times\cancel{11}\times\cancel{11}\times 11}{3630=2\times\cancel{3}\times\ \cancel{5}\times\cancel{11}\times\cancel{11}}=\frac{11}{2}=5\tfrac{1}{2}\text{ quotient.}$$

(*b.*)

$$\begin{array}{r}\text{Dividend}\\ \text{Divisor}\end{array}\frac{\overset{2}{\cancel{22}}\times\overset{3}{\cancel{9}}\times\overset{2}{\cancel{12}}\times 5}{\cancel{3}\times\cancel{11}\times\cancel{6}\times\underset{2}{\cancel{4}}}=\frac{3\times 5}{1}=15\text{ quotient.}$$

ANALYSIS.—(*a.*) Resolve the dividend 19965 into its prime factors, 3, 5, 11, 11, 11; also resolve the divisor 3630 into its prime factors, 2, 3, 5, 11, 11. Since, if both dividend and divisor are diminished in the same ratio, the value of the quotient is not changed, and, since 3, 5, 11, 11 and 11, are the factors of the dividend, and 2, 3, 5, and 11, are the factors of the divisor; therefore, if the dividend is diminished by rejecting the factors 3, 5, 11, 11, and the divisor is diminished by rejecting the equal factors, 3, 5, 11, 11, the quotient will not be changed.

Therefore, 19965 divided by 3630 gives a quotient of $5\frac{1}{2}$.

(*b.*) By inspection we find the factor 11 in the divisor, and

22 in the dividend; but 22 is composed of the two factors, 2 and 11.

Rejecting the common factor 11, we have the 2 remaining in the dividend.

Proceed in the same manner, until all the common factors are rejected.

EXERCISES.

76. What is the value of the following expression? 4×8×5×6×7×3÷4÷6÷8÷3÷5.

MODEL OPERATION.*

$$\frac{\cancel{4}\times\cancel{8}\times\cancel{5}\times\cancel{6}\times7\times\cancel{3}}{\cancel{4}\times\cancel{6}\times\cancel{8}\times\cancel{3}\times\cancel{5}},=\frac{7}{1},=7.\ \text{Ans.}$$

77. 3×4÷3÷2×8÷2÷6×7÷6×9÷4÷9=what?

78. (*a.*) 4÷3÷(8÷12)=(*b.*) 4÷3÷8×12.

The quotient of 4÷3 is to be divided by the *quotient* of 8÷12. If 4÷3 is divided by 8, the divisor being 12 times too large, the quotient will be 12 times too small; hence the quotient of (4÷3÷8) must be multiplied by 12. Therefore,

$$(a.)\ 4\div3\div(8\div12)=(b.)\ 4\div3\div8\times12,=\frac{\overset{2}{\cancel{4}}\times\overset{\cancel{4}}{\cancel{12}}}{\cancel{3}\times\underset{\cancel{2}}{\cancel{8}}}=2,\ \text{Ans.}$$

LESSON X.

79. 3×4÷3÷4×6÷3×5=what?

80. 4÷2÷8×3×3÷4÷4×6=what?

81. 3÷4×6÷7×14÷3×2=what?

82. 3÷2×2×4÷3×24×4=what?

83. 4×3÷4×2÷2×3×4÷7=what?

84. 9×24×3÷9÷63÷4=what?

85. 8×89÷67×38÷43=what?

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*NOTE.—Since the product of the multipliers is to be divided by 4, and the quotient by the divisor, 6; and each following quotient by the next divisor in turn; it is evident that the product of the multipliers is to be divided by the product of all the divisors; thus, 4×8×5×6×7×3÷(4×6×8×3×5), which may be otherwise expressed; thus, $\frac{4\times8\times5\times6\times7\times3}{4\times6\times8\times3\times5}$.

86. $41\times67\div89\div41\times3\times3=$ what?
87. $56\times7\div46\times867\div82=$ what?
88. $916\times4\times3\div7\div976\times8756=$ what?
89. $4\div3\times6\div4\times3\div4\times3\div7=$ what?
90. $43\times2\div(4\div8)=43\times2\div4\times8=$ what?
91. $23\times4\div(4\times6\times3)=23\times4\div4\div6\div3=$ what?
92. $436\times24\times3\div(4\times3\times2)\div4=$ what?
93. $3\times2\div(4\times3\div3)=3\times2\div4\div3\times3=$ what?
94. $3\times2\div(46\times3\div4)=3\times2\div46\div3\times4=$ what?
95. $41\times2\times(6\div7\times3\div4)=41\times2\times6\div7\times3\div4=$ what?
96. $5\div6\times7\div(3\div4\times2)\div3\times2=$ what?
97. $(4\div2\times3)\div(2\times4)=4\div2\times3\div2\div4=$ what?
98. $43\times4\div(6\times3\div2)=$ what?
99. $4\div6\times7\div8\times9\div7\times8\div(9\div2)=$ what?
100. $3\times6\div7\div9\div9\times8\times3\times7\div(6\times3)=$ what?
101. $5\times2\div14\div21\times16\div(4+5)\div2=$ what?
102. $54\div(4\times5)\times(6\div2)\div42\times34\div5=$ what?
103. $48\div2\times5\times(5\times6)\div47\div405\times2=$ what?
104. $356\times20\div4050\div25\times63\div72=$ what?
105. $759\times2\times547\div43\div22\times11=$ what?
106. $33\div330\times\overline{10\div42}\times84=$ what?
107. $59\times60\div\overline{33\times46}\times25\div75\times2=$ what?
108. $42\times42\div21\div2\times7\times6\times2\div4\div21=$ what?
109. $32\div\overline{2\times4}\times2\div8\times\overline{7\div21}\times49=$ what?

QUESTIONS.—If the divisor is *increased* in any ratio, how will the quotient be affected? (66., *h.*) If the divisor is *decreased* in any ratio, how will the quotient be affected? (66., *i.*) If the divisor is any number of times too small, how will the quotient be affected? If the divisor is any number of times too large, how will the quotient be affected? If the dividend is increased or diminished in any ratio, how will the quotient be affected? (66., *j.*) (66., *k.*) The product of what two numbers *plus* the remainder is equal to the dividend? (66., *l.*) What number divided by the multiplier will equal the multiplicand? (66., *m.*) When the sum and difference of two numbers are given, how are the numbers found? (66., *n.*)

LESSON XI.

118. A **Common Divisor** of two or more numbers is any number greater than 1, that will divide each of them without a remainder.

119. The **Greatest Common Divisor** of two or more numbers is the greatest number that will divide them without a remainder.

(*a.*) What is the greatest common divisor of 84 and 132?

(*b.*) What is the greatest common divisor of 84 and 203?

MODEL OPERATION.

(*a.*)

$84=[2\times2\times3]\times7.$

$132=[2\times2\times3]\times11.$

Common factors.

$2\times2\times3=12$, Greatest Com. Divisor.

(*b.*)*

84)203(2
168
35)84(2
70
14)35(2
28
7)14(2

ANALYSIS.—(*a.*) By resolving the numbers into their prime factors, we find that those of 84 are 2, 2, 3, 7; and those of 132 are 2, 2, 3, 11.

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*NOTE.—For the benefit of those who prefer this method, we give the analysis of it here. It depends upon the following principles:—

I. A divisor of any number is a divisor of any multiple of that number.
II. A divisor of any two numbers is a divisor of their sum.
III. A divisor of any two numbers is a divisor of their difference.

First ascertain whether 84 is a divisor of 203, if so, it is the divisor sought.

As any divisor of 84 is a divisor of 168 (Prin. I.) therefore, any divisor of 84 and 203 is also a divisor of 203 and 168.

As any divisor of 203 and 168 is a divisor of 35 (Prin. III.) therefore, any divisor of 84 and 203 is a divisor of 84 and 35.

Again, as any divisor of 84 is a divisor of 168 (Prin. I.) therefore, any divisor of 84 and 35 is a divisor of 168 and 35; but any divisor of 168 and 35 is also a divisor of 203: (Prin. II.) consequently, *any* divisor of 84 and 35 is a divisor of 84 and 203; and hence, the *greatest common divisor* of 35 and 84 is the greatest common divisor of 84 and 203

In the same manner the greatest common divisor of 14 and 35 is found to be the greatest common divisor of 35 and 84; also, the greatest common divisor of 7 and 14 is found to be the greatest common divisor of 14 and 35. As 7 is the greatest common divisor of itself and 14, it is, therefore, the greatest common divisor of 84 and 203

By inspection, we find that the factors 2, 2, 3, are common to both 84 and 132. Since only common factors will divide both numbers, the *product* of all the common factors must be the greatest factor that will divide both without a remainder.

Twelve is the product of the common factors 2, 2, 3, and is, therefore, the greatest common divisor of 84 and 132.

I. RULE.—(*a*.) *Require the pupil to write the rule from the analysis.*

II. RULE.—(*b*.) *Divide the greater number by the less, and, if there be a remainder, divide the preceding divisor by it, and so continue dividing, until there is no remainder. The last divisor will be the greatest common divisor sought.*

If there are more than two numbers, find the greatest common divisor of the first two, then of the divisor thus found and one of the remaining numbers, and so continue until the greatest common divisor of all the given numbers is found.

LESSON XII.

110. What is the greatest common divisor of 85 and 95?

111. What is the greatest common divisor of 72 and 168?

112. What is the greatest common divisor of 119 and 121?

113. What is the greatest common divisor of 12, 18, 24, and 30?

114. What is the greatest common divisor of 14, 28, and 21?

115. What is the greatest common divisor of 20, 16, and 48?

116. What is the greatest common divisor of 28, 16, 12, and 8?

117. What is the greatest common divisor of 11, 88, and 99?

118. What is the greatest common divisor of 28, 63, 47, and 93?

119. I have three rooms respectively 15, 18, and 24 feet in width; what must be the width of the widest oil-cloth that will exactly fit without cutting?

QUESTIONS.—How does annexing a cipher to a number affect it? (67., *a.*) Why? Illustrate the contracted method of dividing by 10, 100, &c. (67., *f.*) What is United States money? (68.) What are the Gold coins? (69., *a.*) Silver? (69., *b.*) Nickel? (69., *c.*) What is the use of the decimal point? (70.) How are dollars changed to mills? (71., *d.*) Cents to mills? (71., *e.*) Mills to dollars? (71., *g.*) Cents to dollars? (71., *f.*) What is a bill of parcels? (76.) What is a simple number? (77.) A compound denominate number? (78.)

LESSON XIII.

COMMON MULTIPLE.

120. A **Multiple** of a number is a number that can be divided by it without a remainder.

121. A **Common Multiple** of two or more numbers, is a number that can be divided by each of them without a remainder.

122. The **Least Common Multiple** of two or more numbers, is the *least* number that can be divided by each of them without a remainder.

What is the least common multiple of 6, 9, and 12?

MODEL OPERATIONS.

(*a.*)

Prime factors

$6=\cancel{2}\times\cancel{3}.$

$9=3\times3,=3^2.$

$12=2\times2\times\cancel{3},=2^2\times\cancel{3}.$

$3^2\times2^2=36.$

(*b.*)

$$\begin{array}{r|rrr} 2 & 6 & 9 & 12 \\ \hline 3 & 3 & 9 & 6 \\ \hline & 1 & 3 & 2 \end{array}$$

$2\times3\times3\times2=36.$

Analysis.—(*a.*) 1. The least number divisible by 6 must contain only the highest powers of all the different factors of 6, which are 2 and 3.

2. The least number divisible by 6 and 9 must contain only the highest powers of all the different factors of 6 and 9, which are 3^2 and 2. We reject the *first* power of 3 found in the given number 6, because we have a higher power of 3 in the given number 9.

3. The least number divisible by 6, 9, and 12, must contain only the highest powers of all the different factors of 6, 9, and 12, which are 2^2, 3^2. We reject the first power of 2 in the number 6, and the first power of 3 in the number 12, because we find higher powers of each factor in the numbers 9, and 12.

Therefore, $2^2 \times 3^2, = 36$, is the least common multiple of 6, 9, and 12.

(*b.*) This abbreviated method of operation is generally preferred, and the analysis being essentially the same as that of the method preceding, it will not be repeated.

Rule.—I. (*a.*) *Resolve the numbers into their prime factors. The product of the highest powers of all the different factors will be the least common multiple.*

Rule.—II. (*b.*) *Arrange the numbers on a horizontal line, divide by the least prime number that will divide more than one of them without a remainder, and write the quotients and the undivided numbers in a line beneath. Continue dividing as before, until there is no prime number greater than a unit that will divide two or more of them without a remainder. The product of the divisors and the undivided numbers is the least common multiple required.*

LESSON XIV.

EXERCISES FOR PRACTICE.

Find the least common multiple of each of the following series of numbers.

120. 4, 9, and 12.
121. 16, 12, and 24.
122. 15, 9, 6, and 5.
123. 10, 6, 18, and 15.
124. 24, 16, 15, and 20.
125. 25, 60, 72, 35.
126. 63, 12, 84, 72.
127. 54, 81, 14, 63.
128. 12, 72, 36, 144.
129. 1, 2, 3, 4, 5, 6, 7, 8, 9.
130. 17, 29, 53, 37.
131. 8, 9, 55, 49.
132. 720, 336, 1736.
133. 8, 12, 16, 24, 36, 48, 72, 144.
134. 3, 7, 5, 9, 186.
135. 5, 3, 7, 9, 91.
136. 4, 3, 6, 7, 8, 16, 19.
137. 4, 8, 12, 24, 48, 84.
138. 4, 6, 7, 4, 5, 9, 13, 12.
139. 45, 78, 37, 42, 63.

140. What is the least number that can be divided by 9, 11, 17, 18, and 6?

141. What is the greatest number that will divide 48, 96, 74, and 108?

142. What is the least number that will contain 43, 12, 25, and 30, without a remainder?

143. What is the difference between the least common multiple and the greatest common divisor of 6, 8, 24, and 12?

144. What is the sum of the greatest common divisor and the least common multiple of 25, 10, and 30?

SECTION X.

FRACTIONS.

LESSON I.

DEFINITIONS.

123. A **Fraction** is an expression denoting a part or several equal parts of a quantity.*

Fractions are of two kinds, Common and Decimal.

124. A **Common Fraction**† is one whose denominator is other than *ten* or some power of *ten;* as, $\frac{4}{7}$, $\frac{3}{8}$, $\frac{4}{11}$, $\frac{3}{105}$.

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*Note —When the quantity is not mentioned unity is understood. Thus, $\frac{1}{4}$ signifies $\frac{1}{4}$ of one

†Note —Decimal fractions are often used in connection with common fractions, when they are written with a denominator in the same manner, and treated as common fractions; as, $\frac{3}{7}\times\frac{6}{10}$, $\frac{3}{8}+\frac{6}{100}$, $\frac{4}{7}-\frac{3}{100}$.

125. A **Decimal Fraction** is one whose denominator is *ten*, or some power of ten; as, $\frac{3}{10}$, $\frac{4}{100}$, $\frac{37}{1000}$, $\frac{346}{100000}$. They are usually expressed by the use of the decimal point; as, .3, .04, .037, .00346.

126. A Common Fraction is expressed by a *Numerator* and a *Denominator*, which are called the *terms* of the fraction.

127. The **Denominator** of a fraction shows the number of equal parts into which the unit is divided, and gives the name to the fraction. It may be written at the right of the numerator in words; as, 4 *tenths*, 9 *sevenths*, 8 *elevenths;* or under the numerator with a short horizontal line between them; as, $\frac{3}{9}\,\begin{matrix}\text{numerator.}\\ \text{denominator.}\end{matrix}$

128. The **Numerator** of a fraction shows how many there are of the equal parts into which a unit is divided. It may be written either at the left of the denominator; as, 3 tenths, 4 sevenths, or over the denominator with a short horizontal line between them; as, $\frac{3}{10}$, $\frac{4}{7}$.

129. A **Proper Fraction** is one whose value is less than a unit; as, $\frac{5}{7}$, $\frac{3}{8}$, $\frac{2}{9}$.

130. An **Improper Fraction** is one whose value is equal to or greater than a unit; as, $\frac{15}{7}$, $\frac{13}{11}$, $\frac{8}{8}$, $\frac{16}{8}$.

131. A **Mixed Number** is an integral number added to a fractional number; as, $3+\frac{4}{7}$, $7+\frac{8}{9}$. The sign of addition is usually omitted; as, $3\frac{4}{7}$, $7\frac{8}{9}$.

132. A **Simple Fraction** is one whose numerator and denominator are both integral numbers.

133. **Complex Fractions** are those which have fractions or mixed numbers for their numerators or denominators, or for both; as, $\frac{\frac{3}{4}}{\frac{6}{7}}$, $\frac{3}{4\frac{2}{7}}$, $\frac{9\frac{2}{3}}{8}$, $\frac{\frac{16}{7}}{8\frac{5}{9}}$.

134. A Compound Fraction is a fraction of a fraction, or several fractions connected by the word, *of*, or the sign of multiplication, ×; as, $\frac{1}{2}$ of $\frac{3}{7}$, $\frac{5}{7}\times\frac{9}{8}$, $\frac{3}{4}$ of $\frac{6}{7}\times\frac{8}{9}$.

Questions.—What is reduction of compound denominate numbers? (79.) What is reduction descending? (80.) What is reduction ascending? (81.) What is the use of English money? (82., *a.*) Repeat the table. (82., *b.*) Which are the gold coins? (82., *c.*) Which the silver? (82., *d.*) Which the copper? (82., *e.*) What is weight? (83.) What are the uses of Troy weight? (84.) To what is the term carat applied? (84., note.) For what is Avoirdupois weight used? (85.) Repeat the table. (85., *a.*) Repeat the Long Ton table. (86.) Repeat the Miscellaneous table. (87.)

LESSON II.

135. Fractions may be considered as indications of unperformed divisions; as, $\frac{8}{4}=8\div4=2$, $\frac{5}{9}=5\div9$.

136. The numerator of the fraction is the dividend, and the denominator the divisor.

137. Principles.—(*a.*) If both denominator and numerator be *increased* in an equal ratio, the quotient will not be changed.

MODEL OPERATIONS.

(*a.*)

Numerator	Demominator.	Quotient
36 ÷	12 =	3.
2	2	
72 ÷	24 =	3.

(*b.*)

$$\frac{36\ \text{Numerator.}}{12\ \text{Denominator.}}=3,\ \text{Quotient.}$$

$$\begin{array}{r}\text{Numerator.}\\\text{Denominator.}\end{array}\frac{36\times2}{12\times2}=\frac{2}{24}=3,\ \text{Quotient.}$$

Illustration.*—(*a.*) 36 is the numerator, 12 the denominator, and 3 the quotient. Multiplying both numerator and denominator by 2, we have 72 for the numerator, and 24 for the denominator, which gives 3 for the quotient, as before.

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*Note.—The teacher should not fail to require the pupil to illustrate each principle in the same manner on the blackboard.

(*b.*) If both numerator and denominator be *decreased* in an equal ratio, the quotient will not be changed.

(*c.*) If the denominator alone is *increased* in any ratio, the quotient will be *decreased* in the same ratio.

(*d.*) If the denominator alone is *decreased* in any ratio, the quotient will be increased in the same ratio.

(*e.*) If the numerator alone be *increased* in any ratio, the quotient will be *increased* in the same ratio.

(*f.*) If the numerator alone be *decreased* in any ratio, the quotient will be *decreased* in the same ratio.

QUESTIONS.—For what is Apothecaries' weight used? (88.) Repeat the table. (88., *a.*) How much heavier is a pound Avoirdupois than a pound Troy? (88., *b.*) How many dimensions has extension? (89.) How many has a line? (89., *a.*) A surface? (89., *b.*) A solid? (89., *c.*) What are the uses of Long Measure? (90.) Repeat the table. (90., *a.*) Repeat the Miscellaneous table. (90., *b.*) What is the use of Gunter's Chain? (91.) Repeat the table. (91., *a.*) What is a square? (92.) Repeat the rule for finding the contents of a Square. (92., *b.*) How do Artificers estimate their work? (92., *d.*) Repeat the table of Square Measure. (92., *f.*) What are the uses of Surveyors' Square Measure? (93.) Repeat the table. (93., *a.*) What is a cube? (94.) What is the use of Cubic Measure? (94., *c.*) Repeat the table. (94., *e.*) What are measures of capacity? (95.) What is the use of Liquid Measure? (95., *a.*) Repeat the table. (95., *b.*) What is the use of Dry Measure? (96.) Repeat the table. (96., *a.*) How many cubic inches in 4 qts., or one gallon, Wine Measure? (96., *c.*) In $\frac{1}{2}$ peck, or 4 quarts, Dry Measure? (96., *c.*) In 4 qts., or one gallon, Beer Measure? (96., *c.*) In one bushel, Dry Measure? (96., *c.*)

LESSON III.

Simplify the fractional expressions (*a.*) $\frac{3}{4} \div \frac{7}{9}$, (*b.*) $\frac{3}{4} \times \frac{7}{9}$.

MODEL OPERATIONS.

(*a.*) $\frac{3}{4}\div\frac{7}{9}=3\div4\div(7\div9),=3\div4\div7\times9$, Ans.

(*b.*) $\frac{3}{4}\times\frac{7}{9}=3\div4\times(7\div9),=3\div4\times7\div9$, Ans.

ANALYSIS.—(*a.*) $\frac{3}{4}$ is an expression which indicates that 3 is to be divided by 4; $\frac{7}{9}$ is an expression which indicates that 7 is to be divided by 9; and $\frac{3}{4}\div\frac{7}{9}$ is an expression which indicates that $3\div4$ is to be divided by $7\div9$; thus, $3\div4\div(7\div9)$.

If the quotient of $3\div4$ be divided by 7, the quotient will be divided by a number 9 times too large; hence the last quotient will be 9 times too small, and must be multiplied by 9; thus, $3\div4\div7\times9$. Therefore, $\frac{3}{4}\div\frac{7}{9}=3\div4\div7\times9$.

(*b.*) The same process of reasoning may be applied to this example.

EXAMPLES FOR PRACTICE.

Simplify each of the following expressions?

1. $\frac{3}{4}\times\frac{4}{5}$.
2. $\frac{4}{3}\times\frac{5}{6}\times\frac{7}{8}$.
3. $\frac{41}{3}\times\frac{5}{7}\times\frac{6}{7}$.
4. $\frac{3}{4}\times\frac{4}{7}\times\frac{6}{7}$.
5. $\frac{5}{4}\times\frac{8}{6}\times\frac{3}{7}$.
6. $\frac{3}{7}\times\frac{4}{7}\times\frac{3}{9}$.
7. $\frac{4}{7}\times\frac{3}{6}\times\frac{4}{9}$.
8. $\frac{68}{37}\times\frac{41}{37}\times\frac{4}{9}$.
9. $\frac{42}{24}\times\frac{41}{27}\times\frac{4}{6}$.
10. $\frac{3}{7}\times\frac{4}{9}$.
11. $\frac{4}{5}\div\frac{3}{7}$.
12. $\frac{5}{7}\div\frac{6}{7}$.
13. $\frac{5}{8}\div\frac{9}{6}$.
14. $\frac{5}{8}\div\frac{3}{7}$.
15. $\frac{4}{5}\div\frac{6}{8}$.
16. $\frac{5}{7}\div\frac{6}{7}$.
17. $\frac{4}{7}\div\frac{4}{5}$.
18. $\frac{3}{9}\div\frac{6}{7}$.
19. $\frac{3}{4}\times\frac{4}{9}\div\frac{6}{8}$.
20. $\frac{3}{4}\times\frac{5}{7}\div\frac{6}{8}$.
21. $\frac{3}{7}\times\frac{5}{7}\div\frac{3}{6}$.
22. $\frac{4}{7}\div\frac{4}{9}\times\frac{3}{7}$.
23. $\frac{5}{7}\div\frac{6}{7}\times\frac{3}{8}$.
24. $\frac{4}{7}\div\frac{8}{3}\times\frac{6}{7}$.
25. $\frac{4}{9}\div\frac{4}{7}\times\frac{8}{7}$.
26. $\frac{4}{9}\div\frac{6}{13}\times\frac{4}{9}$.
27. $\frac{5}{3}\div\frac{7}{4}\times\frac{6}{7}$.

QUESTIONS.—What is time? (97.) Repeat the table. (97., *a.*) Repeat the divisions of the calendar year. (97., *b.*) What is the use of the table of circular motion? (98.) Repeat the table. (98., *a.*) Repeat the Miscellaneous table. (99.) What is Reduction Descending? (80.) Give rule. (100., *b.*) What is Reduction Ascending? (81.) Give rule. (101., *b.*) What is the rule for Compound Addition? (102., *h.*) What is the rule for Compound Subtraction? (103., *g.*) Give the analysis of an example in Multiplication of Compound Numbers. (104., *a.*) In Division of Compound Numbers. (105., *a.*)

LESSON IV.

Simplify the following expressions, and cancel the common factors:—

ILLUSTRATION.

$$\overset{(a.)}{\tfrac{3}{4}\times\tfrac{6}{7}\div\tfrac{4}{7}\times\tfrac{4}{9}\div\tfrac{3}{8}\times\tfrac{5}{6}}=\overset{(b.)}{3\div4\times6\div7\div4\times7\times4\div9\div3\times8\times5\div6};$$

but $\overset{(b.)}{3\div4\times6\div7\div4\times7\times4\div9\div3\times8\times5\div6}=\overset{(c.)}{\tfrac{3}{4}\times\tfrac{6}{7}\times\tfrac{7}{4}\times\tfrac{4}{9}\times\tfrac{8}{3}\times\tfrac{5}{6}}$.

Comparing (*a.*) $\tfrac{3}{4}\times\tfrac{6}{7}\div\tfrac{4}{7}\times\tfrac{4}{9}\div\tfrac{3}{8}\times\tfrac{5}{6}$ with

(*c.*) $\tfrac{3}{4}\times\tfrac{6}{7}\times\tfrac{7}{4}\times\tfrac{4}{9}\times\tfrac{8}{3}\times\tfrac{5}{6}$, we find that by inverting the fractional divisors of (*a.*) and changing them to multipliers, we obtain the equivalent expression (*c.*); hence the following

Rule.*—*Simplify the expression by inverting the fractional divisors and changing them to multipliers; then cancel, and multiply the remaining terms together.*

MODEL OPERATION.

$$\frac{3}{4}\times\frac{6}{7}\div\frac{4}{7}\times\frac{4}{9}\div\frac{3}{8}\times\frac{5}{6}=\frac{\cancel{3}}{\cancel{4}}\times\frac{\cancel{6}}{\cancel{7}}\times\frac{\cancel{7}}{\cancel{4}}\times\frac{\cancel{4}}{9}\times\frac{\cancel{8}^2}{\cancel{3}}\times\frac{5}{\cancel{6}},=\frac{10}{9},=1\tfrac{1}{9}, \text{ Ans.}$$

EXAMPLES FOR PRACTICE.

28. $\tfrac{3}{4}\times\tfrac{6}{7}\div\tfrac{4}{7}\times\tfrac{4}{8}\div\tfrac{3}{7}\times\tfrac{4}{6}=$what?
29. $\tfrac{4}{6}\times\tfrac{3}{7}\div\tfrac{6}{7}\times\tfrac{4}{9}\div\tfrac{6}{7}\times\tfrac{8}{7}=$what?
30. $\tfrac{3}{5}\times\tfrac{3}{6}\times\tfrac{4}{9}\div\tfrac{5}{8}\div\tfrac{8}{9}\times\tfrac{3}{7}=$what?
31. $\tfrac{6}{7}\times\tfrac{3}{7}\div\tfrac{4}{8}\div\tfrac{3}{8}\div\tfrac{8}{9}\times\tfrac{7}{5}=$what?
32. $\tfrac{6}{7}\times\tfrac{5}{8}\div\tfrac{8}{9}\times\tfrac{3}{5}\times\tfrac{5}{9}\times\tfrac{3}{8}=$what?
33. $\tfrac{5}{9}\times\tfrac{3}{4}\div\tfrac{6}{7}\times\tfrac{8}{3}\times\tfrac{5}{7}\times\tfrac{6}{3}=$what?
34. $4\div6\times\tfrac{8}{9}\div3\times5\div6\times7\div\tfrac{3}{9}=$what?
35. $4\times\tfrac{3}{4}\div\tfrac{6}{7}\times\tfrac{5}{8}\div\tfrac{3}{4}\times\tfrac{7}{8}\times\tfrac{2}{3}=$what?
36. $\tfrac{5}{7}\div\tfrac{8}{3}\times\tfrac{9}{7}\times\tfrac{5}{8}\div\tfrac{8}{3}\times\tfrac{6}{7}\div\tfrac{4}{3}=$what?
37. $\tfrac{4}{4}\div\tfrac{8}{3}\times\tfrac{5}{7}\times\tfrac{3}{7}\times\tfrac{8}{9}\times\tfrac{3}{7}\div\tfrac{6}{3}=$what?

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*Note.—The analysis from which this rule is deduced is given in Lesson III

38. $\frac{6}{7}\div\frac{8}{9}\div\frac{6}{7}\times\frac{3}{8}\div\frac{7}{5}\times\frac{6}{8}\div\frac{8}{3}$=what?
39. $\frac{4}{7}\div\frac{3}{9}\div\frac{8}{7}\times\frac{6}{8}\div\frac{3}{7}\times\frac{5}{8}\div\frac{8}{3}$=what?
40. $\frac{6}{7}\times\frac{8}{3}\div\frac{5}{9}\div\frac{6}{7}\times3\div5\times3\times2$=what?
41. $\frac{8}{9}\div6\times3\div8\div9\times3\times4\div\frac{6}{7}$=what?
42. $4\div\frac{3}{7}\times\frac{8}{9}\div\frac{6}{7}\times\frac{3}{8}\div\frac{5}{9}\div\frac{6}{7}\div8$=what?
43. $5\times\frac{3}{4}\times\frac{5}{8}\div\frac{6}{8}\div\frac{5}{9}\div\frac{8}{6}\times\frac{5}{9}\div\frac{7}{9}$=what?
44. $3\times\frac{4}{7}\div\frac{6}{8}\div\frac{4}{7}\div\frac{8}{9}\times\frac{8}{3}\times\frac{4}{7}\times\frac{8}{9}$=what?
45. $\frac{3}{7}\times\frac{5}{8}\div\frac{6}{7}\div\frac{3}{9}\times\frac{5}{7}\times\frac{8}{7}\times\frac{8}{6}\times\frac{8}{7}$=what?
46. $\frac{3}{5}\times\frac{6}{8}\div7\times\frac{8}{7}\times\frac{3}{4}\times\frac{3}{7}\times\frac{5}{8}\times\frac{6}{7}$=what?
47. $\frac{3}{8}\times\frac{9}{7}\times\frac{21}{3}\div\frac{21}{3}\times\frac{46}{3}\div\frac{46}{3}$=what?
48. $\frac{4}{5}\div\frac{4}{5}\times\frac{3}{11}\div\frac{3}{11}\times\frac{37}{5}\div\frac{37}{5}$=what?
49. $\frac{4}{8}\div\frac{3}{7}\times\frac{4}{3}\div\frac{8}{6}\times\frac{5}{11}\times\frac{11}{5}$=what?
50. $\frac{4}{8}\times\frac{3}{7}\div\frac{4}{3}\times\frac{8}{6}\div\frac{5}{11}\div\frac{11}{5}$=what?
51. $\frac{3}{9}\times\frac{6}{7}\times\frac{8}{9}\div\frac{6}{7}\times\frac{3}{8}\times\frac{47}{12}$=what?
52. $\frac{3}{9}\div\frac{6}{7}\times89\times\frac{6}{7}\div\frac{3}{8}\div\frac{47}{12}$=what?
53. $\frac{4}{7}\div\frac{8}{9}\div\frac{6}{7}\div\frac{4}{7}\div\frac{8}{9}\div\frac{6}{7}$=what?
54. $\frac{4}{7}\times\frac{8}{9}\times\frac{6}{7}\times\frac{4}{7}\times\frac{8}{5}\times\frac{6}{7}$=what?
55. $3\div3\times\frac{46}{3}\div\frac{3}{46}\times\frac{3}{8}\div\frac{8}{3}$=what?

QUESTIONS.—What is an integer? (106.) What is an odd number? (107.) What is an even number? (108.) What is a prime number? (109.) What is a composite number? (110.) When are numbers said to be prime to each other? (111.) What is a prime factor? (112.) What is a composite factor? (113.) Is unity considered a factor? (113., *a.*)

LESSON V.

138. A **Fraction** may be considered as a number of equal parts of a unit.

139. The **Denomination** of a fraction is determined by the number of equal parts into which the unit is divided; as, *halves*, *fourths*, &c.

140. The **Value** of a fraction of any denomination depends upon the number of equal parts expressed by the numerator.

141. The **Reduction** of fractions is the process of changing their denominations without altering their values.

142. Reduction Ascending is the process of changing a fraction of a lower denomination to that of a higher; as, *fourths* to *halves*, *thirty-seconds* to *sixteenths*, &c.

143. Reduction Descending is the process of changing a fraction of a higher denomination to that of a lower; as, *halves* to *fourths*, *fifths* to *tenths*, &c.

144. A Fraction is said to be expressed in its *lowest terms* (or highest denomination*), when its numerator and denominator are prime to each other.

QUESTIONS.—What numbers are divisible by 2? (113., *b.*, 1.) What numbers are divisible by 3? (113., *b.*, 2.) What numbers are divisible by 5? (113., *b.*, 3.) What numbers are divisible by 7? (113., *b.*, 4.) (113., *b.*, 5.) What numbers are divisible by 7, 11, 13? (113., *b.*, 6.) Give the rule for finding the prime factors of a composite number? (114., *d.*) Give proof. (114., *e.*) What does the parenthesis denote? (116.)

LESSON VI.

MENTAL EXERCISES.

The teacher should by no means omit these mental exercises.

1. If a unit is divided into 3 equal parts, what are the parts called? If into 8 equal parts? 7? 11? 15? 18? 13? 26? 37? 95? 106? 38?

2. If a unit is divided into 75 equal parts, what will *one* of the parts be called? What will 3 parts be called? 5? 16? 23? 19? 56? 89? 45? 37? 25? 18? 37? 62? 307? 59? 32?

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*NOTE FOR THE TEACHER.—The teacher should not fail to make it plain to the pupil that the *lowest terms* of a fraction really are the *highest denomination* to which a fraction can be reduced and have an *integral* number of equal parts.

3. If a unit, or thing, be divided into 13 equal parts, what is *one* of the parts called? If into 11 equal parts? 37? 18? 29? 71? 36? 54? 86? 39? 47?

4. Which is the higher denomination, 8ths, or 3rds? 4ths, or 5ths? *Thirteenths*, or *fourteenths*? *Elevenths*, or *fifteenths*? 35ths, or 47ths? 105ths, or 104ths? 8ths, or 5ths? 13ths, or *halves*?

5. Which is the higher denomination, one *third*, or one *fourth*? $\frac{1}{9}$, or $\frac{1}{16}$? $\frac{1}{59}$, or $\frac{1}{58}$? $\frac{1}{3}$, or $\frac{1}{2}$? $\frac{4}{9}$, or $\frac{4}{17}$? $\frac{3}{8}$, or $\frac{3}{25}$? $\frac{2}{5}$, or $\frac{5}{8}$? $\frac{3}{7}$, or $\frac{8}{9}$?

6. Which fraction has the greater value, $\frac{4}{8}$, or $\frac{3}{8}$? $\frac{5}{3}$, or $\frac{9}{3}$? $\frac{3}{14}$, or $\frac{7}{14}$? $\frac{4}{63}$, or $\frac{25}{63}$? $\frac{3}{17}$, or $\frac{16}{17}$? $\frac{6}{5}$, or $\frac{6}{7}$? $\frac{9}{6}$, or $\frac{16}{6}$?

7. How many *quarters* in a *half*? How many 8ths? 16ths? 32nds? 64ths? 128ths?

Solution.—Since there are 4 *quarters* in *one* unit, in one-*half* of a unit there is one-half of 4 *quarters*, which is 2 quarters. Therefore, in 1 *half* there are 2 *quarters*.

Note.—The teacher should require the analysis of each question in the following exercises.

8. How many 8ths in a quarter? How many 16ths? 24ths? 36ths? 20ths? 28ths? 48ths? 44ths? 12ths?

9. Change $\frac{1}{2}$ to a series of lower denominations; as, $\frac{1}{2}$, $\frac{2}{4}$, $\frac{4}{8}$, $\frac{8}{16}$, $\frac{16}{32}$, $\frac{32}{64}$, $\frac{64}{128}$, &c.; change $\frac{1}{3}$ in the same manner; change $\frac{1}{4}$, $\frac{2}{3}$, $\frac{3}{4}$.

10. Which is the greater in value, $\frac{2}{8}$, or $\frac{2}{16}$? $\frac{7}{8}$, or $\frac{7}{24}$? $\frac{6}{95}$, or $\frac{30}{95}$? $\frac{9}{60}$, or $\frac{36}{60}$? $\frac{2}{21}$, or $\frac{6}{21}$? $\frac{6}{104}$, or $\frac{36}{104}$? $\frac{12}{64}$, or $\frac{24}{64}$? $\frac{4}{27}$, or $\frac{12}{27}$? $\frac{4}{16}$, or $\frac{8}{16}$? $\frac{32}{64}$, or $\frac{16}{64}$?

11. Change $\frac{2}{3}$ to an equivalent fraction of the denomination 18ths.

Solution.—Since in *one* unit there are 18 *eighteenths*, in one *third* of a unit there is one-*third* of 18 *eighteenths*, which is 6 eighteenths, and in 2 *thirds* of a unit there are 2 times 6 eighteenths, which are 12 eighteenths.

Therefore, there are $\frac{12}{18}$ in $\frac{2}{3}$ of a unit.

12. Change $\frac{2}{3}$ to an equivalent fraction of the denomination of 27ths; of 24ths; of 15ths; of 21sts; of 30ths; of 36ths; of 33rds.

13. How many tenths are equivalent to $\frac{3}{5}$? How many 15ths? 20ths? 35ths? 45ths? 50ths?

14. Reduce $\frac{3}{7}$ to 28ths; to 49ths; to 56ths; to 63rds; to 70ths.

15. Change $\frac{3}{4}$ and $\frac{2}{3}$ to equivalent fractions of the denomination *twelfths;* change $\frac{3}{4}$ and $\frac{2}{3}$ to 36ths; to 60ths; to 96ths; to 84ths; to 24ths.

16. Change $\frac{25}{36}$ to an equivalent fraction of the denomination *twelfths*.

SOLUTION.[*]—As a *twelfth* is 3 times greater than a thirty-sixth there can be only one-*third* as many *twelfths* as thirty-sixths; but one-third of 25 is $8\frac{1}{3}$.

Therefore, in $\frac{25}{36}$ there are $\frac{8\frac{1}{3}}{12}$.

17. How many 3rds in $\frac{8}{12}$? in $\frac{18}{27}$? in $\frac{14}{21}$? in $\frac{22}{33}$?

18. How many *fifths* in $\frac{9}{15}$? in $\frac{24}{40}$? in $\frac{21}{35}$? in $\frac{18}{30}$?

19. How many 7ths in $\frac{9}{21}$? in $\frac{15}{35}$? in $\frac{12}{28}$? in $\frac{24}{56}$?

20. How many 8ths in $\frac{14}{16}$? in $\frac{21}{24}$? in $\frac{42}{48}$? in $\frac{84}{96}$?

21. Change $\frac{16}{32}$ to an equivalent fraction of the *highest denomination* having an integral number of equal parts.

ANALYSIS.[†]—1. 16 is the greatest number that will divide both 16 and 32 without a remainder, and, therefore, 32nds can be changed to a denomination 16 times greater than 32nds, which is *halves*.

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[*] The teacher will find that it is difficult for the scholar to understand the *reason* of this solution unless it is illustrated by visible objects. If blocks are not at hand, an apple, potatoe, or turnip, cut into equal portions, will answer very well.

[†] Changing a fraction to the *highest denomination* is equivalent to reducing it to its *lowest terms*. From the fact that it is very confusing to the pupil to tell him that, in order to change a fraction from a *lower* to a higher denomination he must "*reduce it to lower terms*," arises much of the difficulty in the reduction of fractions. The teacher will also perceive that the reduction of fractions is analagous to the reduction of denominate numbers, for as the individual parts of a unit or thing are decreased in size or *value* the *number* of parts is increased in the same ratio. When these distinctions are properly presented, the subject of fractions will be divested of many of its perplexities.

2. Since *halves* are 16 times greater than 32nds, there can be but one-*sixteenth* as many *halves* as *thirty-seconds*, which is one *half*.

Therefore, in $\frac{16}{32}$ there is $\frac{1}{2}$.

22. In the same manner reduce the following fractions to their *lowest terms:* $\frac{25}{30}$; $\frac{2}{10}$; $\frac{5}{15}$; $\frac{6}{18}$; $\frac{11}{33}$; $\frac{8}{24}$; $\frac{9}{36}$; $\frac{9}{12}$; $\frac{15}{25}$; $\frac{8}{10}$; $\frac{14}{18}$; $\frac{8}{20}$; $\frac{22}{66}$.

23. Change the following fractions to their lowest terms: $\frac{14}{20}$; $\frac{5}{30}$; $\frac{16}{18}$, $\frac{14}{21}$; $\frac{55}{60}$; $\frac{36}{28}$; $\frac{5}{100}$.

QUESTIONS.—What is cancellation? (117.) What is a common divisor of two or more numbers? (118.) What is the greatest common divisor of two or more numbers? (119.) What is a multiple of a number? (120.) What is a common multiple of two or more numbers? (121.) What is the least common multiple of two or more numbers? (122.) What is a fraction? (123.) What is a common fraction? (124.) What is a decimal fraction? (125.) What are the terms of a fraction called? (126.)

LESSON VII.

1. How many 7ths in $5\frac{3}{7}$?

SOLUTION.—Since in 1 unit there are 7 *sevenths*, in 5 units there are 5 times 7 *sevenths*, which are 35 *sevenths*, which with 3 *sevenths* added make 38 *sevenths*.

Therefore, in $5\frac{3}{7}$ there are $\frac{38}{7}$.

2. How many 5ths in $9\frac{3}{5}$? in $8\frac{4}{5}$? in $5\frac{2}{5}$? in 7 units? in 8 units? in 10 units?

3. How many 8ths in $6\frac{3}{8}$? in $12\frac{2}{8}$? in $5\frac{5}{8}$? in 9 units?

4. How many 9ths in $5\frac{3}{9}$? in $6\frac{1}{9}$? in $7\frac{8}{9}$? in $11\frac{5}{9}$?

5. How many 16ths in $3\frac{5}{8}$?

ANALYSIS.—(1.) Change 3 units to sixteenths.

Since there are 16 *sixteenths* in one unit, in 3 units there are 3 times 16 *sixteenths*, which are 48 *sixteenths.*

(2.) Change $\frac{5}{8}$ to *sixteenths.*

Since there are 16 *sixteenths* in one unit, in 1 *eighth* of a unit there is one *eighth* of 16 *sixteenths*, which are 2 *sixteenths*, and in 5 eighths of a unit there are 5 times 2 *sixteenths*, which are 10 sixteenths.

(3.) Find the sum of the sixteenths.

Since in 3 units there are 48 sixteenths, and in $\frac{5}{8}$ there are 10 sixteenths, in both there are the sum of 48 sixteenths and 10 sixteenths, which are 58 sixteenths.

Therefore, in $3\frac{5}{8}$ there are $\frac{58}{16}$.

6. How many 15ths in $4\frac{2}{5}$? in $9\frac{2}{3}$? in $11\frac{3}{5}$? in $12\frac{1}{3}$? in $8\frac{3}{5}$? in 8 units? in 1 unit? in $4\frac{3}{15}$?

7. How many 18ths in $3\frac{5}{6}$? in $8\frac{1}{6}$? in $11\frac{2}{3}$? in $5\frac{1}{3}$? in $12\frac{7}{9}$? in $2\frac{5}{9}$? in $1\frac{8}{9}$? in 4 units? in $\frac{4}{9}$ of a unit?

8. How many units in $\frac{35}{8}$?

Solution.—Since in 8 eighths there is *one* unit, in 35 *eighths* there are as many units as 8 eighths are contained times in 35 eighths, which are 4, with a remainder of 3 eighths of a unit.

Therefore, in $\frac{35}{8}$ there are $4\frac{3}{8}$.

9. How many units in $\frac{47}{3}$? in $\frac{89}{7}$? in $\frac{63}{5}$? in $\frac{11}{2}$? in $\frac{68}{9}$? in $\frac{5}{8}$? in $\frac{13}{7}$? in $\frac{21}{4}$? in $\frac{26}{5}$? in $\frac{36}{5}$? in $\frac{27}{1}$?

10. How many units in $\frac{37}{3}$? in $\frac{46}{8}$? in $\frac{44}{11}$? in $\frac{38}{3}$? in $\frac{28}{7}$?

11. How many oranges in $\frac{86}{12}$ of an orange? in $\frac{29}{7}$? in $\frac{63}{9}$? in $\frac{51}{8}$? in $\frac{48}{6}$? in $\frac{37}{4}$?

12. How many 7ths of an apple in $3\frac{3}{7}$ apples? in $5\frac{2}{7}$ apples? in $4\frac{5}{7}$? in $9\frac{2}{7}$? in $11\frac{5}{7}$? in $6\frac{4}{7}$?

13. How many *twelfths* of a dollar in $\$3\frac{1}{4}$? in $\$5\frac{5}{4}$? in $\$11\frac{1}{2}$? in $\$8\frac{1}{2}$? in $\$6\frac{5}{6}$? in $\$7\frac{3}{4}$? in \$8? in \$11?

14. How many halves in $\frac{4}{8}$ of a dollar? in $\$3\frac{4}{5}$? in $\$11\frac{8}{16}$? in $\$12\frac{3}{8}$? in $\$9\frac{5}{10}$? in $\$4\frac{7}{14}$? in $\$6\frac{3}{6}$?

15. How many 3rds in $\frac{3}{9}$ of a dollar? in $\$\frac{6}{18}$? in $\$\frac{5}{15}$? in $\$8\frac{18}{54}$? in $\$6\frac{8}{24}$? in $\$\frac{7}{21}$? in $11? in $\$9\frac{12}{36}$?

QUESTIONS.—What does the denominator of a fraction show? (127.) What does the numerator show? (128.) What is a proper fraction? (129.) What is an improper fraction? (130.) What is a mixed number? (131.) What is a simple fraction? (131., *a.*) What is a complex fraction? (132.) What is a compound fraction? (133.) Of what are fractions indications? (134.) To what does the numerator answer in division? (135.) The denominator? (135.)

LESSON VIII.

REDUCTION OF FRACTIONS.

145. To reduce a fraction to its *lowest terms*, or to a simple fraction of the *highest denomination* that can be expressed by an integral number of equal parts

Change $\frac{48}{54}$ to its lowest terms.*

MODEL OPERATION.

$$\frac{48 \div 3}{54 \div 3} = \frac{16 \div 2}{18 \div 2} = \frac{8}{9} \text{ Ans.}$$

(*a.*) ANALYSIS.—$\frac{48}{54}$ is not expressed in its lowest terms, because the numerator and denominator are not prime to each other, being divisible by the common factor 3; therefore:—

1. Change $\frac{48}{54}$ to a denomination 3 times greater than 54ths, which is 18ths.

Since 18ths are 3 times greater than 54ths, there can be only one *third* as many 18ths as 54ths, which is 16 eighteenths.

$\frac{16}{18}$ is not expressed in its *lowest terms* because both the numerator and denominator can be divided by the common factor 2; therefore:—

2. Change $\frac{16}{18}$ to a denomination 2 times larger than 18ths, which is *ninths.*

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* **NOTE FOR THE TEACHER.**—The teacher should, perhaps, use the expression lowest terms, in order to prevent the use of a long and awkward phrase.

Since 9ths are 2 times greater than 18ths, there can be but half as many 9ths as 18ths, which is 8 *ninths.*

As there is no factor common to both terms of the fraction $\frac{8}{9}$, it is, therefore, expressed in its *lowest terms.*

(*b.*) RULE.—(The pupil should be required to write a rule for each of the following cases in fractions, to be read in the class, or presented to the teacher for inspection.)

EXAMPLES FOR PRACTICE.

56. $\frac{5}{10}$
57. $\frac{16}{18}$
58. $\frac{48}{72}$
59. $\frac{63}{105}$
60. $\frac{6}{9}$
61. $\frac{32}{48}$
62. $\frac{55}{121}$
63. $\frac{126}{162}$
64. $\frac{240}{312}$
65. $\frac{272}{425}$
66. $\frac{384}{1152}$
67. $\frac{435}{957}$
68. $\frac{1740}{2900}$
69. $\frac{504}{2142}$
70. $\frac{6465}{7335}$
71. $\frac{1001}{3003}$
72. $\frac{402}{926}$
73. $\frac{5484}{9168}$

QUESTIONS.—When both the numerator and denominator are increased in an equal ratio, how is the value of the fraction affected? (136., *a.*) When both are decreased? (136., *b.*) When the denominator alone is increased? (136., *c.*) When the denominator alone is decreased? (136., *d.*) When the numerator alone is increased? (136., *e.*) When the numerator alone is decreased? (136., *f.*) How is the denomination of a fraction determined? (138.) What is the reduction of fractions? (140.) What is reduction ascending? (141.) What is reduction descending? (142.) When is a fraction said to be reduced to its lowest terms? (143.) What is the difference between an odd and an even number? (107.) (108.) Between prime and expressed in its lowest terms? (143.) What is the difference between an odd and an even number? (107.) (108.) Between prime and composite numbers? (109.) (110.)

LESSON IX.

146. To reduce an improper fraction to an integral or mixed number.

Change $\frac{178}{17}$ to a mixed number.

MODEL OPERATION.

$$\frac{178}{17} = \frac{178}{17} \div \frac{17}{17}, = 10\frac{8}{17}.\ \text{Ans.}$$

Solution.—Since in 17 *seventeenths* there is one unit, in 178 seventeenths there are as many units as 17 *seventeenths* are contained times in 178 seventeenths, which are $10\frac{8}{17}$.

Therefore, $\frac{178}{17}$ are equal to $10\frac{8}{17}$.

74. Reduce $\frac{96}{8}$ to an integral number.

75. Reduce $\frac{1476}{3}$ to an integral or mixed number.

76. Change $\frac{374}{6}$, $\frac{463}{7}$, $\frac{864}{9}$, and $\frac{341}{7}$, to integral or mixed numbers.

77. How many whole apples in $\frac{374}{4}$? in $\frac{5786}{91}$? in $\frac{347}{13}$?

78. How many whole oranges in $\frac{4678}{9}$? in $\frac{3967}{4}$? in $\frac{3478}{361}$?

79. Change $\frac{347}{4}$, $\frac{384}{6}$, $\frac{91874}{364}$, and $\frac{386428}{41162}$, to integral or mixed numbers.

80. Change $\frac{307}{4}$, $\frac{30702}{910}$, $\frac{30869}{4183}$, and $\frac{3042}{61}$ to integral or mixed numbers.

Questions.—Are composite numbers ever prime to each other? (111.) What is the difference between prime, and composite factors? (112.) (113.) Is unity ever considered as a factor? (113., *a.*) What is the difference between an integral number and a fractional number? (106.) (123.) What does the parenthesis, or vinculum denote? (116.) What is the difference between the common and the greatest common divisor of two or more numbers? (118.) (119.) Does a multiple of a number contain all the factors of that number? (120.) Does the least common multiple of two or more numbers contain the highest powers of all the different factors of these numbers? (122.)

LESSON X.

147. To reduce an integral or mixed number to an improper fraction.

Change $43\frac{3}{4}$ to quarters, or reduce $43\frac{3}{4}$ to an improper fraction.

MODEL OPERATION.

$$\begin{array}{r} 43+\frac{3}{4} \\ 4 \text{ qr.} \\ \hline 172 \text{ “} \\ 3 \text{ “} \\ \hline 175 \text{ “} = \frac{175}{4} \text{ ans.} \end{array}$$

(b.)

$$\frac{4\times43}{4}=\frac{172}{4}+\frac{3}{4}=\frac{175}{4}$$

ANALYSIS.—Since in one unit there are 4 *quarters*, in 43 units there are 43 times 4 *quarters*, which are 172 quarters, which with 3 quarters added, are 175 quarters.

Therefore, in 43$\frac{3}{4}$ there are $\frac{175}{4}$.

81. Reduce 39$\frac{4}{7}$ to 7ths.
82. Reduce 46$\frac{3}{8}$ to 8ths.
83. Reduce 97$\frac{4}{6}$ to 6ths.
84. Reduce 157$\frac{3}{16}$ to 16ths.
85. Reduce 416$\frac{3}{11}$ to 11ths.
86. Reduce 301$\frac{3}{8}$ to 8ths.
87. Reduce 507$\frac{4}{26}$ to 26ths.

88. Reduce 46$\frac{3}{7}$ to an improper fraction.
89. Reduce 387$\frac{4}{7}$ to an improper fraction.
90. Reduce 471$\frac{3}{11}$ to an improper fraction.
91. Reduce 317$\frac{5}{36}$ to an improper fraction.
92. Reduce 417$\frac{3}{11}$ to an improper fraction.
93. Reduce 371$\frac{3}{5}$ to an improper fraction.
94. Reduce 416$\frac{13}{75}$ to an improper fraction.

QUESTIONS.—What is the rule for finding the least common multiple of two or more numbers when part are factors of the rest? ANS.—*Cancel the factors, and find the least common multiple of the largest number and the other numbers remaining.* What is the least common multiple of 18, 24, 9, 4, 12, 10, 6, and 5? ANS.—*Cancel* 9, 4, 12, 6, *and* 5, *and find the least common multiple of* 18, 24, *and* 10; *for* 9 *is a factor of* 18; 4, 12, 6, *are factors of* 24; *and* 5 *is a factor of* 10.

LESSON XI.

147. To change an integral or mixed number to a given denomination.

Reduce 5$\frac{4}{9}$ *to* 27ths.

MODEL OPERATION.

$5\frac{4}{9}$

27 twenty-sevenths.

135 " "

12 " "

147 " " $= \frac{147}{27}$, Ans.

ANALYSIS.—1. Change 5 units to 27ths.

Since in one unit there are 27 *twenty-sevenths*, in 5 units there are 5 times 27 *twenty-sevenths*, which are 135 *twenty-sevenths*.

2. Change $\frac{4}{9}$ of a unit to 27ths.

Since in one unit there are 27 *twenty-sevenths*, in one *ninth* of a unit there is one *ninth* of 27 *twenty-sevenths*, which is 3 *twenty-sevenths;* and in 4 *ninths* there are 4 times 3 *twenty-sevenths*, which are 12 *twenty-sevenths*, which added to 135 *twenty-sevenths* make 147 *twenty-sevenths*.

Therefore, in $5\frac{4}{9}$ there are $\frac{147}{27}$.

95. Reduce $33\frac{4}{9}$ to 27ths.
96. Reduce $15\frac{3}{7}$ to 21sts.
97. Reduce $11\frac{9}{12}$ to 108ths.
98. Reduce 437 to 11ths.
99. Reduce $434\frac{2}{9}$ to 603rds.
100. Reduce 4137 to 986ths.
101. Reduce $\frac{4037}{2}$ to 98ths.
102. Reduce 104 to 1104ths.
103. Reduce $307\frac{22}{68}$ to 340ths.
104. Reduce $37\frac{5}{9}$ to 63rds.
105. Reduce $417\frac{3}{25}$ to 450ths.
106. Reduce $417\frac{3}{25}$ to 450ths.

107. How many 232nds in $\frac{3}{29}$ of an apple?
108. How many 2800ths in $\frac{4}{112}$ of an orange?
109. How many 121sts in $\frac{196}{11}$?
110. How many 36ths in $\frac{34}{9}$ of a peach?
111. How many 7533rds in $417\frac{2}{9}$?
112. How many 588ths in $217\frac{3}{7}$?
113. How many 16ths in $14\frac{3}{4}$ lemons?
114. How many 28ths in $63\frac{5}{7}$ melons?

QUESTIONS.—What is said of a fraction when no number will divide both the numerator and denominator without a remainder? (143.) What is said of a fraction when the numerator and denominator are prime to each other? (143.) What is reduction ascending? (141.) What is reduction descending? (142.) Is reducing a fraction to its lowest terms reduction ascending or descending? Which is the higher denomination, 5ths, or 7ths? 9ths or 13ths? Which is of the higher denomination, $\frac{4}{8}$, or $\frac{7}{13}$? What is a simple fraction? (131., *a.*) When the numerator and denominator have a common factor, can the fraction be changed to a higher denomination? In reduction ascending, do we multiply, or divide the terms of the fraction? In reduction descending? Are 8ths greater, or less than 7ths?

LESSON XII.

149. ADDITION OF FRACTIONS.

150. Fractions are said to have a *common denominator* when they are of the same denomination.

MENTAL EXERCISES.

1. What is the sum of $\frac{3}{4}$, $\frac{2}{4}$, $\frac{1}{4}$, $\frac{2}{4}$, and $\frac{2}{4}$?
2. What is the sum of $\frac{1}{9}$, $\frac{7}{9}$, $\frac{16}{9}$, $\frac{11}{9}$, and $\frac{5}{9}$?
3. Add together $\frac{3}{11}$, $\frac{7}{11}$, $\frac{2}{11}$, and $\frac{4}{11}$.
4. What is the sum of $\frac{3}{4}$ and $\frac{4}{5}$?

ANALYSIS.—1. Change the fractions to the same denomination.

As 4ths can not be added to 5ths, change both to the same denomination. For convenience, change them to 20ths, as 20 is the least number that can be divided by both 4 and 5 without a remainder. $\frac{3}{4}$ equals $\frac{15}{20}$, and $\frac{4}{5}$ equals $\frac{16}{20}$.

2. Find the sum.

The sum of $\frac{15}{20}$ and $\frac{16}{20}$, is $\frac{31}{20}$, equal to $1\frac{11}{20}$.

Therefore, the sum of $\frac{3}{4}$ and $\frac{4}{5}$, is $1\frac{11}{20}$.

5. Add together $\frac{3}{4}$ and $\frac{4}{5}$; $\frac{3}{2}$ and $\frac{5}{8}$; $\frac{2}{5}$ and $\frac{3}{4}$; $\frac{2}{3}$ and $\frac{1}{2}$.

6. Add together 1 and $\frac{3}{4}$; 3 and $\frac{4}{5}$; $\frac{3}{4}$ and $\frac{7}{8}$; $5\frac{2}{3}$ and 4.

7. What is the sum of $\frac{3}{4}$ of an apple, $2\frac{1}{2}$ apples, 1 apple, and $5\frac{1}{3}$ apples?

8. What is the sum of $5\frac{2}{3}$ peaches, $11\frac{5}{7}$ peaches, and 3 peaches?

LESSON XIII.

What is the sum of $\frac{3}{4}$, $\frac{5}{12}$, $\frac{9}{10}$, $\frac{5}{9}$, and $\frac{3}{5}$?

MODEL OPERATION.

(*a*.)

$$\begin{array}{l}
2)4,\ 12,\ 10,\ 9,\ 5. \\ \hline
2)2,\ \ 6,\ \ 5,\ 9,\ 5. \\ \hline
3)1,\ \ 3,\ \ 5,\ 9,\ 5. \\ \hline
5)1,\ \ 1,\ \ 5,\ 3,\ 5. \\ \hline
\quad 1,\ \ 1,\ \ 1,\ 3,\ 1.
\end{array}$$

$$2\times2\times3\times5\times3=180.$$

(*b*)

$$\begin{array}{l}
\frac{3}{4}=135 \text{ 180ths.} \\
\frac{5}{12}=\ \ 75 \quad \text{“} \\
\frac{9}{10}=162 \quad \text{“} \\
\frac{5}{9}=100 \quad \text{“} \\
\frac{3}{5}=108 \quad \text{“} \\ \hline
\quad \frac{580}{180}=3\frac{40}{180},=3\frac{2}{9}.
\end{array}$$

ANALYSIS.—The least common multiple of the denominators of several fractions indicates the most convenient denomination to which they can all be reduced*; therefore,

1. (*a*.) Find the least common multiple of the denominators, 4, 12, 10, 9, and 5, which is 180.

2. (*b*.) Change the fractions $\frac{3}{4}$, $\frac{5}{12}$, $\frac{9}{10}$, $\frac{5}{9}$, and $\frac{3}{5}$, to equivalent fractions of the denomination 180ths; $\frac{3}{4}$ equals $\frac{135}{180}$; $\frac{5}{12}$ equals $\frac{75}{180}$; $\frac{9}{10}$ equals $\frac{162}{180}$; $\frac{5}{9}$ equals $\frac{100}{180}$; and $\frac{3}{5}$ equals $\frac{108}{180}$.

3. Find the sum total.

The sum of $\frac{135}{180}$, $\frac{75}{180}$, $\frac{162}{180}$, $\frac{100}{180}$, and $\frac{108}{180}$, is $\frac{580}{180}$, equal to $3\frac{40}{180}$, equal to $3\frac{2}{9}$.

Therefore, the sum of $\frac{3}{4}$, $\frac{5}{12}$, $\frac{9}{10}$, $\frac{5}{9}$, and $\frac{3}{5}$, is $3\frac{2}{9}$.

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*NOTE.—Since the denominators of several fractions must be the divisors of the number indicating the denomination, it is evident that the least common multiple of the denominators indicates the highest denomination to which they can all be reduced, each having an integral number of equal parts.

115. $\frac{5}{8}+\frac{1}{12}+\frac{3}{7}=$what?
116. $\frac{3}{5}+\frac{4}{6}+\frac{3}{9}=$what?
117. $4+\frac{5}{6}+\frac{3}{4}+\frac{3}{8}=$what?
118. $5+\frac{3}{9}+4\frac{6}{7}+\frac{3}{7}=$what?
119. $5+\frac{3}{7}+\frac{4}{23}+\frac{3}{7}=$what?
120. $\frac{8}{3}+\frac{46}{3}+8+3\frac{2}{7}=$what?
121. $\frac{5}{9}+\frac{46}{38}+7\frac{4}{9}+4\frac{2}{17}=$what?
122. $\frac{3}{7}+\frac{68}{7}+\frac{3}{4}+\frac{8}{3}=$what?
123. $4+\frac{37}{4}+\frac{48}{3}+\frac{6}{7}=$what?
124. $\frac{5}{3}+\frac{3}{6}+\frac{7}{12}+\frac{6}{12}=$what?
125. $\frac{5}{7}+\frac{3}{9}+\frac{8}{21}+\frac{2}{14}=$what?
126. $11+\frac{5}{9}+\frac{8}{3}+\frac{5}{8}=$what?
127. $34+\frac{2}{3}+\frac{4}{11}+\frac{3}{17}=$what?
128. $\frac{6}{27}+\frac{5}{31}+\frac{6}{51}+\frac{3}{61}=$what?

QUESTIONS.—What is a common fraction? (124.) What is a decimal fraction? (125.) What are the terms of a fraction? (126.) What does the denominator of a fraction show? (127.) What does the numerator show? (128.) What is a proper fraction? (129.) An improper fraction? (130.) A mixed number? (131.) A simple fraction? (131., *a.*) A complex fraction? (132.) A compound fraction? (133.)

LESSON XIV.

PRACTICAL EXAMPLES IN REDUCTION AND ADDITION OF FRACTIONS.

A man bought a horse for \$73$\frac{3}{9}$, a carriage for \$97$\frac{5}{9}$, a set of harness for \$37$\frac{5}{12}$; what was the cost of the whole?

MODEL OPERATION.

$$\begin{array}{rcll}
\$73\frac{3}{9} & = & 73\frac{12}{36}, & \text{cost of horse.}\\
97\frac{5}{9} & = & 97\frac{20}{36}, & \text{" carriage.}\\
37\frac{5}{12} & = & \underline{37\frac{15}{36}}, & \text{" harness.}\\
 & & 207\frac{47}{36} = \$208\frac{11}{36}, & \text{total cost.}
\end{array}$$

ANALYSIS.—1. Reduce the several fractions to others of a common denomination.

\$73$\frac{3}{9}$=73$\frac{12}{36}$; \$97$\frac{5}{9}$=97$\frac{20}{36}$; \$37$\frac{5}{12}$=37$\frac{15}{36}$.

2. Find the sum total.

The sum of \$73$\frac{12}{36}$, \$97$\frac{20}{36}$, and \$37$\frac{15}{36}$, is \$207$\frac{47}{36}$. \$$\frac{47}{36}$ equals \$1$\frac{11}{36}$, which added to \$207 make \$208$\frac{11}{36}$.

Therefore, if a man bought a horse for \$73$\frac{3}{9}$, a carriage, &c.

129. A merchant purchased 4 pieces of silk; the first contained $37\frac{5}{9}$ yds., the second $43\frac{4}{7}$ yds., the third $27\frac{3}{11}$ yds., and the fourth $38\frac{1}{12}$ yds.; how many yards did all the pieces contain?

130. If a book cost $12\frac{5}{9}$ cts., a slate $10\frac{3}{8}$ cts., a pencil $\frac{5}{12}$ of a cent, and a sponge 5 cts.; how much did they all cost?

131. A butcher bought, at one time, $129\frac{3}{7}$ lbs. of beef; at another, $439\frac{4}{11}$ lbs.; at another, 372 lbs.; and, at another, $473\frac{4}{13}$ lbs.; how many pounds did he purchase in all?

132. A boy spent $\frac{5}{13}$ of a dollar for a geography, $\$\frac{8}{17}$ for a spelling-book, and $\$\frac{10}{87}$ for a slate; how much did he spend for all?

133. If I have $\frac{3}{7}$ of a pound of sugar in one paper, $\frac{2}{11}$ lbs. in another, 3 lbs. in another, and $2\frac{4}{5}$ lbs. in another; how many 385ths of a pound have I in all?

134. I have three strings; the first is $2\frac{3}{4}$ inches long, the second is $2\frac{1}{2}$ inches long, and the third is $\frac{5}{8}$ of an inch long; how many *twelfths* of an inch are there in the three strings?

135. I have three apples; the 1st weighs $5\frac{2}{9}$ oz., the 2nd $11\frac{7}{8}$ oz., and the 3rd $12\frac{3}{4}$ oz.; how many 72nds of an ounce do the three apples weigh?

136. What is the size of the largest equal pieces into which $\frac{3}{7}$, $\frac{4}{5}$, and $\frac{1}{2}$ of a pear can be cut? How many pieces in each part? How many pieces in all the parts?

137. What is the size of the largest equal pieces into which $\frac{3}{4}$, $\frac{7}{8}$, and $\frac{4}{9}$ of an orange, can be divided? How many pieces in all the parts?

138. Reduce $\frac{3}{7}$, $4\frac{2}{6}$, $\frac{5}{9}$, $37\frac{1}{4}$, to equivalent fractions having a common denominator.

139. What is the size of the largest pieces that can be made from $\frac{36}{144}$ of an apple, each piece to be of equal size? How many pieces will there be?

QUESTIONS.—What do fractions indicate? (134.) Which term of a fraction answers to the dividend? (135.) Which to the divisor? (135.) What, then, is the meaning of $\frac{13}{4}$? ANS.—13÷4. What is the meaning of $\frac{5}{11}$? ANS.—5÷11. In what other way can 15÷8 be expressed? ANS.—$\frac{15}{8}$. What is the value of a fractional expression? ANS.—*The number of times the denominator is contained in the numerator.* What is the value of an expression of unperformed division; as, 13÷6? ANS.—*The number of times the divisor is contained in the dividend.* Give the fractional forms of the following expressions:—3÷4; 4÷8; 9÷8; 13÷11; 6÷27; 9÷16; 9÷3; 23÷37.

LESSON XV.

151. SUBTRACTION OF FRACTIONS.

From $\frac{13}{16}$ take $\frac{7}{12}$.

MODEL OPERATION.

(a.)	(b.)
2)16,12	$\frac{13}{16}$=39 48ths, minuend.
2)8, 6	$\frac{7}{12}$=28 " subtrahend.
4, 3	$\frac{11}{48}$, remainder, or difference.

2×2×4×3=48, l. c. mult.

ANALYSIS.—1. (*a.*) For convenience find the least common multiple of the denominators, 16 and 12, which is 48.

2. (*b.*) Change $\frac{13}{16}$ and $\frac{7}{12}$ to equivalent fractions having the denomination of 48ths, and subtract.

Since *twelfths* can not be subtracted from *sixteenths*, change both 16ths and 12ths to 48ths; $\frac{13}{16}$=39 48ths; $\frac{7}{12}$=28 48ths; and $\frac{39}{48}-\frac{28}{48}$ leaves a remainder of $\frac{11}{48}$.

Therefore, the difference between $\frac{13}{16}$ and $\frac{7}{12}$ is $\frac{11}{48}$.

140. From $\frac{7}{12}$ take $\frac{2}{11}$.
141. From $\frac{11}{19}$ take $\frac{7}{19}$.
142. From $\frac{13}{18}$ take $\frac{7}{9}$.
143. From $\frac{15}{11}$ take $\frac{3}{8}$.
144. From $\frac{8}{13}$ take $\frac{11}{43}$.
145. From $\frac{47}{2}$ take $\frac{3}{4}$.
146. $\frac{4}{9}-\frac{3}{15}=$how many?
147. $\frac{4}{7}-\frac{4}{8}=$how many?
148. $\frac{16}{17}-\frac{17}{19}=$how many?
149. $\frac{5}{11}-\frac{5}{12}=$how many?
150. $\frac{6}{12}-\frac{5}{11}=$how many?
151. *$3\frac{1}{4}-\frac{4}{13}=$how many?

Questions.—If the divisor is *less* than the dividend, is the value of the expression greater, or less, than a unit? Ans.—*Greater.* If the divisor is *greater* than the dividend, is the value greater, or less, than a unit? Ans.—*Less.* If the numerator is greater than the denominator, is the value of the expression greater, or less, than a unit? (130.) If the denominator is greater than the numerator, is the value of the expression greater, or less, than a unit? (129.) If both numerator and denominator be increased in an equal ratio, how is the value of the fraction affected? (136., *a.*)

LESSON XVI.

	(*a.*)	(*b.*)	(*c.*)	(*d.*)	(*e.*)
From	13	14	$15\frac{3}{4}=15\frac{6}{8}$	$13\frac{4}{5}=13\frac{24}{30}$	$47\frac{5}{11}$
Take	$3\frac{4}{7}$	$9\frac{3}{11}$	$6\frac{2}{8}=\ 6\frac{2}{8}$	$12\frac{5}{6}=12\frac{25}{30}$	13
Rem.	$9\frac{3}{7}$	$4\frac{8}{11}$	$9\frac{4}{8}$	$\frac{29}{30}$	$34\frac{5}{11}$

Analysis.—(*a.*) As $\frac{4}{7}$ can not be taken from no *sevenths*, take 1 unit from 13 units, which is equal to 7 sevenths; 4 sevenths from 7 sevenths leave 3 sevenths, which write in the remainder; 3 units from 12 units leave 9 units.

Therefore, $3\frac{4}{7}$ taken from 13 leave $9\frac{3}{7}$.

(*d.*) $13\frac{4}{5}$ equals $13\frac{24}{30}$, and $12\frac{5}{6}=12\frac{25}{30}$.

Since 25 30ths can not be taken from 24 30ths, take one unit from the 13 units; one unit is equal to 30 30ths; 30 30ths and 24 30ths equal 54 30ths; and 25 30ths from 54 30ths leave 29 30ths, which write in the remainder; 12 units from 12 units leave no units.

Therefore, $12\frac{5}{6}$ taken from $13\frac{4}{5}$ leave $\frac{29}{30}$.

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* Note.—Reduce mixed numbers to improper fractions.

EXAMPLES FOR PRACTICE.

152. $46\frac{2}{3}-2\frac{3}{11}$=how many?
153. $37\frac{4}{11}-6\frac{3}{7}$=how many?
154. $14\frac{21}{17}-5\frac{3}{9}$=how many?
155. $11-3\frac{4}{7}$=how many?
156. $37-16\frac{5}{7}$=how many?
157. $25-14\frac{5}{8}$=how many?
158. $37\frac{4}{7}-16$=how many?
159. $36\frac{5}{27}-\frac{8}{96}$=how many?
160. $49\frac{37}{468}-\frac{3}{27}$=how many?
161. $55\frac{12}{4}-3\frac{31}{7}$=how many?
162. $57-3\frac{4}{2}+\frac{6}{13}$=how many?
163. $14\frac{3}{7}-6\frac{2}{9}+\frac{5}{7}$=how many?
164. $5\frac{4}{7}-3\frac{2}{9}+4\frac{3}{7}$=how many?
165. $37\frac{4}{9}-5\frac{2}{7}+8\frac{6}{7}$=how many?
166. $42\frac{3}{13}-4\frac{7}{12}+3\frac{8}{7}$=how many?
167. $67\frac{3}{11}-15\frac{3}{7}+\frac{11}{14}$=how many?
168. $147\frac{3}{8}-86\frac{4}{9}+3\frac{4}{7}$=how many?
169. $387\frac{41}{286}-\frac{347}{132}+4\frac{173}{86}$=how many?
170. $467\frac{3}{41}-34\frac{483}{917}+2\frac{916}{372}$=how many?
171. $834\frac{3}{712}-41\frac{384}{721}+6\frac{417}{2183}$=how many?

LESSON XVII.

152. To add any two fractions, or find their difference, when each has a unit for its numerator.

What is the sum of $\frac{1}{5}$ and $\frac{1}{7}$?

MODEL OPERATIONS.

(*a.*)

$$\begin{array}{lr} \text{Sum of denominators} & 5+7=12 \\ \text{Product of } `` & 5\times7=\overline{35} \end{array} \text{ Ans.}$$

What is the difference between $\frac{1}{5}$ and $\frac{1}{7}$?

(*b.*)

$$\begin{array}{lr} \text{Difference of denominators} & 7-5=\ 2 \\ \text{Product of } `` & 7\times5=\overline{35} \end{array} \text{ Ans.}$$

ANALYSIS.—(*a.*) $\frac{1}{5}=\frac{7}{35}$; $\frac{1}{7}=\frac{5}{35}$; and the sum of $\frac{7}{35}$ and $\frac{5}{35}$ is $\frac{12}{35}$.

(*b.*) $\frac{1}{5}=\frac{7}{35}$; $\frac{1}{7}=\frac{5}{35}$; and the difference between $\frac{7}{35}$ and $\frac{5}{35}$ is $\frac{2}{35}$.

Hence the following rules:—

I. To add two fractions having units for their numerators:—*Write the sum of the denominators over their product.*

II. To find the difference between two fractions having units for numerators:—*Write the difference of the denominators over their product.*

EXERCISES FOR PRACTICE.

What is the value of each of the following expressions?

172. $\frac{1}{7}+\frac{1}{9}$.
173. $\frac{1}{13}+\frac{1}{12}$.
174. $\frac{1}{16}+\frac{1}{13}$.
175. $\frac{1}{19}+\frac{1}{86}$.
176. $\frac{1}{5}-\frac{1}{9}$.
177. $\frac{1}{8}-\frac{1}{9}$.
178. $\frac{1}{3}-\frac{1}{4}$.
179. $\frac{1}{15}-\frac{1}{18}$.
180. $\frac{1}{3}-12$.
181. $\frac{1}{187}+\frac{1}{37}$.
182. $\frac{1}{37}-\frac{1}{867}$.
183. $\frac{1}{57}+\frac{1}{89}$.
184. $\frac{1}{47}-\frac{1}{346}$.
185. $\frac{1}{67}+\frac{1}{52}$.
186. $\frac{1}{371}-\frac{1}{674}$.
187. $\frac{1}{8}+\frac{1}{872}$.

QUESTIONS.—If both numerator and denominator be decreased in an equal ratio, how is the value of the fraction affected? (136., *b.*) If the denominator alone is increased in any ratio, how is the value of the fraction affected? (136., *c.*) If the denominator alone is decreased in any ratio, how is the value of the fraction affected? (136., *d.*) If the numerator alone is increased in any ratio, how is the value of the fraction affected? (136., *e.*) If the numerator alone is decreased in any ratio, how is the value of the fraction affected? (136., *f.*)

LESSON XVIII.

PRACTICAL EXAMPLES IN ADDITION AND SUBTRACTION OF FRACTIONS.*

188. A man bought a horse for $\$39\frac{3}{18}$, and sold it for $146; what was his profit?

189. John bought $371\frac{4}{7}$ lbs. of butter, and then sold $147\frac{5}{7}$ lbs.; how many pounds had he left?

190. A man bought at one time $34\frac{3}{5}$ bbls. of flour, at another $47\frac{3}{15}$ bbls., and at another $44\frac{3}{7}$ bbls.; how many barrels did he buy in all?

191. I sold one lot of books for $\$43.75\frac{2}{3}$, another lot for $\$37.62\frac{1}{5}$, and another for $\$38.27\frac{4}{9}$; how much do my books amount to?

192. If I have $\$437\frac{3}{9}$, and pay out $\$341\frac{4}{15}$, how much have I left?

193. 11 horses are worth $\$1501\frac{3}{7}$; 41 cows are worth $\$923\frac{3}{11}$; how many dollars are the horses worth more than the cows?

194. A drover bought a yoke of oxen for $\$89\frac{3}{17}$, and paid $\$14\frac{3}{5}$ for pasture; he afterwards sold them for $\$89\frac{5}{11}$; did he gain, or lose, and how much?

195. A merchant bought 37 bales of cloth for $\$347\frac{3}{9}$, and 48 bales for $\$402\frac{9}{11}$; he then sold the whole for $1000; how much did he gain?

196. A merchant bought $37\frac{3}{7}$ barrels of flour for $\$157\frac{3}{11}$, and $143\frac{4}{7}$ barrels for $\$597\frac{3}{22}$; how many barrels did he buy, and how much did they cost him?

197. A gentleman owns a farm worth $3397, and a store worth $\$1413.47\frac{3}{8}$; how much more is the farm worth than the store?

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*NOTE.—The analysis is the same as in the simple rules.

QUESTIONS.—How is the denomination of a fraction determined? (138.) Upon what does the value of a fraction of any denomination depend? (139.) What is the reduction of fractions? (140.) What is reduction ascending? (141.) What is reduction descending? (142.) When are fractions said to be expressed in their lowest terms? (143.) What is the difference between a common multiple and the least common multiple of two or more numbers? (121.) (122.)

LESSON XIX.

198. A gentleman gave for a house and lot $7384, and for a cotton factory 9503.13\frac{4}{7}$; how much more did the factory cost than the house and lot?

199. A man valued a factory at $9410.30, and a farm at 8964.37\frac{2}{5}$; what is the difference of their values?

200. A merchant had 1987 yds. of cloth, and sold 847$\frac{3}{17}$ yds. of it; how many yards had he remaining?

201. A speculator sold a quantity of flour for $7897, and, by so doing, gained 332.463\frac{5}{19}$; what did it cost him?

202. A man sold an estate for $9873, and gained 3896.46\frac{3}{71}$; how much did the estate cost him?

203. A man sold an estate for $9873, and, by so doing, lost 3896.46\frac{3}{11}$; how much did the estate cost him?

204. A gentleman having 57897\frac{3}{8}$, gave his eldest son 26805\frac{3}{11}$; how much had he remaining?

205. Joseph had 47\frac{3}{8}$, and Weston as many lacking 8\frac{4}{15}$; how many dollars had Weston?

206. The sum of two numbers is 34172$\frac{4}{9}$; one of them is 2417$\frac{3}{7}$; what is the other?

207. If I have in different banks 141\frac{3}{7}$, 467\frac{7}{8}$, 417\frac{1}{10}$, and 348\frac{6}{11}$, how many dollars have I in all the banks?

208. If I sell 3 horses for 430\frac{3}{19}$, 3 cows for 324\frac{4}{7}$, and 55 sheep for 223\frac{3}{14}$, for how many dollars do I sell them all?

QUESTIONS.—What is a fraction? (124.) What is the difference between a common fraction and a decimal fraction? (124.) (125.) What is the difference between the uses of the numerator and those of the denominator? (127.)(128.) Are decimal fractions ever used in connection with common fractions? (124.*) Are they written differently? (124.*) Are decimal fractions usually written with a denominator? (125.) What determines the denomination of a common and a decimal fraction? (138.) What is a rule in Arithmetic? (12.) Write, on the blackboard, a rule for the reduction of a fraction to its lowest terms. (144., *b*.) Write a rule for the reduction of an improper fraction to an integral or a mixed number. (145.)

LESSON XX.

209. A farmer bought 3 loads of hay; the first contained $1473\frac{3}{7}$ lbs., the second $1834\frac{4}{2}$ lbs., and the third $2171\frac{3}{14}$ lbs.; how many pounds in the three loads?

210. The same farmer afterward sold $3479\frac{3}{11}$ lbs.; how many had he remaining?

211. James Smith had two lots of wild land; the first contained $387\frac{3}{16}$ acres, the second $417\frac{3}{11}$ acres; he sold $521\frac{3}{4}$ acres; how many acres had he remaining?

212. A lumber merchant bought $4347\frac{3}{5}$ feet of pine lumber; he then sold $2417\frac{5}{11}$ feet, and afterward bought $3874\frac{2}{22}$ feet; how many feet had he then?

213. The hind-quarters of an ox weighed $375\frac{3}{5}$ lbs. each; the two fore-quarters weighed $621\frac{3}{8}$; the hide and tallow weighed $112\frac{3}{16}$ lbs.; what was the entire weight?

214. There are two numbers whose difference is $8764\frac{3}{7}$; the greater number is $15287\frac{5}{11}$; what is the less?

215. What number is that, which, if taken from $3794\frac{3}{4}$, will leave $865\frac{5}{12}$?

216. The subtrahend is $3417\frac{4}{7}$, the difference is $9137\frac{4}{14}$; what is the minuend?

QUESTIONS.—Write on the blackboard a rule for the reduction of an integral number to a given denomination. (147.) ANS.—*Multiply the number of equal parts of the given denomination in one unit by the integral number, and the product will be the numerator of a fraction of the denomination required.* What is the rule for the reduction of a fraction to a given denomination? ANS.—*Divide the number of equal parts of the given denomination in one unit by the denominator of the fraction, and multiply the quotient by the numerator, and the product will be the numerator of a fraction of the denomination required.* Write a rule for the reduction of a mixed number to a given denomination. (147.) What is the addition of fractions? When is a fraction said to have a common denominator? (149.) What is the subtraction of fractions?

LESSON XXI.

217. The sum of two numbers is $1408\frac{3}{7}$; the less is $216\frac{5}{14}$: what is the greater?

218. The minuend is 4183, the difference is $421\frac{3}{47}$; what is the subtrahend?

219. The minuend is 4 million and 3 *seventeenths*, the difference is 3 million and 5 *thirteenths;* what is the subtrahend?

220. A man pays $300 for 100 sheep, 95^3 for a yoke of oxen, 60\frac{3}{7}$ for a horse, and 124\frac{3}{11}$ for a buggy; how much do they all cost?

221. A man bought a barrel of sugar for $30, 25 yards of broadcloth for 53\frac{4}{7}$, two barrels of molasses for 47\frac{3}{7}$; in return for these he gave 100 bushels of oats worth 65\frac{5}{12}$, a colt worth 40\frac{1}{11}$, and the balance in money; how much money did he pay?

222. A flour merchant, having 700 bbls. of flour on hand, sold $371\frac{1}{15}$ bbls. to one man, and $142\frac{1}{15}$ bbls. to another; how many barrels had he left?

223. What is the sum of $137\frac{1}{13}$ and $46\frac{1}{15}$?

224. What is the sum of $46\frac{1}{12}$ and $37\frac{1}{13}$?

225. What is the difference between $38\frac{1}{11}$ and $22\frac{1}{9}$?

226. What is the difference between $37\frac{1}{15}$ and $24\frac{1}{11}$?

227. Give the sum and the difference of $43\frac{1}{7}$ and $29\frac{1}{9}$.

228. Give the sum and the difference of $38\frac{5}{9}$ and $32\frac{1}{11}$.

229. What is the amount of the sum and the difference of $43\frac{1}{11}$ and $37\frac{1}{13}$?

Questions.—What are the rules for adding and subtracting fractions which have units for their numerators? (151.) What is a unit? (1.) What is the difference between abstract and concrete numbers? (3.) (4.) What is the difference between simple and complex problems? (8.) (9.) What is an analytical step? (10.) Illustrate the definition of a rule. (12.) Illustrate on the blackboard the definition of a sign. (13.) What is the answer called in Addition? (30.) In Subtraction? (42.) In Multiplication? (49.) In Division? (57.)

LESSON XXII.

153. MULTIPLICATION AND DIVISION OF FRACTIONS.

154. To multiply a fraction by an integral number.

(*a.*) What is the product of $\frac{3}{8}$ multiplied by 4?

(*b.*) What is the product of $\frac{4}{5}$ multiplied by 4?

MODEL OPERATIONS.

$$(a.)\ \frac{3}{8}\times4=\frac{3}{8\div4},=\frac{3}{2},=1\tfrac{1}{2},\ \text{Ans.}$$

$$(b.)\ \frac{4}{5}\times4=\frac{4\times4}{5},=\frac{16}{5},=3\tfrac{1}{5},\ \text{Ans.}$$

Analysis.—(*a.*) Decreasing the denominator of a fraction in any ratio, increases the value of the fraction in the same ratio.*

If, then, the denominator of $\frac{3}{8}$ be divided by 4, the result, $\frac{3}{2}$, will be 4 times greater than $\frac{3}{8}$.

Therefore, $\frac{3}{8}$ multiplied by 4 equals $1\frac{1}{2}$.

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*Note.—Since the denominator is a divisor, decreasing it in any ratio must increase the quotient in the same ratio.

(*b.*) 4 times 4 *fifths* are 16 *fifths*, equal to $3\frac{1}{5}$.

Therefore, $\frac{4}{5}$ multiplied by 4 equals $3\frac{1}{5}$.

Hence the

RULE.—I. *Divide the denominator of the fraction by the integral number, when it can be done without a remainder; if this is not possible, multiply the numerator, and write the result over the denominator.*

II. *Cancel common factors, and multiply as in cancellation.*

EXAMPLES FOR PRACTICE.

230. $\frac{5}{36}\times 4=$ how many?
231. $\frac{5}{37}\times 16=$ "
232. $\frac{5}{37}\times 83=$ "
233. $\frac{273}{28}\times 21=$ "
234. $\frac{383}{371}\times 32=$ "
235. $\frac{486}{21}\times 42=$ "
236. $\frac{41}{92}\times 46=$ how many?
237. $\frac{37}{56}\times 28=$ "
238. $\frac{42}{48}\times 12=$ "
239. $\frac{83}{216}\times 72=$ "
240. $\frac{47}{437}\times 21=$ "
241. $\frac{41}{50}\times 25=$ "

QUESTIONS.—How is addition proved? (36., *f.*) Subtraction? (44., *i.*) Multiplication? (53., *n.*) Division? (65., *i.*) What is the difference between addition and multiplication? (31.) (46.) Between subtraction and division? (39.) (54.) How does addition differ from multiplication? ANS.—*In addition, the numbers to be added are usually different quantities, while in multiplication they are the same, or equal quantities; thus,* 3+8+7=18, 3+3+3+3+3=15, *are examples in addition; but the latter can be contracted by multiplication; as,* 3×5=15. In multiplication, are the numbers to be added or repeated always the same in each example? ANS.—*They are.* In multiplication, what does the multiplier show? (48.) To what does the *product* in multiplication correspond in addition? ANS.—*To the sum.* Has the multiplier any corresponding term in addition? ANS.—*It has none.*

LESSON XXIII.

PRACTICAL EXAMPLES.

What will $37\frac{3}{5}$ lbs. of sugar cost, at 11 cts. per pound?

MODEL OPERATION.

$$\begin{array}{rr}
 & 37\frac{3}{9}\text{ lbs.} \\
 & 11\text{ cts.} \\
\hline
 & 407\text{ ``} \\
\frac{33}{9} = & 3\frac{2}{3} \\
\hline
 & \$4.10\frac{2}{3}\text{, Ans.}
\end{array}$$

ANALYSIS.—1. If 1 pound of sugar cost 11 cts., 37 pounds will cost 37 times 11 cts., which are 407 cts.

2. If 1 pound of sugar costs 11 cts., $\frac{3}{9}$ of a pound will cost $\frac{3}{9}$ of 11 cts.,* which are $\frac{33}{9}$, equal to $3\frac{2}{3}$ cts., which, added to 407 cts., make $410\frac{2}{3}$ cts., equal to $\$4.10\frac{2}{3}$.

Therefore, $37\frac{3}{9}$ lbs. of sugar will cost $\$4.10\frac{2}{3}$, if 1 pound cost 11 cts.

242. What will $463\frac{2}{7}$ bushels of wheat cost, at 125 cts. per bushel? at $1.35 per bushel?

243. What will be the cost of 472 hats, at $\$3.42\frac{2}{9}$ apiece?

244. What will $3412\frac{6}{11}$ yds. of cloth cost, at 50 cts. per yard?

245. What will a firkin of butter containing $90\frac{5}{7}$ pounds cost, at $25\frac{1}{2}$ cts. per pound?

246. What will 341 oranges cost, at $1\frac{4}{7}$ cts. apiece?

247. What will $17\frac{3}{8}$ pounds of flour cost, at 2 cts. per pound?

248. If the interest on one dollar for one year is $6\frac{5}{8}$ of a cent, what will be the interest on a note of $572?

249. If the interest on one dollar for one month is $\frac{7}{8}$ of

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* NOTE.—$\frac{3}{9}$ of 11 is equivalent to $\frac{3}{9}$ *times* 11, or 11 *times* $\frac{3}{9}$; for *of* has the same signification as *times*, and is used when the multiplier is a fractional number. $\frac{3}{9}$ *of* 11 is equivalent to $\frac{1}{9}$ *of* 3 *times* 11. 3 times 11 are 33, and 1 ninth of 33 is $\frac{33}{9}$.

a cent, what will be the interest on $537 for 12 months, or one year?

250. If the interest on one dollar for one month is $\frac{10}{11}$ of a cent, what is the interest on $378 for 3 years?

QUESTIONS.—How does division differ from subtraction? ANS.—*In division the divisor is taken from the dividend as many times as there are units in the quotient, and in subtraction the subtrahend is taken from the minuend only once.* Which are the corresponding terms? ANS.—*The dividend corresponds to the minuend; the divisor, to the subtrahend; the remainder in division, to the remainder in subtraction; and the quotient, to unity, which, though not expressed, indicates the number of times the subtrahend is taken from the minuend.*

LESSON XXIV.

EXAMPLES IN ADDITION, SUBTRACTION AND MULTIPLICATION OF FRACTIONS.

251. I bought, at different times, 36 lbs. of sugar at $6\frac{3}{8}$ cts. per pound, 47 lbs. of tea at $97\frac{3}{5}$ cts. per pound, $48\frac{3}{4}$ lbs. of rice at 6 cts. per pound; what was the entire cost of the articles?

MODEL OPERATION.

(*a.*)	(*b.*)	(*c.*)	(*d.*)
36 lbs.	$97\frac{3}{5}$ cts.	$48\frac{3}{4}$ lbs.	$2.29 $+(\frac{1}{2}=\frac{5}{10}.)$
$6\frac{3}{8}$ cts.	47 lbs.	6 cts.	45.87 $+(\frac{1}{5}=\frac{2}{10}.)$
216 “	679 cts.	288 “	2.92 $+(\frac{1}{2}=\frac{5}{10}.)$
$\frac{108}{8}=13\frac{1}{2}$ “	388 “	$\frac{18}{4}=4\frac{1}{2}$ “	51.09\frac{1}{5}$ $(\frac{12}{10}=1\frac{1}{5}.)$
2.29\frac{1}{2}$	4559 “	2.92\frac{1}{2}$	
	$\frac{141}{5}=28\frac{1}{5}$ “		
	45.87\frac{1}{5}$		

ANALYSIS.—(*a.*) Find the cost of the sugar.
(*b.*) Find the cost of the tea.
(*c.*) Find the cost of the rice.
(*d.*) Find the total cost.

252. The interest on one dollar for one day is $\frac{1}{12}$ of a cent; what is the interest for the same time on $347?

253. The interest on one dollar for one day is $\frac{2}{15}$ of a cent; how much is the interest on one dollar for 30 days, or one month?

254. The interest on one dollar for one day is $\frac{2}{11}$ of a cent; how much is the interest on one dollar for 12 months, or one year?

255. The interest on one dollar for one day is $\frac{3}{16}$ of a cent; how much is the interest on one dollar for 3 years?

256. The interest on one dollar for one day is $\frac{3}{11}$ of a cent; how much is the interest on one dollar for 2 years and 5 months?

257. If the interest on one dollar for one day is $\frac{4}{5}$ of a cent, what is the interest on one dollar for 3 years 4 months 2 days?

258. I bought 347 yds. of linen at $63\frac{5}{12}$ cts. per yard; how much did it cost?

259. I bought 4 barrels of cider, each barrel containing $30\frac{4}{7}$ gallons, and I gave 12 cts. per gallon; how much did it cost?

260. I bought 13 lbs. 4 oz. of gold dust at $\$15\frac{3}{7}$ per ounce; how much did it cost?

QUESTIONS.—Show on the blackboard how the example 8×3=24, may be performed by addition. Show how the example 24÷8=3, can be performed by subtraction. What does the quotient show? ANS.—*How many subtractions can be made.* Can every example in division be performed by subtraction? ANS.—*It can.* Can every example in multiplication be performed by addition? ANS.—*It can.*

How many *primary* rules are there? ANS.—*Three,—notation and numeration, addition, and subtraction.*

LESSON XXV.

261. I bought a gold chain weighing 2 oz. 18 pwt., at \$0.09$\frac{4}{5}$ per grain; what was its cost?

262. I bought a garnet weighing 15 pwt., at 13$\frac{4}{9}$ cts. per grain; what did it cost?

263. A man bought 13 diamond rings, each weighing 11 pwts., at 1£. 13s. 5$\frac{7}{12}$d. per pwt.; what was the cost?

264. A man bought 11 cwt. 3 qr. 21 lbs. of hay; 4 cwt. 4 qr. 17 lbs. became worthless; he sold the remainder at 2$\frac{5}{6}$ cts. per pound; how much did he receive for it?

265. From a hogshead of sugar weighing 9 cwt. 1 qr. 13$\frac{5}{8}$ lbs., I sold 3 cwt. 3 qr. 12$\frac{5}{16}$ lbs.; how much was the remainder worth at 7 cts. per pound?

266. A merchant bought 13 firkins of butter, each containing 97$\frac{3}{11}$ pounds, and sold 647$\frac{5}{8}$ pounds; how much remained unsold?

267. Mr. David Hurd bought two hogsheads of molasses; the first contained 44$\frac{3}{7}$ gallons, the second 51$\frac{4}{9}$ gallons; how much did he receive for it at \$0.36 per gallon?

268. Mr. S. J. Brundige had a piece of cloth measuring 24$\frac{11}{13}$ yds.; he sold 13$\frac{5}{9}$ yards; what will the remainder be worth at \$1.42 per yard?

269. A man, on settling with his butcher, finds that he is charged with 156$\frac{3}{4}$ lbs. of veal, at 11 cts. per pound, and 37 lbs. of beef, at 14$\frac{3}{5}$ cts. per pound; how much does he owe him?

270. Mr. Otis Lyon bought 4783 feet of boards, at 2$\frac{3}{7}$d. per foot; in payment, he sold 15 doz. fruit cans, at 1s. 6$\frac{2}{3}$d.

apiece, and paid the balance in cash; how much cash did he pay?

271. What will 341$\frac{2}{3}$ bushels of flax-seed cost, at a half-sovereign per bushel?

272. What will 478$\frac{5}{8}$ lbs. of tea cost, at 3 half-crowns per pound?

273. What will 387 pounds of lead cost, at 6$\frac{4}{7}$d. per pound?

274. Two men start from the same place, and travel in opposite directions: the first travels at the rate of 17 m. 2 fur. 16 rds. 3 yds. 1 ft. per day; the second travels at the rate of 23 mi. 7 fur. 9 rds. per day: how far will they be apart at the end of 15$\frac{4}{7}$ days?

QUESTIONS.—What is notation? (17.) What notation is used in Arithmetical operations? (19.) What is the difference between the simple and the local value of a figure? (20.) (21.) What is the difference between reading numbers, and numeration? (24.) (23.) Has the cipher any value? (22.) How are the periods usually separated, in writing numbers? (27., *a.*) What is the sign of equality? (33.) The dollar sign? (34.) The sign of Subtraction? (43.) Of Multiplication? (51.) Of Division? (62.)

LESSON XXVI.

155. To divide a fractional by an integral number.

(*a.*) Divide $\frac{14}{9}$ by 7.

(*b.*) Divide $\frac{4}{9}$ by 7.

MODEL OPERATIONS.

$$(a.)\ \frac{14}{9}\div7=\frac{14\div7}{9}=\frac{2}{9},\ \text{Ans.}$$

$$(b.)\ \frac{4}{9}\div7=\frac{4}{9\times4}=\frac{4}{63},\ \text{Ans.}$$

ANALYSIS.—(*a.*) 1 *seventh* of 14 *ninths* is 2 *ninths.* Therefore, $\frac{14}{9}$ divided by 7 equals $\frac{2}{9}$.

(*b.*) Increasing the denominator of a fraction in any ratio, decreases the value of the fraction in the same ratio.*

If, then, the denominator of $\frac{4}{9}$ be multiplied by 7, the result, $\frac{4}{63}$, will be 9 times less than $\frac{4}{9}$.

Therefore, $\frac{4}{9} \div 7$ equals $\frac{4}{63}$; or, $\frac{1}{7}$ of $\frac{4}{9}$ is $\frac{4}{68}$†.

Rule.—*Divide the numerator of the fraction by the integral number, if it can be done without a remainder; if not, multiply the denominator by the integral number and write the product under the numerator.*

EXAMPLES FOR PRACTICE.

275. $\frac{4}{39} \div 7 =$ how many?
276. $\frac{43}{7} \div 14 =$ how many?
277. $\frac{37}{42} \div 16 =$ how many?
‡278. $1\frac{37}{61} \div 17 =$ how many?
279. $\frac{83}{71} \div 16 =$ how many?
280. $6\frac{3}{5} \div 3 =$ how many?
281. $9\frac{4}{7} \div 4 =$ how many?
282. $\frac{48}{37} \div 12 =$ how many?
283. $\frac{56}{37} \div 8 =$ how many?
284. $\frac{48}{131} \div 12 =$ how many?

Questions.—What are the terms in addition? (30.) In subtraction? (40.) (41.) (42.) In multiplication? (47.) (48.) (49.) Which are called factors? (50.) What are the terms used in division? (55.) (56.) (57.) (61.) Express (on the blackboard) 23 in the Roman notation. (18., *a.*) Express 78 in the same manner; 86; 37; 23; 46; 32; 21; 11; 8; 325; 312000. How many characters in the Arabic notation? (19.) What is the largest number that can be expressed by the digits 7 and 2? What is the smallest?

LESSON XXVII.

156. To divide a mixed number by an integral number.

Divide $86\frac{5}{18}$ by 4.

Notes for the Teacher.—* Since the denominator is a divisor, increasing it in any ratio must decrease the quotient in the same ratio.

† The expression "$\frac{4}{9}$ divided by 7" is equivalent to $\frac{1}{7}$ of $\frac{4}{9}$, but is not identical with it The latter comprehends an operation in multiplication, the former an operation in division. They are often used as synonymous; but their difference should be explained to the pupil, to prevent confusion.

‡ Change the mixed numbers to improper fractions.

MODEL OPERATION.

$$\begin{array}{r} 4)86\frac{5}{16} \\ \hline 21\frac{37}{64} \end{array}$$

ANALYSIS.—4 is contained in 8 tens, 2 tens' times which are to be written in the place of tens.

4 is contained in 6 units once, with a remainder of 2 units; write 1 in the place of units, and add the 2 units to the 16ths of a unit.

In 1 unit there are 16 *sixteenths*, and in 2 units there are 2 times 16 sixteenths, which are 32 sixteenths, which, with 5 sixteenths added, are 37 sixteenths.

As 37 sixteenths can not be divided into 4 equal parts without a remainder, change the sixteenths to a denomination *one-fourth* as great as sixteenths, which is sixty-fourths.

Since sixty-fourths are *one-fourth* as great as sixteenths, then $\frac{37}{64}$ are one-fourth as much as $\frac{37}{16}$.

Therefore, 4 is contained in $86\frac{5}{16}$ $21\frac{37}{64}$ times.

285. Divide $416\frac{3}{5}$ by 9, by 8, by 7, by 6.

286. Divide $347\frac{3}{11}$ by 5, by 6, by 3, by 12.

287. Divide $241\frac{3}{4}$ by 6, by 8, by 5, by 9.

288. Divide $4134\frac{6}{7}$ by 15, by 36, by 25, by 84.

QUESTIONS.—How is Addition proved? (36., *f.*) Subtraction? (44., *i.*) Multiplication? (53., *n.*) Division? (65., *i.*) What is the difference between U. S. money and English money? (68.) (82., *a.*) What are the U. S. gold coins? (69., *a.*) The English? (82., *c.*) What are the U. S. silver coins? (69., *b.*) The English? (82., *d.*) What is the difference between a pound English money, and a pound Troy weight? (82., *c.*) (84.) What is a knot? (90., *c.*) What is the difference in the use of Troy and of Avoirdupois weight? (84.) (85.) By which would you weigh hay? Flour? Pork? Gold? Ice? Jewels? Coal at the mines? Coffee? Tea? Goods at the Custom House? Silver? When are medicines weighed by Avoirdupois?

When by Apothecaries'? Is there anything gained in buying by Avoirdupois weight and selling by Apothecaries'? How many grains in a pound Troy? (88., *b.*)

LESSON XXVIII.

PRACTICAL EXAMPLES.

289. What is the cost of one pound of tea, if 12 pounds cost $\$16.42\frac{4}{7}$?

290. What is the cost of one pound of tea, if 12 pounds cost $\$15\frac{4}{7}$?

291. What is the cost of one pound of tea, if 12 pounds cost $\$15.463\frac{5}{8}$?

292. What is the cost of one pound of coffee, if 48 pounds cost $\$20.34\frac{5}{9}$?

293. What is the cost of one pound of spice, if 29 pounds cost $\$6.383\frac{5}{12}$?

294. If I give $\$5.85\frac{3}{19}$ for 9 pounds of tea, what will one pound cost?

295. A man received $\$29.25\frac{3}{11}$ for a month's work; how much is that a day, allowing 26 working days to a month?

296. A man bought 13 pieces of cloth, for which he paid $\$88.304\frac{3}{14}$; each piece contained 46 yds.; how much did he pay a yard?

297. A drover pays $\$1250\frac{4}{13}$ for 500 sheep; for how much must he sell them apiece, so that he may neither make nor lose by the bargain?

298. The dairy of a farmer produces $\$600\frac{4}{11}$; he has 25 cows; how much is his average gain from each cow?

299. A man traveled 160 mi. 4 fur. 37 rds. $4\frac{5}{8}$ yds. in 16 days; how many yards did he travel in one day?

QUESTIONS.—Repeat the table of English money. (82., *b.*) Of

Troy weight. (84., *a.*) Of Avoirdupois weight. (85., *a.*) Repeat the Long Ton table. (86.) The table of Apothecaries' weight. (88., *a.*) Of Long Measure. (90., *a.*) Of Gunter's Chain. (91., *a.*) Of Square Measure. (92., *f.*) Surveyors' Square Measure. (93., *a.*) Of cubic measure. (94., *e.*) Of Liquid Measure. (95., *b.*) Of Dry Measure. (96., *a.*) Of Time. (97., *a.*) Of Circular Motion. (98., *a.*) The Miscellaneous Tables. (99.)

LESSON XXIX.

300. A railroad car moves 300 yds. 4 ft. $6\frac{8}{9}$ inches per minute; at that rate, how many inches does it move in one second?

301. A fox runs $3360\frac{4}{11}$ feet in 120 seconds; at that rate, how far does he run in one second?

302. A steamboat moves $115\frac{3}{4}$ miles in 12 hours; how many miles is that per hour?

303. If 30 tons of hay cost $\$436\frac{5}{7}$, how much will 5 tons cost?

ANALYTICAL STEPS.—1. Find the cost of one ton.
2. Find the cost of 5 tons.

304. If 40 bbls. of molasses cost $\$249\frac{4}{21}$, what will 25 bbls. cost?

305. If 130 bushels of oats cost $\$260\frac{4}{7}$, what will 147 bushels cost?

306. If 347 yards of broadcloth are worth $\$1496\frac{5}{11}$, how much are 372 yards of the same kind of cloth worth?

307. A merchant bought 347 yards of silk for $471\frac{3}{9}$; how much did 40 yards of the cloth cost?

308. A merchant bought 294 yds. of calico for $\$69.47\frac{2}{9}$; he sold 25 yards for $\$7.32\frac{4}{11}$; did he make, or lose, on what he sold, and how much?

ANALYTICAL STEPS.—1. Find the cost of one yard.
2. Find what he received for one yard.

3. Find how much he gained or lost on one yard.
4. Find how much he gained or lost on 25 yards.

309. A farmer bought 725 acres of land for \$$14500\frac{3}{8}$, and sold it for \$$18472\frac{5}{7}$; how much did he gain per acre?

310. A man, dying, left an estate of \$$33463\frac{4}{15}$ to be equally divided between his four children, after his wife had taken a third; what did each child receive?

QUESTIONS.—What is the difference between Reduction Ascending and Reduction Descending? (80.) (81.) Of what is every number composed? (112., *a.*) What is the least common multiple of two or more numbers? (122.) What is the difference between a common and a decimal fraction? (124.) (125.) Are common fractions ever written without a denominator expressed?

LESSON XXX.

157. To multiply a fraction by a fraction.

Multiply $\frac{3}{7}$ by $\frac{5}{8}$.

MODEL OPERATION.

(*a.*)

1. $\frac{3}{7}\times 1$ or $\frac{8}{8}=\frac{3}{7}$,
2. $\frac{3}{7}\times\frac{1}{8}=\frac{3}{56}$,
3. $\frac{3}{7}\times\frac{5}{8}=\frac{15}{56}$, Ans.

(*b.*)

$\frac{3}{7}\times\frac{5}{3}=\frac{15}{56}$, Ans.

ANALYSIS.*—(*a.*) 1. $\frac{3}{7}$ multiplied by $\frac{8}{8}$, or 1 unit, equals $\frac{3}{7}$.

2. If $\frac{3}{7}\times\frac{8}{8}=\frac{3}{7}$, then $\frac{3}{7}\times\frac{1}{8}$ will equal one *eighth* of $\frac{3}{7}$, which is $\frac{3}{56}$.

3. If $\frac{3}{7}\times\frac{1}{8}=\frac{3}{56}$, then $\frac{3}{7}\times\frac{5}{8}$ will equal 5 times $\frac{3}{56}$, which is $\frac{15}{56}$. Therefore, $\frac{3}{7}\times\frac{5}{8}=\frac{15}{56}$.

†RULE.—(*b.*) *Cancel all the common factors, and then multiply the numerators together for a new numerator, and the denominators for a new denominator.*

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*NOTE.—Those who prefer, may use the following *analysis*:—$\frac{3}{7}\times\frac{5}{8}=\frac{5}{8}$ of $\frac{3}{7}$; $\frac{5}{8}$ of $\frac{3}{7}=\frac{1}{8}$ of 5 times $\frac{3}{7}$; 5 times $\frac{3}{7}=\frac{15}{7}$, and $\frac{1}{8}$ of $\frac{15}{7}=\frac{15}{56}$, Ans.

†NOTE FOR THE TEACHER.—Rules being often concise directions for contracted operations, they, consequently, do not always comprehend every step of the analysis, and may not seem to be properly deduced from it.

EXAMPLES FOR PRACTICE.

311. $\frac{4}{5}\times\frac{6}{7}$.
312. $\frac{8}{9}\times\frac{3}{8}$.
313. $\frac{56}{4}\times\frac{3}{8}$.
314. $\frac{41}{8}\times\frac{3}{7}$.
315. $4\frac{7}{2}\times8\frac{9}{4}$.
316. $\frac{8}{11}\times\frac{5}{37}$.
317. $\frac{4}{16}\times\frac{5}{7}$.
318. $\frac{8}{7}\times\frac{6}{17}$.
319.* $3\frac{4}{7}\times\frac{8}{7}$.
320. $\frac{3}{4}\times8\frac{3}{4}$.
321. $6\frac{3}{7}\times\frac{42}{8}$.
322. $4\frac{2}{9}\times\frac{63}{7}$.
323. $\frac{6}{7}\times91\frac{3}{5}$.
324. $\frac{6}{5}\times83\frac{4}{7}$.
325. $9\frac{4}{5}\times6\frac{1}{867}$.
326. $41\frac{2}{4}\times4\frac{1}{846}$.
327. $\frac{3}{9}\times4\frac{1}{4}$.
328. $\frac{5}{9}\times6\frac{3}{7}$.
329. $\frac{8}{6}\times7\frac{3}{2}$.
330. $\frac{4}{7}\times9\frac{3}{17}$.

QUESTIONS.—In how many ways may the denominator of a fraction be written? (127.) The numerator of a fraction? (128.) How are decimal fractions usually written? (125.) In a compound fraction what is the meaning of the word *of?* (133.) What is the difference between a simple fraction and a complex fraction? (131., *a.*) (132.) In the expression $\frac{3}{4}$, which is the dividend? Which the divisor? What effect has it on the value of a fraction to decrease both numerator and denominator in the same ratio? (136., *b.*) To increase them? (136., *a.*) When are fractions said to have a common denominator? (149.)

LESSON XXXI.

158. To multiply a mixed number by a fractional or mixed number.

What will $473\frac{3}{8}$ pounds of cheese cost at $4\frac{3}{7}$ cts. per pound?

MODEL OPERATION.

(*a.*)	(*b.*)
$473\frac{3}{8}$	$473\frac{3}{8}$
3	$4\frac{3}{7}$
$7)1420\frac{1}{8}$	$202+(\frac{7}{8}=\frac{7}{8})$ cts.
	$1893+(\frac{1}{2}=\frac{4}{8})$ "
$202\frac{7}{8}$ cts.	$2095\quad+(\frac{11}{8}=1\frac{3}{8})=\$20.96\frac{3}{8}$.

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*Mixed numbers may be changed to improper fractions before multiplying.

ANALYSIS.*

General Step.—If one pound of cheese costs $4\frac{3}{7}$ cts., $473\frac{3}{8}$ pounds will cost $473\frac{3}{8}$ times $4\frac{3}{7}$ cts., which are $2096\frac{3}{8}$ cts., equal to \$$20.96\frac{3}{8}$.

Analytical Steps.—(*a.*) 1. Find $\frac{3}{7}$ of $473\frac{3}{8}$.

$\frac{3}{7}$ of $473\frac{3}{8}$ is equivalent to $\frac{1}{7}$ of 3 times $473\frac{3}{8}$; 3 times $473\frac{3}{8}$ is $1420\frac{1}{8}$, and $\frac{1}{7}$ of $1420\frac{1}{8}$ is $202\frac{7}{8}$.

2. (*b.*) Find the product of 4 and $473\frac{3}{8}$.

4 times $\frac{3}{8}$ are $\frac{3}{2}$, equal to $1\frac{1}{2}$; 4 times 473=1892, which, with $1\frac{1}{2}$ added, equals $1893\frac{1}{2}$.

3. Reduce and add the fractions of the partial products.

As *halves* and *eighths* can not be added, change *halves* to *eighths*; $\frac{1}{2}=\frac{4}{8}$. The sum of $\frac{7}{8}$ and $\frac{4}{8}$ is $\frac{11}{8}$, $=1\frac{3}{8}$.

4. Add the integral partial products.

The sum of the partial products 1893, 202 and $1\frac{3}{8}$, equals $2096\frac{3}{8}$ cts., equal to \$$20.96\frac{3}{8}$.

Therefore, $473\frac{3}{8}$ pounds of cheese will cost \$$20.96\frac{3}{8}$ at $4\frac{3}{7}$ cts. per pound.

Questions.—On what does the value of a fraction of any denomination depend? (139.) What is the value of 3 divided by 4? **Ans.**—$\frac{3}{4}$ *of one unit..* 8÷9? **Ans.**—$\frac{8}{9}$ *of one unit.* 7÷6? **Ans.**—$1\frac{1}{6}$ *of a unit.* 4÷9? 3÷11? 24÷2? 87÷40? What is the reduction of fractions? (140.) What is the difference between Reduction Ascending and Reduction Descending? (141.) (142.) When is a fraction said to be reduced to its lowest terms? (143.) What is, in effect, equivalent to dividing both the numerator and the denominator of a fraction by any number that will divide them without a remainder? **Ans.**—*It is simply rejecting a factor common to both.*

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*Note.—When the integral numbers are large, and the fractional numbers are small, the following method is, without doubt, the most convenient.

LESSON XXXII.

PRACTICAL EXAMPLES.

331. What costs $473\frac{4}{7}$ lbs. of sugar at $3\frac{7}{8}$ cts. per pound?

332. What costs $343\frac{3}{7}$ lbs. of rice at $2\frac{5}{8}$ cts. per pound?

333. What will $3476\frac{2}{5}$ lbs. of iron cost at $4\frac{5}{8}$ cts. per pound?

334. A merchant bought 4 bales of cloth; each bale contained 46 pieces, and each piece was $36\frac{3}{4}$ yards long; how much did it cost him, at \$$2.37\frac{7}{8}$ per yard?

335. A publisher bought 46 bundles of paper; each bundle contained 2 reams, and each ream weighed $6\frac{5}{8}$ lbs.; how much did it amount to, at $18\frac{3}{7}$ cts. per pound?

336. A person sells 3 cows at \$$37\frac{4}{7}$ each, and a yoke of oxen for \$$68\frac{4}{5}$; he agrees to take in payment 60 sheep; what do his sheep cost him a head?

337. A farmer sold a yoke of oxen for \$$80.75\frac{2}{5}$, 6 cows at \$$29.42\frac{3}{4}$ each, 30 sheep at \$$2.50\frac{2}{5}$ each, and 3 colts at \$47 each; how much did he receive for them all?

338. Mr. Jones bought 9 barrels of sugar, each weighing 216 lbs., for which he paid \$$116.37\frac{2}{3}$; how much did he pay per pound?

339. The interest on \$478 for one year is \$$24.37\frac{2}{3}$; what is the interest on one dollar for the same time?

QUESTIONS.—What is an improper fraction? (130.) What is a mixed number? (131.) Give the analysis of changing $\frac{8}{3}$ to a mixed number. (145.) Give the rule for the same. Give the analysis of changing $2\frac{2}{7}$ to an improper fraction. (146.) Give the rule. Give the analysis of changing $4\frac{3}{7}$ to *fourteenths.* How many *ninths* in a unit? *Sixteenths?* 23rds? 11ths? 115ths? 5ths? 17ths? How many 8ths in $\frac{1}{2}$ a unit? In $\frac{1}{4}$ of a unit? In $\frac{1}{8}$? In $\frac{3}{4}$? In $2\frac{1}{4}$ units? In $\frac{1}{3}$ of a unit?

LESSON XXXIII.

159. To divide an integral number by a fraction.

Divide 9 by $\frac{5}{7}$.

MODEL OPERATIONS.

(*a*.)

1. $9\div\frac{7}{7}=9$.
2. $9\div\frac{1}{7}=63$.
3. $9\div\frac{5}{7}=\frac{63}{5}=12\frac{3}{5}$, Ans.

(*b*.)

$9\div\frac{5}{7}=9\times\frac{7}{5}=\frac{63}{5}=12\frac{3}{5}$, Ans.

Analysis.—(*a*.) 1. $9\div\frac{7}{7}$, or 1 unit, equals 9.

2. If $9\div\frac{7}{7}$ equals 9, then $9\div\frac{1}{7}$ will equal 7 times 9, which is 63.

3. If $9\div\frac{1}{7}$ equals 63, then $9\div\frac{5}{7}$ will equal one *fifth* of 63, which is $\frac{63}{5}=12\frac{3}{5}$.

Therefore, $9\div\frac{5}{7}=12\frac{3}{5}$.

Rule.*—(*b*.) *Invert the divisor, and proceed as in the multiplication of fractions.*

What are the values of the following expressions?

340. $364\div\frac{4}{7}$.
341. $241\div\frac{3}{19}$.
342. $413\div\frac{41}{63}$.
343. $213\div\frac{5}{7}$.
344. $413\div\frac{6}{7}$.
345. $8134\div7\frac{2}{3}$†.
346. $4134\div3\frac{2}{3}$.
347. $5147\div3\frac{5}{7}$.
348. $4134\div8\frac{3}{11}$.
349. $2416\div4\frac{3}{17}$.
350. $4186\div5\frac{7}{8}$.
351. $4187\div41\frac{3}{4}$.
352. $864\div\frac{5}{7}$.
353. $9186\div8\frac{3}{91}$.
354. $2164\div3\frac{7}{9}$.
355. $4183\div64\frac{3}{22}$.

LESSON XXXIV.

160. To divide a fraction by a fraction.

Divide $\frac{4}{9}$ by $\frac{3}{7}$.

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* Notes.—The teacher will observe, that in the analysis, the dividend, 9, is multiplied by a number equal to the denominator of the divisor, and that this product is divided by a number equal to the numerator of the divisor; hence the rule.

† Change the mixed numbers to improper fractions, before dividing.

MODEL OPERATIONS.

(*a.*)

1. $\frac{4}{9} \div \frac{7}{7} = \frac{4}{9}$.
2. $\frac{4}{9} \div \frac{1}{7} = \frac{28}{9}$.
3. $\frac{4}{9} \div \frac{3}{7} = \frac{28}{27} = 1\frac{1}{27}$, Ans.

(*b.*)

$\frac{4}{9} \div \frac{3}{7} = \frac{4}{9} \times \frac{7}{3} = \frac{28}{27} = 1\frac{1}{27}$, Ans.

Analysis.—(*a.*) 1. $\frac{4}{9} \div \frac{7}{7}$, or 1 unit, equals $\frac{4}{9}$.

2. If $\frac{4}{9} \div \frac{7}{7}$ equals $\frac{4}{9}$, then $\frac{4}{9} \div \frac{1}{7}$ will equal 7 times $\frac{4}{9}$, which is $\frac{28}{9}$.

3. If $\frac{4}{9} \div \frac{1}{7}$ equals $\frac{28}{9}$, then $\frac{4}{9} \div \frac{3}{7}$ will equal one *third* of $\frac{28}{9}$, which is $\frac{28}{27}$, equal to $1\frac{1}{27}$.

Therefore, $\frac{4}{9} \div \frac{3}{7} = 1\frac{1}{27}$.

Rule.—(*b.*) *Invert the divisor, and proceed as in the multiplication of fractions.*

EXAMPLES FOR PRACTICE.

356. $\frac{4}{3} \div \frac{3}{7}$ = how many?
357. $\frac{3}{5} \div 4\frac{3}{7}$ = "
358. $6\frac{3}{7} \div 4\frac{3}{5}$ = "
359. $\frac{47}{3} \div 4\frac{6}{7}$ = "
360. $\frac{3}{41} \div 8\frac{6}{5}$ = "
361. $\frac{2}{5} \div 3\frac{6}{7}$ = how many?
362. $\frac{4}{7} \div \frac{46}{32}$ = "
363. $\frac{14}{16} \div 38\frac{2}{7}$ = "
364. $\frac{63}{87} \div 46\frac{4}{7}$ = "
365. $\frac{5}{7} \div 83\frac{5}{7}$ = "

Questions.—What is the addition of fractions? When are fractions said to have a common denominator? (149.) Can fractions of different denominations be added or subtracted? Why? (16., *a.* and *b.*) Can 3ds of a bushel and 4ths of a bushel be added? Why? (16., *a.* and *b.*) Can 4ths of a lb. and 4ths of an oz. be added? Why? Is it necessary to find the least common multiple of the denominators of fractions to be added? Ans.—*It is not; but it is generally more convenient, when the fractions are numerous, or of*

LESSON XXXV.

PRACTICAL EXAMPLES.

If $9\frac{1}{2}$ pounds of cheese cost 456 cts., what will one pound cost?

1. $9\frac{1}{2}$ lbs. $=\frac{19}{2}$ lbs.

$$2.\ 456\div\frac{19}{2}=\frac{2}{\cancel{19}}\times\frac{\overset{24}{\cancel{456}}}{1}=48 \text{ cts.}$$

ANALYSIS.—1. $9\frac{1}{2}$ lbs. equals 19 *half pounds.*

2. If $\frac{19}{2}$ lbs. of cheese cost 456 cts., one half pound will cost one *nineteenth* of 456 cts., and 2 *halves*, or one pound, will cost 2 times $\frac{1}{19}$ of 456 cts., which is $\frac{2}{19}$ of 456 cts., equal to 48 cts.

Therefore, if $9\frac{1}{2}$ lbs. of cheese cost 456 cts., one pound will cost 48 cts.

366. $13\frac{5}{8}$ pounds of rice cost 136 cts.; how much is that a pound?

367. $137\frac{4}{7}$ pounds of butter cost \$27.43; how much is that a pound?

368. A man paid \$43.74 for $153\frac{4}{9}$ pounds of butter; what did it cost per pound?

369. A man paid \$3,894.624 for $20\frac{3}{8}$ shares in a factory; what did each share cost?

370. Mr. Wilson paid \$400 for transporting 34 tons 13 qrs. $19\frac{3}{4}$ pounds of freight from Springfield to Baltimore; how much is that per pound?

371. A man traveled 528000 feet in $16\frac{2}{3}$ days; how many miles was that a day?

372. Horace Jones traveled 417060 feet in $11\frac{4}{9}$ days; how many miles was that a day?

373. A fox ran 105600 feet in $30\frac{4}{7}$ minutes; how many feet is that a minute?

374. The interest on \$$30\frac{4}{7}$ for a certain time is \$15.36; what is the interest on one dollar for the same time?

QUESTIONS.—Can fractions of unlike denominations be added? (16., *a.*) Why? Can fractions of unlike denominations be subtracted? (16., *b.*) Multiplied? Divided? What is the multiplication of fractions? What is the division of fractions? What is the rule for multiplying a fraction by an integral number? (153.) What is the rule for dividing a fraction by an integral number? (154.) What is an integer? (106.) What effect does dividing the denominator have on the denomination of a fraction? Why? What effect does dividing the numerator have on the value of a fraction? (186., *f.*)

LESSON XXXVI.

If $46\frac{3}{4}$ pounds of tea cost $\$47.86\frac{3}{7}$, what will one pound cost?

MODEL OPERATIONS.*

(*a.*)

$$46\tfrac{3}{4}\text{ lbs.}=\tfrac{187}{4}\text{ lbs.}$$

$$\begin{array}{l} 187\overline{)4786\tfrac{3}{7}}\text{ cts.} \\ \quad 25\tfrac{780}{1309} \\ \quad\ \ 4 \\ \hline 100(\tfrac{3120}{1309}=2\tfrac{502}{1309})=\$1.02\tfrac{502}{1309}. \end{array}$$

(*b.*)†

$$\begin{array}{r} 4786\tfrac{3}{7} \\ 4 \\ \hline 187)19145\tfrac{5}{7} \\ \hline \$1.02\tfrac{502}{1309}. \end{array}$$

ANALYSIS.—(*a.*) 1. $46\frac{3}{4}$ lbs. equals $\frac{187}{4}$ lbs.; $\$47.86\frac{3}{7}=4786\frac{3}{7}$ cts.

2. If $\frac{187}{4}$ lbs. of tea cost $4786\frac{3}{7}$ cts., 1 fourth of a pound will cost $\frac{1}{187}$ of $4786\frac{3}{7}$ cts., which is $25\frac{780}{1309}$ cts., and 4 *fourths*, or one pound, will cost 4 times $25\frac{780}{1309}$ cts., which is $102\frac{502}{1309}$ cts., equal to $\$1.02\frac{502}{1309}$.

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*NOTE.—When the divisor is small and the dividend large, MODEL OPERATION (*a.*) is preferable; but when the dividend and divisor are about equal, MODEL OPERATION (*c.*) may be used.

† As 4 times $\frac{1}{187}$ of any number is equal to $\frac{1}{187}$ of 4 times the same number, it is generally more convenient to multiply before dividing, as in operation (*b.*)

Therefore, if $46\frac{3}{4}$ pounds of tea cost $4786\frac{3}{7}$ cts., one pound will cost $\$1.02\frac{502}{1309}$.

At $\$5\frac{3}{4}$ a ream, how much paper can be bought for $\$175\frac{4}{7}$?

MODEL OPERATION.

(*c.*)

$\$5\frac{3}{4}=\$\frac{23}{4}$.

$\$175\frac{4}{7}=\frac{1229}{7}$.

$\$\frac{1229}{7}\div\$\frac{23}{4}=\frac{1229}{7}\times\frac{4}{23}=\frac{4916}{161}=30\frac{86}{161}$ Reams.

ANALYSIS.—$\$5\frac{3}{4}=\frac{23}{4}$; $\$175\frac{4}{7}=\$\frac{1229}{7}$.

If for $\$\frac{23}{4}$ I can purchase one ream of paper, for $\$\frac{1229}{7}$ I can purchase as many reams as $\$\frac{23}{4}$ are contained times in $\$\frac{1229}{7}$, which is $30\frac{86}{161}$.

Therefore, $30\frac{86}{161}$ reams of paper can be bought for $\$175\frac{4}{7}$, if one ream costs $\$5\frac{3}{4}$.

QUESTIONS.—What effect does the multiplication of the denominator have on the denomination of the fraction? Why? What is the rule for multiplying a fraction by a fraction? (156., *b.*) What is the rule for dividing an integral number by a fraction? (158., *b.*) What is the rule for dividing a fraction by a fraction? (159., *b.*) What effect has the division of the numerator on the value of the fraction? What effect has the multiplication of the numerator on the value of fractions? What two methods are there for dividing a fraction by an integral number? (154.) What two for multiplying a fraction by an integral number? (153.)

LESSON XXXVII.

375. At $\$\frac{1}{8}$ a pound, how much butter can be bought for $\$5\frac{11}{16}$?

376. If a man consume $1\frac{1}{5}$ pounds of meat a day, how many men would $472\frac{4}{5}$ pounds supply for the same time?

377. If $6\frac{3}{8}$ pounds of tea cost $\$14\frac{1}{2}$, what will one pound cost?

378. At $\frac{4}{5}$ of a dollar a basket, how many baskets of peaches can be bought for $\$113\frac{1}{5}$?

379. If $\frac{5}{8}$ of a ton of coal cost $6\frac{5}{7}$ dollars, what will 1 ton cost at the same rate?

380. For $\$4\frac{12}{124}$, how much cheese can be bought, at $\frac{3}{8}$ of a dollar a pound?

381. A man divided $\$2\frac{4}{5}$ among his children, giving each $\frac{7}{10}$ of a dollar; how many children had he?

382. How many times will $42\frac{1}{12}$ gallons of ale fill a vessel holding $\frac{3}{7}$ of a gallon?

383. A farmer has $18\frac{3}{4}$ bushels of wheat worth $396\frac{5}{7}$ shillings, and $16\frac{5}{7}$ bushels of rye worth $128\frac{3}{11}$ shillings; he sold 1 bushel of each; how much did he receive for both?

384. A butcher purchased 3 sheep; one from a flock of 45 valued at $1080\frac{3}{4}$ shillings, one from a flock of 112 valued at $3589\frac{3}{7}$ shillings, and one from a flock of 350 valued at $11200\frac{3}{11}$ shillings; how much did they cost?

385. A grocer has 2 boxes of tea, each containing $12\frac{3}{7}$ pounds; one box is valued at $\$6.43\frac{1}{4}$, the other at $\$7.83\frac{4}{7}$; he sells one pound of each; how much does he receive?

Questions.—What is an integer? (106.) What is an odd number? (107.) What is an even number? (108.) What is the difference between prime and composite numbers? (109.) (110.) When are numbers said to be prime to each other? (111.) What is the difference between a prime and a composite factor? (112.) (113.)

LESSON XXXVIII.

161. To simplify compound and complex fractions.

(*a.*) Simplify $\frac{3}{8}$ of $\frac{4}{7}$ of $3\frac{2}{3}$.

(*b.*) Simplify $\frac{4\frac{2}{7}}{8}$.

(*c.*) Simplify $\frac{3}{4\frac{2}{3}}$ of $\frac{3\frac{2}{3}}{\frac{6}{7}}$.

MODEL OPERATIONS.

(*a.*)

$$\frac{3}{8} \text{ of } \frac{4}{7} \text{ of } 3\tfrac{2}{3}=\frac{3}{8}\times\frac{4}{7}\times\frac{11}{3},=\frac{\cancel{3}}{\underset{2}{\cancel{8}}}\times\frac{\cancel{4}}{7}\times\frac{11}{\cancel{3}},=\frac{11}{14}, \text{ Ans.}$$

(*b.*)

$$\frac{4\frac{2}{7}}{8}=\frac{\frac{30}{\cancel{7}}\times 7}{8\times 7},=\frac{30}{56},=\frac{15}{28}, \text{ Ans.}$$

(*c.*)

$$\frac{3}{4\frac{2}{3}} \text{ of } \frac{3\frac{2}{3}}{\frac{6}{7}}=\frac{3}{\frac{14}{3}}\times\frac{\frac{11}{3}}{\frac{6}{7}},=\frac{\cancel{3}}{1}\times\frac{\cancel{3}}{\underset{2}{\cancel{14}}}\times\frac{11}{\cancel{3}}\times\frac{\cancel{7}}{\underset{2}{\cancel{6}}},=\frac{11}{4},=2\tfrac{3}{4}, \text{ Ans.}$$

ANALYSIS.—(*a.*) As *of* in this connection signifies *times*, the expression is simply an example in the multiplication of fractions, and may be treated as such.

(*b.*) As the numerator $4\frac{2}{7}$ is a dividend, and the denominator 8 is a divisor, according to the principle (136., *a.*) that, if both divisor and dividend be increased in an equal ratio, the quotient will not be changed, it follows that $\frac{4\frac{2}{7}}{8}=\frac{4\frac{2}{7}\times 7}{8\times 7}$ which is $\frac{30}{56},=\frac{15}{28}$.

Therefore, $\frac{4\frac{2}{7}}{8}=\frac{15}{28}$; hence, for simplifying complex fractions, the following:—

RULES.—I. *Multiply both numerator and denominator by any number that will cause the fractions to disappear, or*

II. *Consider the expressions as unperformed operations in the multiplication and division of fractions, and treat them as such.*

EXAMPLES FOR PRACTICE.

Simplify the following expressions:

386. $\frac{12}{\frac{3}{7}}$.

387. $\frac{\frac{3}{7}}{14}$.

388. $\frac{4\frac{7}{8}}{9}$.

389. $\frac{\frac{3}{4}}{1\frac{1}{2}}$.

390. $\frac{\frac{5}{6}}{7\frac{3}{4}}$.

391. $\frac{8\frac{3}{4}}{\frac{2}{3}}$.

392. $\frac{9\frac{3}{5}}{12\frac{1}{2}}$.

393. $\frac{\frac{4}{4}}{3\frac{3}{4}}$.

394. $\frac{\frac{4}{7}}{4}$ of $\frac{3}{4}$.

395. $\frac{\frac{4}{3}}{3\frac{4}{7}}$ of $\frac{2}{3\frac{2}{3}}$.

396. $\frac{\frac{4}{5}}{4\frac{6}{7}}$ of $\frac{4\frac{1}{2}}{41}$.

397. $\frac{\frac{2}{3}}{4\frac{1}{2}} \div \frac{3}{4}$.

398. $\frac{\frac{2}{3}}{4\frac{1}{2}} \div \frac{3\frac{1}{4}}{42}$.

399. $\frac{3}{4} \div \frac{4\frac{2}{3}}{4\frac{2}{7}}$.

400. $\frac{2}{3} \times \frac{\frac{4}{7}}{42}$.

401. $\frac{3}{4\frac{3}{4}}$ of $\frac{\frac{37}{4\frac{2}{3}}}{9}$.

NOTE.—To add or subtract complex or compound fractions, it is necessary first to simplify them, and then to perform the required additions and subtractions.

402. $\frac{\frac{1}{3}}{\frac{3}{7}} + \frac{4\frac{1}{7}}{12\frac{1}{2}}$=how many?

403. $\frac{7\frac{3}{4}}{\frac{4}{7}} + \frac{7\frac{1}{2}}{\frac{7}{12}}$=how many?

404. $\frac{3\frac{3}{4}}{7\frac{1}{2}} - \frac{\frac{4}{7}}{35}$=how many?

405. $\frac{\frac{2}{3}}{4\frac{1}{2}} + \frac{\frac{3}{4}}{\frac{4}{4}} \div \frac{5\frac{1}{2}}{3}$=how many?

406. $\dfrac{\frac{3}{9}}{4\frac{1}{2}}\times\dfrac{\frac{4}{6}}{3}\div\dfrac{5\frac{3}{4}}{4}$=how many?

407. $\dfrac{\frac{3}{7}}{4\frac{1}{2}}\times\dfrac{\frac{3}{7}}{4\frac{1}{3}}\div\dfrac{2\frac{4}{7}}{3\frac{1}{2}}$=how many?

LESSON XXXIX.

162. To find the Greatest Common Divisor of two or more fractions.

What is the greatest common divisor of $\frac{4}{9}$, $\frac{2}{3}$ and $\frac{4}{5}$?

MODEL OPERATION.

$$\left.\begin{array}{l}\dfrac{4}{9}=\dfrac{20}{45},=\dfrac{2\times2\times5}{45};\\[2ex]\dfrac{2}{3}=\dfrac{30}{45},=\dfrac{2\times3\times5}{45};\\[2ex]\dfrac{4}{5}=\dfrac{36}{45},=\dfrac{2\times2\times3\times3}{45}.\end{array}\right\}\dfrac{2}{45},\text{ Ans.}$$

Analysis.—As the greatest common divisor of $\frac{4}{9}$, $\frac{2}{3}$ and $\frac{4}{5}$ *can not be conveniently found* in their present forms, change them to equivalent fractions having the least common denominator. $\frac{4}{9}=\frac{20}{45}$, $\frac{2}{3}=\frac{30}{45}$, and $\frac{4}{5}=\frac{36}{45}$; and the product of all the common factors of $\frac{20}{45}$, $\frac{30}{45}$, and $\frac{36}{45}$, is $\frac{2}{45}$, which is the greatest common divisor.

Therefore, $\frac{2}{45}$ is the greatest common divisor of $\frac{4}{9}$, $\frac{2}{3}$ and $\frac{4}{5}$.

408. What is the greatest common divisor of $\frac{3}{4}$, $\frac{7}{8}$ and $1\frac{3}{8}$?

409. What is the greatest common divisor of $\frac{4}{5}$, $3\frac{4}{7}$ and $7\frac{5}{7}$?

410. What is the greatest number that will divide $\frac{4}{7}$, $3\frac{4}{5}$ and $\dfrac{6}{4\frac{3}{7}}$?

411. In a certain triangular field, the first side is $97\frac{1}{2}$

feet; the second is 71⅔ feet; and the third is 130 feet; what is the length of the longest rails that can be used in fencing it, making no allowance for lapping?

412. I have three rooms: the first is 16¼ feet wide; the second 26 feet wide; and the third is 19½ wide: what is the width of the widest carpeting that can be used without cutting from the width of any piece?

QUESTIONS.—Is unity to be considered a factor of numbers? (113., *a.*) What is the rule for the analysis of composite numbers? (114., *d.*) What is the proof? (114., *e.*) Of what is every number composed? ANS.—*Prime numbers.* If two numbers have the same prime factors, are they equal to each other? Can they be divided by the same numbers? Why? Illustrate the use of the parenthesis, or vinculum. (116.) What is the difference of their forms?

LESSON XL.

163. To find the Least Common Multiple of two or more fractions.

What is the least number that will exactly contain $\frac{1}{6}$, $1\frac{1}{2}$ and $5\frac{1}{4}$.

MODEL OPERATION.

$$\left.\begin{array}{l} \frac{1}{6}=\frac{2}{12},=\frac{2}{12}. \\ 1\frac{1}{2}=\frac{18}{12},=\frac{2\times3\times3}{12}. \\ 5\frac{1}{4}=\frac{63}{12},=\frac{3\times3\times7}{12}. \end{array}\right\} \begin{array}{l} \text{The highest powers of the } \textit{different} \text{ factors.} \\ \frac{\overset{3}{\cancel{2}\times\cancel{3}^2\times7}}{\underset{2}{\cancel{12}}},=\frac{21}{2},=10\frac{1}{2},\ \text{Ans.} \end{array}$$

ANALYSIS.—* For convenience change, $\frac{1}{6}$, $1\frac{1}{2}$ and $5\frac{1}{4}$ to equivalent fractions having the least common denominator. $\frac{1}{6}=\frac{2}{12}$, $1\frac{1}{2}=\frac{18}{12}$, and $5\frac{1}{4}=\frac{63}{12}$; again, $\frac{2}{12}=\frac{2}{12}$, $\frac{18}{12}=\frac{2\times3\times3}{12}$, $\frac{63}{12}=\frac{3\times3\times7}{12}$. The highest powers of all the different factors in 2, 18 and 63, twelfths, are 2, 3^2 and 7, *twelfths*, the product of which by cancellation equals $\frac{21}{2}$, or $10\frac{1}{2}$, the least common multiple.

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*NOTE.—On account of its simplicity this method of analysis is considered preferable to the one usually given.

Therefore, $10\frac{1}{2}$ is the least common multiple of $\frac{1}{6}$, $1\frac{1}{2}$ and $5\frac{1}{4}$.

413. What is the least common multiple of $\frac{3}{4}$, $2\frac{3}{8}$ and $2\frac{1}{10}$?

414. What is the least common multiple of $\frac{4}{7}$, $\frac{3}{8}$ and $\frac{3}{16}$?

415. Four men start from the same place to go around an island. The first can go around it in $5\frac{1}{2}$ days; the second in $3\frac{3}{8}$ days; the third in $9\frac{3}{7}$ days: in what time will they all meet at the place from which they start?

416. On the dial of a clock there is an hour-hand, a minute-hand, and a second-hand; the second-hand can complete the circuit in $60\frac{4}{7}$ seconds, the minute-hand in 60 minutes, the hour-hand in 12 hours. They are all exactly together at 12 o'clock M., Jan. 1st, 1864; when will they all meet at the same place again?

417. What is the smallest number of shillings for which I can purchase a number of pounds of sugar at $\frac{4}{5}$ of a shilling per pound, the same number of pounds of coffee at $2\frac{4}{7}$ shillings per pound, and an equal number of pounds of rice at $\frac{7}{9}$ shillings per pound? How many pounds of each must I purchase?

QUESTIONS.—What is cancellation? (117.) What are the conditions essential to cancellation? ANS.—*There must be a dividend and a divisor, and each must contain a common factor or factors.* What is meant by a common factor? ANS.—*A number that will divide both dividend and divisor without a remainder.* What is meant by $\frac{3}{7}$ of $\frac{14}{9}$? ANS.—*$\frac{3}{7}$ of $\frac{14}{9}$ means 3 times one* SEVENTH *of $\frac{14}{9}$; or, that $\frac{14}{9}$ is to be divided by 7, and that quotient to be multiplied by 3;*

$$\textit{thus, } \frac{14}{9}\div 7\times 3=\frac{14\times 3}{9\times 7}, =\frac{\overset{2}{\cancel{14}}\times\cancel{3}}{\underset{3}{\cancel{9}}\times\cancel{7}}, =\frac{2}{3}, \textit{ Ans.}$$

LESSON XLI.

MISCELLANEOUS EXAMPLES IN FRACTIONS.

418. Multiply 46 by 3, and divide the product by 8.

419. Multiply $\frac{32}{4}$ by 9, and divide the product by 5.

420. Multiply 42 by $\frac{3}{7}$, and divide the product by 6.

421. Multiply $\frac{3}{7}$ by $\frac{4}{5}$, and divide the product by $\frac{6}{5}$.

422. Multiply $\frac{3}{4}$ of $\frac{7}{8}$ by 4, and divide the product by 19.

423. Multiply $\frac{3}{7}$ of 4 by $\frac{3}{4}$, and divide the product by $\frac{3}{5}$.

424. Multiply $\frac{4\times3}{9}$ of $\frac{3}{8}$ by $\frac{3}{4}$, and divide the product by $\frac{3}{8}$.

425. Divide $\frac{4\times3}{46}$ of $\frac{3}{7}$ by $\frac{44}{7}$, and multiply the quotient by $\frac{7\div8}{4}$.

426. Divide $\frac{\frac{3}{7}}{4}$ of $\frac{4}{\frac{4}{7}}$ by $\frac{3\times7}{4\div8}$, and multiply the quotient by $\frac{3}{7}$.

427. Divide $\frac{6}{8}\div\frac{8}{5}$ by $\frac{4\div6}{8\div9}$, and multiply the quotient by $\frac{4}{5}$.

428. Divide $\frac{3}{7}$ of $\frac{4}{8}$ by $\frac{4}{8}\times\frac{4}{7}$, and multiply the quotient by $\frac{4}{7}$.

429. If one pound of rice costs 9 cents, what will $\frac{5}{18}$ of a pound cost?

430. If one barrel of flour costs $\frac{72}{8}$ of a dollar, how much is $\frac{32}{8}$ of a barrel worth? $\frac{3}{8}$ of a barrel?

431. One pair of boots is worth $\frac{46}{7}$ dollars; what will 46 pairs be worth?

432. One gallon of molasses costs $\frac{137}{8}$ cents; at that rate what will $\frac{16}{8}$ of a gallon cost? $\frac{4}{7}$ of a gallon?

433. A certain ship is worth \$46070; how much is $\frac{9}{16}$ of the ship worth? $\frac{5}{38}$? $\frac{9}{80}$?

434. My property is worth \$4680; how much is $\frac{5}{18}$ of it worth? $\frac{3}{20}$?

435. If one barrel of salt is worth $\frac{4}{10}$ of 8 dollars, what is $\frac{5}{9}$ of a barrel worth? $\frac{3}{8}$ of a barrel? $\frac{3}{15}$ of a barrel?

436. If one barrel of salt is worth $\frac{7}{25}$ of 10 dollars, what is $\frac{7}{15}$ of 50 barrels worth? $\frac{10}{11}$ of 9 barrels?

QUESTIONS.—To what does the denominator of a fraction correspond in division? (135.) To what the numerator? (135.) If the dividend and divisor are both divided by the same number, how is the quotient affected? If the same factor is rejected from both numerator and denominator, how is the quotient affected? If both nnmerator and denominator are divided by the same number, how is the value of the fraction affected? (136., *b.*) If a factor be rejected from the numerator, how is the value of the fraction affected? (136., *b.*)

LESSON XLII.

437. How many books at $\frac{5}{9}$ of a dollar apiece can I purchase for $\frac{15}{18}$ of 12 dollars? for $\frac{3}{9}$ of 18 dollars?

438. How many quires of paper at $\frac{125}{10}$ cents a quire can I purchase for $\frac{34}{7}$ of 25 dollars?

439. I have 25 times $\frac{3}{9}$ of a dollar with which to purchase inkstands at $\frac{41}{3}$ cents apiece; how many can I buy?

440. I have 124 times $\frac{4}{18}$ of a dollar with which to purchase matches at $\frac{2}{9}$ of a cent a box; how many boxes can I buy?

441. $\frac{34}{7}$ of a pound of tea cost $\frac{328}{56}$ shillings; how much is that per pound?

MODEL OPERATION.

$$\frac{328}{56}\text{s.}\div\frac{34}{7}=\frac{\overset{41}{\cancel{328}}}{\underset{8}{\cancel{56}}}\text{s.}\times\frac{\cancel{7}}{34},=\frac{41}{34}\text{s.},=1\tfrac{7}{34}\text{s.}$$

Solution.—If $\frac{34}{7}$ pounds of tea cost $\frac{328}{56}$ shillings, $\frac{1}{7}$ of a pound will cost $\frac{1}{34}$ of $\frac{328}{56}$ shillings, and $\frac{7}{7}$, or one pound, will cost $\frac{7}{34}$ of $\frac{328}{56}$ shillings, which is $\frac{41}{34}$s.,$=1\frac{7}{34}$ of a shilling.

Therefore, if $\frac{34}{7}$ pounds of tea cost $\frac{328}{56}$ shillings, one pound costs $1\frac{7}{34}$ of a shilling.

442. $\frac{1}{5}$ of a pound of tea costs $\frac{21}{15}$ of a shilling; what does one pound cost?

443. $\frac{3}{7}$ of a gallon of vinegar costs $\frac{27}{21}$ cents; what does one gallon cost?

444. If $\frac{43}{9}$ of a quart of molasses costs $\frac{25}{27}$ of a shilling, what will one quart cost?

445. What will one gallon of wine be worth, if $\frac{5}{18}$ of a hogshead (63 gal.) costs $\frac{61}{5}$ dollars?

446. What will one quart of wine be worth, if $\frac{4}{18}$ of a hogshead is worth $\frac{65}{4}$ dollars?

Questions.—What is a quotient? (57.) What is the value of a fraction? (139.) What is the reduction of fractions? (140.) What is a common divisor? (118.) What is the greatest common divisor? (119.) What is a multiple of a number? (120.) What is the difference between a common and the least common multiple of two or more numbers? (121.) (122.)

LESSON XLIII.

447. What will $\frac{26}{5}$ pounds of beef cost, at $\frac{36}{7}$ cents per pound? at $\frac{42}{9}$ cents? at $\frac{11}{3}$ cents?

448. I sold $\frac{42}{7}$ cords of wood, at $\frac{12}{3}$ dollars a cord; what did I receive?

449. A man having $\frac{4863}{9}$ bushels of potatoes, wished to put them into barrels holding $\frac{42}{11}$ bushels each; how many barrels did he need?

450. A farmer had $\frac{4172}{11}$ pounds of butter, which he wished to put into pots holding $\frac{111}{4}$ pounds; how many pots did it require?

451. 16 horses cost $\frac{11374}{8}$ dollars; what does one horse cost?

452. $\frac{47}{3}$ bushels of beans cost $\frac{83}{5}$ dollars; what does one bushel cost?

453. 14 times $\frac{3}{8}$ of a pound of candles costs 19 shillings; what does one pound cost?

454. $\frac{4}{5}$ of 14 pounds of soap cost 9 times $\frac{4}{7}$ of a dollar; what does one pound cost?

455. $\frac{3}{9}$ of 5 gallons of vinegar is worth 4 times $\frac{11}{3}$ cents; what is one gallon worth?

456. $\frac{4}{9}$ of 18 pounds of pork is worth $\frac{2}{5}$ of 350 cents; what is one pound worth?

QUESTIONS.—What is the difference between a common and a decimal fraction? (124.) (125.) Is a common fraction ever used without a denominator expressed either by words or figures? Is a decimal? How is the denomination of a decimal fraction usually expressed? Is a decimal fraction ever written in the same way as a common fraction? Is it ever used in the same way? What are the terms of a fraction? (126.)

LESSON XLIV.

MISCELLANEOUS EXAMPLES FOR ANALYSIS.

If $4\frac{3}{4}$ cwt. of hay cost \$5.40, what is the value of $3\frac{7}{9}$ cwt.?

MODEL OPERATION.

ANAL. STEPS. 1. $4\frac{3}{4}=\frac{19}{4}$ cwt.

2. $3\frac{7}{9}=\frac{34}{9}$ cwt.

3. $\$5.40\div19=\$0.28\frac{8}{19}$, cost of $\frac{1}{4}$ cwt.

4. $\$0.28\frac{8}{19}\times4=\$1.13\frac{13}{19}$, " $\frac{4}{4}$ cwt.

5. $\$1.13\frac{13}{19}\div9=\$0.12\frac{12}{19}$, " $\frac{1}{9}$ cwt.

6. $\$0.12\frac{12}{19}\times34=\$4.29\frac{9}{19}$, " $\frac{34}{9}$ cwt.

ANALYSIS.—1st. Change $4\frac{3}{4}$ to 4ths.

2nd. Change $3\frac{7}{9}$ to 9ths.

3rd. Find the cost of $\frac{1}{4}$ cwt.

If $\frac{19}{4}$ of a cwt. of hay costs \$5.40, $\frac{1}{4}$ will cost one *nineteenth* of \$5.40, which is \$0.28$\frac{8}{19}$.

4th. Find the cost of 1 cwt.

If $\frac{1}{4}$ of a cwt. costs \$0.28$\frac{8}{19}$, $\frac{4}{4}$, or 1 cwt., will cost 4 times \$0.28$\frac{8}{19}$, which is \$1.13$\frac{13}{19}$.

5th. Find the cost of $\frac{1}{9}$ cwt.

If $\frac{9}{9}$, or 1 cwt., costs \$1.13$\frac{13}{19}$, $\frac{1}{9}$ cwt. will cost one *ninth* of \$1.13$\frac{13}{19}$, which is \$0.12$\frac{12}{19}$.

6th. Find the cost of $\frac{34}{9}$ cwt.

If $\frac{1}{9}$ cwt. costs \$0.12$\frac{12}{19}$, $\frac{34}{9}$ will cost 34 times \$0.12$\frac{12}{19}$, which is \$4.29$\frac{9}{19}$.

Therefore, if 4$\frac{3}{4}$ cwt. of hay cost \$5.40, 3$\frac{7}{9}$ cwt. will cost \$4.29$\frac{9}{19}$.

QUESTIONS.—What is the difference of the uses of the terms of a fraction? (127.) (128.) What is the difference between a proper and an improper fraction? (129.) (130.) What is a mixed number? (131.) What is the difference between a simple and a compound fraction? (131., *a.*) (133.) Of what may fractions be considered as indications? (134.) What are the corresponding terms in division and fractions? (135.)

LESSON XLV.

457. If one yard of cloth costs \$5.40, what will $\frac{3}{7}$ of a yard cost?

458. If a barrel of flour costs \$8.50, what will $\frac{3}{7}$ of a barrel cost?

459. If a load of hay costs \$17.30, what will $\frac{3}{7}$ of a load cost?

460. If \$786.47 are paid for a cargo of wheat, what is the cost of $\frac{3}{11}$ of a cargo?

461. What is $\frac{3}{4}$ of 15£. 18s. 9d.?*

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*NOTE.—$\frac{3}{4}$ of any thing or number equals one *fourth* of *three* times the thing or number; hence:—1. Find 3 times the number. 2. Find $\frac{1}{4}$ of *three* times the number.

462. If 1£. 17s. are paid for 7 yards of cloth, what is the price of one yard?

463. If $\frac{3}{10}$ of a barrel of flour costs 1£. 13s. 7d., what will be the cost of a whole barrel?

464. When $13.746 are paid for $\frac{7}{8}$ of a ton of hay, what will 1 ton cost?

465. When $\frac{5}{7}$ of a cargo of wheat costs $887.40, what sum will pay for the whole cargo?

466. If 73.60\frac{3}{7}$ are paid for $\frac{5}{12}$ of a ton of potash, what will 1 ton cost?

467. If $\frac{4}{17}$ of an acre produce 18 cwt. 12 lbs. of hay, what quantity will a whole acre produce?

468. If $\frac{7}{11}$ of a certain lot of land equals 12 A. 2 R. 30$\frac{7}{8}$ sq. rds., what is the area of the field?

469. $\frac{11}{13}$ of a ton of potash costs 83.40\frac{6}{7}$; what is the value of 3 tons?

Questions.—When both numerator and denominator are increased or decreased in any ratio, how is the value of the fraction affected? (136., *a.*, *b.*) Why does dividing the numerator of a fraction decrease its value? (66., *k.*) Why does dividing the denominator of a fraction increase its value? (66., *i.*) Why does multiplying the numerator of a fraction increase its value? (66., *j.*) Why does multiplying the denominator of a fraction decrease its value? (66., *h.*)

LESSON XLVI.

470. If $\frac{3}{4}$ of a cwt. of sugar costs $7.40, what is the value of $\frac{3}{17}$ of a cwt.?

471. If $\frac{7}{11}$ of a pound of opium costs $2.71, what is the value of $\frac{5}{7}$ of a pound?

472. When 80\frac{3}{4}$ are paid for $\frac{3}{4}$ of an acre of land, what will $\frac{7}{9}$ of an acre cost?

473. If $\frac{9}{13}$ of a cotton factory is worth 4370\frac{9}{11}$, what is $\frac{5}{14}$ of it worth?

474. $\frac{3}{7}$ of a ship is worth \$147.37$\frac{4}{7}$; what is $\frac{5}{29}$ of it worth?

475. I sold $\frac{3}{8}$ of a farm for \$178$\frac{4}{5}$; what is the remainder worth?

476. I sold $\frac{3}{4}$ of a farm for \$371$\frac{2}{3}$; what is $\frac{2}{3}$ of the remainder worth, at the same rate?

477. A man bought $\frac{3}{8}$ of a farm containing 178$\frac{4}{7}$ acres for \$1728; what was the cost of one acre?

478. A man bought $\frac{3}{5}$ of a farm containing 23$\frac{4}{11}$ acres for \$4712; what was the cost of $\frac{3}{7}$ of an acre?

479. A man bought $\frac{2}{5}$ of a farm containing 437$\frac{2}{3}$ acres for \$2744.26; how much should he receive for 23$\frac{3}{7}$ acres, in order that he may neither gain nor lose?

480. A man bought $\frac{3}{5}$ of a farm containing 371$\frac{4}{9}$ acres for \$2343.27; how much should he receive for 13$\frac{3}{5}$ acres, so as to gain \$342.42 on the amount sold?

LESSON XLVII.

481. Mr. Jones has a field 13$\frac{4}{7}$ rds. long, 11$\frac{9}{13}$ rds. wide, and he sells it for \$73.46; what does he receive for 3$\frac{3}{7}$ sq. rds.?

482. An English merchant paid 27£. 12s. 6d. for $\frac{8}{9}$ of a bale of cloth; what did $\frac{2}{3}$ of the remainder cost?

483. If 7$\frac{3}{4}$ cwt. of sugar cost \$29.13, what will 9$\frac{3}{7}$ cwt. cost?

484. If 3 tons of hay cost \$49$\frac{7}{9}$, what will 17$\frac{4}{5}$ tons cost?

485. Gave \$40$\frac{3}{8}$ for 5$\frac{4}{7}$ yards of broadcloth; what did 19$\frac{3}{7}$ yards cost?

486. 7$\frac{3}{7}$ bushels of rye cost \$8.73; what did 18$\frac{7}{13}$ bushels cost?

487. If I pay \$13.84 for 2$\frac{3}{5}$ yds. of broadcloth, what must I pay for 11$\frac{4}{7}$ yds.?

488. If $9\frac{3}{8}$ cwt. of sugar cost \$46.47, what must be paid for $7\frac{3}{4}$ cwt.

489. When \$$18\frac{7}{9}$ are paid for 3 cwt. of rice, how much may be purchased for \$1?

490. If $6\frac{4}{7}$ tons of potash cost \$276.43, how many tons can I purchase for \$596.46?

491. If $3\frac{4}{7}$ acres of land cost \$764, what will 13 acres cost?

492. If $4\frac{3}{7}$ tons of coal cost \$$70\frac{3}{4}$, what will one ton cost? what will $46\frac{3}{7}$ tons cost?

QUESTIONS.—What is the reduction of fractions? (140.) What is reduction ascending? (141.) What is reduction descending? (142.) Is the reduction of fractions to their lowest terms reduction ascending or reduction descending? (143.) What is the rule for reducing a fraction to its lowest terms? (144., *b.*) Give the rule for reducing an improper fraction to a mixed number. (145.) A mixed number to an improper fraction. (146.)

LESSON XLVIII.

493. Sold a farm for \$896.50; what was received for $\frac{3}{11}$ of it?

494. I gave \$$27\frac{3}{5}$ for $3\frac{4}{7}$ barrels of flour; what will $37\frac{2}{3}$ barrels cost?

495. Sold a house for \$$3874\frac{2}{3}$; what sum was received for $\frac{7}{8}$ of it?

496. When \$$46\frac{2}{3}$ are paid for 100 gallons of molasses, what will $\frac{3}{7}$ of a gallon cost?

497. When 112 cents are paid for $9\frac{4}{7}$ gallons of molasses, what will $48\frac{3}{13}$ gallons cost?

498. If $3\frac{4}{9}$ barrels of flour cost $10\frac{2}{7}$, what will $6\frac{3}{5}$ barrels cost?

499. Paid, in Liverpool, $197\frac{4}{7}$£. for 50 bales of cloth; how many bales should be received for $4320\frac{7}{11}$£.?

500. $3\frac{5}{7}$ lbs. of coffee cost $50\frac{3}{4}$ cts.; what sum must be paid for $74\frac{2}{3}$ lbs.?

501. If $2\frac{5}{7}$ tons of hay cost \$$63\frac{5}{7}$, what will be the cost of $16\frac{4}{9}$ tons?

502. If a piece of land $\frac{7}{8}$ rods square cost \$$27\frac{5}{7}$, what will be the cost of $4\frac{5}{9}$ square rods?

503. A certain piece of land $3\frac{4}{7}$ rods square is worth \$$57.23\frac{4}{9}$; what will be the worth of 7 acres of land, at that rate?

QUESTIONS.—What is the rule for reducing a mixed number to an equivalent fraction having a given denomination? (147.) What is the addition of fractions? (148.) What is the subtraction of fractions? (150.) What is the rule for the addition or subtraction of fractions, each having a unit for a numerator? (151.) What is the multiplication or the division of fractions? (152.)

LESSON XLIX.

504. $3\frac{5}{7}$ cwt. of iron cost \$$45\frac{7}{8}$; what must be paid for $689\frac{4}{13}$ cwt.?

505. For $6\frac{2}{3}$ cords of wood I paid \$$63.41\frac{2}{3}$; what must I pay, at the same rate, for 1864 cu. ft.?

506. Gave \$$243\frac{1}{11}$ for 96 barrels of tar; how much can I buy for \$2000?

507. Paid \$7888.30 for $72\frac{5}{7}$ acres of land; how much, at that rate, would $39\frac{2}{3}$ acres cost?

508. If for $17\frac{2}{3}$ days' work I pay \$$27\frac{2}{3}$, at that rate, how much should I pay for $47\frac{5}{7}$ days' labor?

509. Sold $5\frac{7}{12}$ bushels of apples, for \$$7.28\frac{2}{3}$; what should I receive for $19\frac{5}{7}$ bushels?

510. $\frac{5}{7}$ of a ship is worth \$$3746\frac{2}{3}$; what is the entire ship worth?

511. $\frac{3}{4}$ of $\frac{2}{3}$ of a ship is worth \$$427\frac{4}{9}$; what is the entire vessel worth?

512. $\frac{5}{8}$ of $\frac{1}{7}$ of a vessel is worth $374.26; what is $\frac{3}{7}$ of the vessel worth?

513. If $\frac{3}{7}$ of 5 times the value of a vessel is $46748, what is its value?

514. If 3 times $\frac{3}{7}$ of the value of a vessel is $4937.24, what is $\frac{2}{5}$ of its value?

515. I bought $\frac{5}{8}$ of a house for $347.22; at that rate, what should I give for $\frac{3}{7}$ of 19 such houses?

QUESTIONS.—What is the rule for multiplying a fraction by an integral number? (153.) What is the rule for dividing a fraction by an integral number? (154.) Give the rule for dividing a mixed number by an integral number. (155.) What is the rule for multiplying a fraction by a fraction? (156.) How can an integral number be changed to a fraction? ANS.—*By placing a unit for its denominator.*

LESSON L.

REDUCTION OF FRACTIONAL DENOMINATE NUMBERS.

164. Reduction Descending of fractional compound numbers is the same as that of integral compound numbers.

Change $\frac{1}{2160}$£. to farthings.

MODEL OPERATION.

(*a.*)

1. $\frac{1}{2160}\pounds.\times 20=\frac{1}{108}\text{s.}$
2. $\frac{1}{108}\text{s.}\times 12=\frac{1}{9}\text{d.}$
3. $\frac{1}{9}\text{d.}\times 4=\frac{4}{9}$ far., Ans.

(*b.*)

By Cancellation.

$$\underset{108}{\frac{1}{\cancel{2160}}}\pounds.\times\frac{\cancel{20}}{1}=\frac{1}{108}\text{s.};\ \underset{9}{\frac{1}{\cancel{108}}}\text{s.}\times\frac{\cancel{12}}{1}=\frac{1}{9}\text{d.};\ \frac{1}{9}\text{d.}\times\frac{4}{1}=\frac{4}{9}\text{ far.,}$$

(*a.*) ANALYSIS.—1. Since in 1 pound there are 20 shillings, in $\frac{1}{2160}$ of a pound there are 20 times as many shillings as pounds, which is $\frac{1}{108}$s.

2. Since in 1 shilling there are 12 pence, in $\frac{1}{108}$s. there are 12 times as many pence as shillings, which is $\frac{1}{9}$d.

3. Since in one penny there are 4 far., in $\frac{1}{9}$d. there are 4 times as many farthings as pence, which is $\frac{4}{9}$ far.

Therefore, $\frac{1}{2160}$£. is equal to $\frac{4}{9}$ of a farthing.

Change $\frac{3}{9}$ of a furlong to its equivalent, integral numbers of lower denominations.

MODEL OPERATION.

1. $\frac{3}{9}$ fur.$\times 40 = \frac{120}{9}$ rds.,$= 13\frac{1}{3}$ rds.
2. $\frac{1}{3}$ rd.$\times \frac{11}{2} = \frac{11}{6}$ yds.,$= 1\frac{5}{6}$ yds.
3. $\frac{5}{6}$ yds.$\times 3 = \frac{5}{2}$ ft.,$= 2\frac{1}{2}$.
4. $\frac{1}{2}$ ft.$\times 12 = \frac{12}{2}$ ft.,$= 6$ in.

Therefore, $\frac{3}{9}$ fur.$=$13 rds. 1 yd. 2 ft. 6 in.

ANALYTICAL STEPS.—1. Change $\frac{3}{9}$ fur. to rds.
2. Change $\frac{1}{3}$ rd. to yds.
3. Change $\frac{5}{6}$ yd. to ft.
4. Change $\frac{1}{2}$ ft. to in.

NOTE.—The analysis for denominate fractions is essentially the same as that for denominate numbers. The teacher should require the scholar to analyze every example until he can explain rapidly and accurately.

QUESTIONS.—What is the reduction of denominate numbers? (79.) What is reduction ascending? (81.) Reduction descending? (80.) Repeat the table of English money. (82., *b.*) Repeat the table of Troy weight. (84., *a.*) Avoirdupois weight. (85., *a.*) Apothecaries' weight. (88., *a.*) Long measure. (90., *a.*) Square measure. (92., *f.*) Surveyors' square measure. (93., *a.*) Cubic measure. (94., *e.*)

LESSON LI.

EXAMPLES FOR PRACTICE.

516. Change $\frac{1}{1400}$ of a pound to farthings.

517. Reduce $\frac{4}{75}$ of a shilling to pence.

518. Reduce $\frac{1}{8640}$ lb. Troy to grains.

519. How many farthings in $3\frac{7}{9}$£.?

520. How many ounces in $\frac{1}{1728}$ cwt.?

521. How many square feet in $\frac{4}{7124}$ of an acre?

522. How many seconds in $\frac{3}{4164}$ of a day?

523. How many drams in $\frac{6}{743}$ cwt.?

524. How many gills in $7\frac{3}{4}$ hhds.?

525. How many grs. in $3\frac{3}{7}$℔. $2\frac{4}{7}$ʒ. $2\frac{3}{4}$℈.?

526. How many in. in $2\frac{4}{7}$ mi. $5\frac{3}{4}$ fur. $2\frac{5}{8}$ rds. $2\frac{1}{4}$ ft.?

QUESTIONS.—What is meant by $\frac{1}{1400}$ of a pound? ANS.—*It means that it is* ONE *of the* 1400 *equal parts into which a pound is divided.* What are the steps in changing acres to square feet? Into how many equal parts is an acre divided, and what are the parts called? How many of them make an acre? Which is larger, a rood, or a square rod? Repeat the table of liquid measure. (95., *a.*)

LESSON LII.

527. How many square feet in $4\frac{3}{7}$ A. $2\frac{9}{124}$ R.?

528. Reduce $5\frac{3}{7}$ wks. $2\frac{4}{5}$ da. to seconds.

529. Reduce $\frac{3}{7}$ T. $2\frac{1}{8}$ qr. $\frac{3}{4}$ lb. to drams.

530. Reduce $\frac{4}{9}$ cd. $6\frac{5}{7}$ cu. ft. to cu. in.

531. Reduce $\frac{5}{7}$ cd. $2\frac{4}{7}$ cu. ft. $1\frac{2}{7}$ cu. in. to cu. in.

532. Reduce $\frac{3}{467}$ cd. to cu. ft.

533. Reduce $34\frac{1}{7}$£. $\frac{1}{12}$s. 2d. $\frac{3}{7}$ far. to far.

534. Reduce $3\frac{3}{8}$ cwt. $1\frac{2}{3}$ qrs. to drams.

535. How many inches in $\frac{3}{7}$ deg. $\frac{3}{4}$ mi. $\frac{3}{4}$ fur. $\frac{3}{4}$ ft.?

536. How many square miles in $\frac{2}{3}$ sq. mi. $\frac{3}{4}$ sq. rd.?

537. How many square links in $\frac{3}{4}$ sq. mi. $\frac{3}{4}$ sq. ch. $2\frac{3}{4}$ sq. rd.?

538. Reduce $\frac{7}{8}$ cwt. to its equivalent, integral numbers of lower denominations.

539. Reduce $\frac{4}{5}$ cwt. to its equivalent in lower denominations.

540. Reduce $\frac{7}{8}$ of a yard to its equivalent in lower denominations.

541. Reduce $\frac{3}{7}$ of an acre to lower denominations.

542. Reduce $\frac{5}{7}$ of an acre to lower denominations.

543. Reduce $\frac{5}{7}$ of a sq. mi. to lower denominations.

544. Reduce $\frac{3}{8}$ of a mi. to lower denominations.

545. Reduce $\frac{3}{7}$ of a yr. to lower denominations.

546. What is the sum of $\frac{5}{8}$ fur. and $\frac{3}{7}$ rds.?

547. What is the sum of $\frac{3}{4}$ bu. and $\frac{5}{8}$ pk.?

QUESTIONS.—What is an integer? (106.) What is the first step in changing a hundred weight to ounces? Is it an operation in reduction ascending, or reduction descending? Why? Which is the greater, a sq. yard, or a sq. rod? How can sq. yards be reduced to sq. rods? How can sq. rods be changed to sq. yards? What is the sum of two or more numbers? (30.) Can ounces be reduced to gallons? Why? Can gallons be changed to gills? Why?

LESSON LIII.

548. Bought $34\frac{2}{3}$ lbs. of gold dust, at $16 per ounce; what did it cost?

549. Bought $21\frac{5}{8}$ lbs. $\frac{3}{4}$ oz. of gold dust, at $0.80 per pwt.; what was the cost?

550. What is the cost of 3 lbs. $9\frac{5}{7}$ oz. of gold dust, at $4\frac{3}{7}$ cts. per grain?

551. Bought a gold chain weighing $2\frac{1}{7}$ oz. $\frac{4}{5}$ gr., at $9\frac{3}{7}$ cts. per grain; what was the cost?

552. What will be the cost of $11\frac{3}{7}$ cwt. of hay, at $2\frac{3}{4}$ cts. per pound?

553. What will be the cost of $3\frac{3}{7}$ T. $4\frac{5}{8}$ cwt. 2 qr. of hay, at $2\frac{2}{3}$ cts. per pound?

554. What will be the cost of a hogshead of sugar, weighing 9 cwt. $1\frac{3}{7}$ qr. 12 lbs., at $0.05 per pound?

555. Bought $4\frac{2}{3}$ hhds. of sugar, each containing $2\frac{2}{3}$ cwt. 2 qr. $12\frac{5}{7}$ lbs., at 0.03\frac{1}{3}$ per pound; what did they cost?

556. Bought $13\frac{5}{7}$ T. of iron, at $2\frac{1}{2}$d. per pound, and sold it so as to gain 17£. 13s. 8d. 3 far.; for how much did I sell it?

557. Bought $365\frac{2}{7}$ cwt. of lead, at $9\frac{3}{5}$d. per pound, and sold it so as to gain 14£. 13s. 8d. 5 far.; for how much did I sell it?

QUESTIONS.—What is English money? (82., *a.*) What are the Gold coins? (82., *c.*) Silver coins? (82., *d.*) Copper? (82., *e.*) What is weight? (83.) What is the use of Troy weight? (84.) When gold has six parts alloy, how many carats fine is it said to be? (84., note.) What is the usual degree of fineness of gold? What is the use of Avoirdupois weight? (85.) Repeat the table. (85., *a.*) Which is the heavier, a pound Troy, or a pound Avoirdupois? (88., *b.*) What is the use of Apothecaries' weight? (88., *a.*)

LESSON LIV.

165. The **Reduction Ascending** of fractional, compound numbers, is the same as that of integral, compound numbers.

Reduce $\frac{4}{9}$ far. to the fraction of a pound.

MODEL OPERATION.

(*a.*)

1. $\frac{4}{9}$ far. $\times \frac{1}{4} = \frac{1}{9}$d.
2. $\frac{1}{9}$d. $\times \frac{1}{12} = \frac{1}{108}$s.
3. $\frac{1}{108}$s. $\times \frac{1}{20} = \frac{1}{2160}$£.

(*b.*)

By Cancellation.

$$\frac{\cancel{4}}{9}\text{ far.} \times \frac{1}{\cancel{4}} = \frac{1}{9}\text{d.};\ \frac{1}{9}\text{d.} \times \frac{1}{12} = \frac{1}{108}\text{s.};\ \frac{1}{108} \times \frac{1}{20} = \frac{1}{2160}\text{£.}$$

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Analysis.—(*a.*) 1. Since in 4 far. there is 1 penny, in $\frac{4}{5}$ of a far. there are $\frac{1}{4}$ as many pence as farthings, which is $\frac{1}{5}$d.

2. Since in 12d. there is one shilling, in $\frac{1}{5}$ of a penny there is $\frac{1}{12}$ as many shillings as pence, which is $\frac{1}{60}$s.

3. Since in 20 shillings there is 1 pound, in $\frac{1}{60}$s. there is $\frac{1}{20}$ as many pounds as shillings, which is $\frac{1}{1200}$£.

Therefore, in $\frac{4}{5}$ of a farthing there is $\frac{1}{1200}$ of a pound.

Questions.—Do Apothecaries buy and sell medicines by Apothecaries' weight? (88.) How many grains in a pound Avoirdupois? (88., *b.*) How can 25 lbs. Troy be reduced to pounds Avoirdupois? **Ans.**—*Reduce 25 lbs. Troy to grains, and then the grains to Avoirdupois pounds and a fraction of a pound.* What can then be done with the fraction of a pound Avoirdupois? **Ans.**—*Reduce it to lower denominations.*

LESSON LV.

558. Reduce $\frac{4}{7}$ of a grain Troy to the fraction of a pound.
559. Reduce $\frac{3}{7}$ of a scruple to pounds.
560. Reduce $\frac{3}{7}$ of an ounce to the fraction of a ton.
561. How many tons in $\frac{5}{9}$ of a pound?
562. What fraction of a mile is $\frac{7}{3}$ rds.?
563. What fraction of a mile is $\frac{434}{3}$ inches?
564. What fraction of an acre is $\frac{472}{8}$ sq. ft.?
565. What fraction of a month is 13 sec.?
566. What fraction of an acre is 25 sq. ft.?
567. What part of a hogshead is 13 gills?

Questions.—In example 567, what is to be done with the 13 gills? **Ans.**—*Reduce them to hogsheads.* What are the steps? What are the dimensions of extension? (89.) What are the dimensions of a line? (89., *a.*) What are the dimensions of a surface? (89., *b.*) What is a solid? (89., *c.*) What is the use of Long measure? (90.) Repeat the table of Long measure. (90., *a.*) What is the use of Gunter's chain measure? (91.) Repeat the table. (91., *a.*)

LESSON LVI.

Reduce 2 mi. 3 fur. 20 rds. 3 yds. 2 ft. 4 in. to a fraction of a mile.

MODEL OPERATION.

ANALYTICAL STEPS.

1. 4 in.$\times\frac{1}{12}=\frac{1}{3}$ ft.; $\frac{1}{3}$ ft.$+$2 ft.$=\frac{7}{3}$ ft.
2. $\frac{7}{3}$ ft.$\times\frac{1}{3}=\frac{7}{9}$ yds.; $\frac{7}{9}$ yds.$+$3 yds.$=\frac{34}{9}$ yds.
3. $\frac{34}{9}$ yds.$\times\frac{2}{11}=\frac{68}{99}$ rds.; $\frac{68}{99}$ rds.$+$20 rds.$=\frac{2048}{99}$.
4. $\frac{2048}{99}$ rds.$\times\frac{1}{40}=\frac{256}{495}$ fur.; $\frac{256}{495}$ fur.$+$3 fur.$=\frac{1741}{495}$ fur.
5. $\frac{1741}{495}$ fur.$\times\frac{1}{8}=\frac{1741}{3960}$ mi.; $\frac{1741}{3960}$ mi.$+$2 mi.$=2\frac{1741}{3960}$ mi., Ans.

ANALYSIS.—1. Reduce 4 inches to feet, and add the 2 ft.

Since in 12 inches there is one foot, in 4 inches there are $\frac{1}{12}$ as many feet as inches, which is $\frac{1}{3}$ of a foot; $\frac{1}{3}$ of a foot added to 2 feet equals $\frac{7}{3}$ feet.

2. Reduce $\frac{7}{3}$ feet to yards, and add the three yards.

Since in 3 feet there is 1 yard, in $\frac{7}{3}$ feet there are $\frac{1}{3}$ as many yards as feet, which is $\frac{7}{9}$ of a yard; $\frac{7}{9}$ of a yard added to 3 yards equals $\frac{34}{9}$ yds.

3. Reduce $\frac{34}{9}$ yards to rods, and add the 20 rods.

Since in $\frac{11}{2}$ yds. ($5\frac{1}{2}$ yds.) there is one rod, in $\frac{34}{9}$ yards there are $\frac{2}{11}$ as many rods as yards, which are $\frac{68}{99}$ of a rod; $\frac{68}{99}$ of a rod added to the 20 rds. equals $\frac{2048}{99}$ rods.

4. Reduce $\frac{2048}{99}$ rds. to furlongs, and add the 3 fur.

5. Reduce $\frac{1741}{495}$ fur. to miles, and add the 2 mi.

Therefore, 2 mi. 3 fur. 20 rds. 3 yds. 2 ft. 4 in. are equal to $2\frac{1741}{3960}$ mi.

NOTE.—Should the teacher deem it necessary, he may require the analysis of the addition.

QUESTIONS.—What is the use of Square measure? (92., *c.*) Repeat the table. (92., *f.*) How do artificers estimate their work? (92., *d.*) For what is Surveyors' square measure used? (93.) Repeat the table. (93., *a.*) If we know the length and breadth of a square or rectangular piece of land, how do we find the surface, or area? Ans.—*Multiply the length by the breadth.* What is a cube? (94.) What is the use of Cubic measure? (94., *c.*) Repeat the table of Cubic measure. (94., *e.*) What is Liquid measure? (95., *a.*) Repeat the table. (95., *b.*)

LESSON LVII.

EXAMPLES FOR PRACTICE.

568. Reduce 4s. 8d. to a fraction of a pound.

569. Reduce 4 cwt. 3 qr. 12 lbs. to tons.

570. Reduce 33 rds. 4 yds. 7 in. to miles.

571. What part of a mile is 23 rds. 3 yds. 6 ft.?

572. What part of a hogshead is 3 gal. $2\frac{2}{3}$ gi.?

573. What part of a cord is 37 cu. ft. $39\frac{4}{7}$ cu. in.?

574. How many cords of wood in a pile containing 474 cu. ft.?

575. How many acres in $3\frac{1}{4}$ R. 9 sq. rds. $7\frac{2}{3}$ sq. yds. 8 sq. in.?

576. How many sq. yds. in a field $15\frac{3}{7}$ ft. long, and $3\frac{2}{7}$ ft. wide?

577. Bought a piece of land $148\frac{3}{7}$ rds. long, $15\frac{3}{7}$ chains wide; how much did it cost, at $37. per acre?

Questions.—What is the use of Dry measure? (96.) Repeat the table. (96., *a.*) How many cubic inches in one gallon wine measure? (96., *c.*) Repeat the comparative table of measures of capacity. (96., *c.*) How can 25 gallons, Liquid measure, be reduced to bushels? Ans.—*Reduce the gallons to cubic inches, and the cubic inches to bushels, and a fraction of a bushel; and the fraction of a bushel to an equivalent of lower denominations.* Reduce 45 bushels to gallons, Liquid measure, and give the analysis.

LESSON LVIII.

578. What will be the cost of 2 bu. 2 pk. 3 qt. of potatoes, at $0.37 per bushel?

ANALYSIS.—1. Change 2 pks. 3 qts. to a fraction of a bushel.

2. Find the cost of the potatoes.

579. What will 20 barrels of potatoes cost, at 75 cts. a bushel, each barrel containing 2 bu. 3 pks. 5¾ qts.?

580. A grocer bought 14 barrels of sugar, each weighing 2 cwt. 1 qr. 15 lbs., at $5.75 per cwt.; how much did it cost him?

581. What will 37 yds. 4⅜ qrs. of silk cost, at $1.37½ per yard?

582. What will 3 lbs. 6 oz. of sugar cost, at $10 per cwt.?

583. What will 1 day and 3 hours' labor be worth, at $17 per month, allowing 26 days to the month, and 10 hours to the day?

584. How much should I pay a man for working 7 hours 42 min. 12 sec. at $1.75 for 10 hours?

585. How many cords of wood in a pile 37 ft. long, 4 ft. wide, and 7$\frac{3}{7}$ ft. high?

586. At $5.75 a cord, what would be the cost of a pile of wood 4 ft. 6 in. wide, 13 ft. 7 in. long, 13 ft. 2 in. high?

587. What is the cost of a pile of oak bark, 378 ft. long, 11 ft. 3 in. wide, 13 ft. 4 in. high, at $10 per cord?

QUESTIONS.—What is the axiom for the comparison of numbers? (16., *c*.) Can ounces be compared with houses? Why? Can pounds be compared with acres? Bushels with pecks? Ounces with drams? Potatoes with corn? Bushels of potatoes with bushels of corn? *Fifths* with *thirteenths?* *Tenths* with *fifteenths?* *Tenths* of an apple, with a *tenth* of a melon? A *fifth* of an apple with a *third* of an apple?

LESSON LIX.

COMPARISON OF NUMBERS.

166. Comparison of Numbers is the process of finding what part one number is of another when compared with it; as, 3 is what part of 7? ANS.—$\frac{3}{7}$. 5 ounces is what part of 10 ounces? ANS.—$\frac{5}{10}=\frac{1}{2}$. $3 is what part of $7? ANS.—$\frac{3}{7}$.

167. Before concrete or fractional numbers can be compared, they must, necessarily, be reduced to the same denomination.

(*a.*) 4s. 8d. is what part of 1£.?

(*b.*) What part of 2 mi. 3 fur. is 7 rds. 5 yds.?

MODEL OPERATION.

(*a.*) 4s. 8d.=56d.; 1£.=240d.; 56d. is $\frac{56}{240}$ of 240d.; $\frac{56}{240}=\frac{7}{30}$, Ans.

(*b.*) 7 rds. 5 yds.=$43\frac{1}{2}$ yds.; 2 mi. 3 fur.=4180 yds.; $43\frac{1}{2}$ yds. is $\frac{43\frac{1}{2}}{4180}$ of 4180 yds.; $\frac{43\frac{1}{2}}{4180}\times\frac{2}{2}=\frac{87}{8360}$, Ans.

Therefore, 7 rds. 5 yds. are $\frac{87}{8360}$ of 2 mi. 3 fur.

EXAMPLES FOR PRACTICE.

588. What part of an ounce is $\frac{3}{5}$ of a scruple?

589. What part of 2 ounces is $3\frac{4}{5}$ of a dram?

590. What part of $2\frac{1}{2}$ mi. is 4 rds. 7 fur.?

591. What part of a surface 5 rds. square, is 5 sq. rds.?

592. What part of $\frac{1}{2}$ wk. is 3 m. $2\frac{4}{7}$ sec.?

593. How many times greater than 5 sq. ft. 34 sq. in. is $6\frac{3}{4}$ square miles?

594. How many times larger than $\frac{4}{7}$ of a quart is 3 hhds.?

595. What part of $\frac{1}{3}$ cu. yd. is $\frac{1}{3}$ cu. ft.?

596. What part of a hogshead of wine is 18 gal. 2 qts.?

LESSON LX.

597. 5 yds. 3 qr. were cut from a piece of cloth containing 13 yds. 2 qrs.; what part of the piece was taken?

598. A man had 4 gal. 3 qts. of wine, and sold 3 qts. 2 gi.; what part of his wine did he sell?

599. Mr. Jones agreed to give a man $43.27 for chopping 37 cds. 48 cu. ft. of wood. The man chopped but 13 cds. 59 cu. ft. To what part of the money was he entitled?

600 If I agree to give a man $64 for carting 46 T. 3 cwt. 8 qr. of hay, and he carts only 15 T. 2 qr. 2 lbs., to how much of the money is he entitled?

601. I agree to give a carman $13.26 for delivering 47 T. 3 qr. 19 lbs. of coal; how much should he receive for delivering 5 T. 2 qr. 11 lbs.?

602. A man sawed 3 cds. 43 cu. ft. of wood for a certain sum of money; at the same rate, how many times this sum of money should be received for sawing 43 cds. 84 cu. ft.?

603. A man ploughed 2 A. 47 sq. rds. of land for $3.47; at the same rate, how much should he receive for ploughing 14 A. 2 sq. rds. of land?

604. A man dug 1 A. 47 sq. rds. of potatoes in a field containing 4 acres; what part of the field did he dig?

605. A certain load of hay weighs 15 cwt. 3 qrs. 21 lbs.; if I sell 2 cwt. 2 qrs. 11 lbs., what part of the load will remain?

606. A certain hogshead of sugar weighs 3 cwt. 19 lbs.; if I sell 1 cwt. 2 qrs. 18 lbs., what part of the hogshead will remain?

QUESTIONS.—What is the reduction of denominate fractions? (164.) How do we prepare numbers for comparison? ANS.—*Reduce them to like denominations.* Why? (16., *c.*) How do we find how many times one number is larger or smaller than another? ANS.—*Divide the mensurable number by the standard, and the quotient will be the number of times required.* How many times larger than 4 is 18? ANS.—18 *is as many times* 4 *as* 4 *is contained in* 18, *which is* $4\frac{2}{4}$ *times; here,* 18 *is the mensurable number, and* 4 *is the* STANDARD *of measure.*

LESSON LXI.

168. To add and subtract fractional compound numbers.

Add $\frac{6}{7}$ of a pound to $\frac{7}{9}$ of a shilling.

(*a.*)

MODEL OPERATION.

$\frac{6}{7}$£.$=17\frac{1}{7}$s.,$=17\frac{9}{63}$s.

$\frac{7}{9}$s. $=$ $\frac{49}{63}$s.

$17\frac{58}{63}$s.$=$17s. 11d. $0\frac{4}{21}$ far., Ans.

Analysis.—1. Reduce the pounds to shillings.

2. Reduce $\frac{1}{7}$s. and $\frac{7}{9}$s. to 63rds of a shilling.

3. Add, and reduce the fractions to equivalent integral numbers of lower denominations.

Hence the general rule for the addition or subtraction of fractional denominate numbers.

(*a.*) Rule.—*Reduce the numbers to the most convenient denomination; then add or subtract as in common fractions, and reduce the result to equivalent integral numbers of lower denominations.*

From $\frac{5}{7}$ of an acre, take $2\frac{4}{9}$ of a square rod.

(*b.*)

MODEL OPERATION.

$\frac{5}{7}$ A.$=$2 R.$+$34 sq. rds.$+$ 8 sq. yds. $+$5 sq. ft.$+113\frac{1}{7}$ sq. in.
$2\frac{4}{9}$ sq. rds.$=$ 2 " $+$13 " $+$4 "
2 R.$+$32 sq. rds.$+25(\frac{1}{4})$* sq. yds.$+$1 sq. ft.$+113\frac{1}{7}$ sq. in.
$\frac{1}{4}$ sq. yd.$=$ 2 " $+$ 36 "
2 R.$+$32 sq. rds.$+$25 sq. yds. $+$4 sq. ft.$+$ $5\frac{1}{7}$ sq. in.

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*In these model operations when numbers are enclosed in a parenthesis, it shows that their value is expressed in another form; thus, $(\frac{1}{4})$ sq. yd.$=$2 sq. ft.$+$36 sq. in.

Analysis.—1. Reduce $\frac{5}{7}$ A. and $\frac{4}{9}$ sq. rds. to equivalent integral numbers of lower denominations.

2. Subtract, as in the subtraction of compound numbers.

Hence the general rule for the addition or subtraction of fractional denominate numbers.

Rule.—*Reduce the fractions to equivalent integral numbers of lower denominations, and then add or subtract as in denominate numbers.*

Questions.—How do we find what part one number is of another? **Ans.**—*Divide the mensurable number by the* STANDARD, *and the quotient will express the ratio. It is required to find what part* 5 *is of* 18. 18 *is the* STANDARD *of measure by which we measure* 5; *consequently,* 5 *divided by* 18, *or* $\frac{5}{18}$, *is the ratio required.*

LESSON LXII.

607. Add $\frac{4}{7}$ of a pound to $\frac{5}{7}$ of a shilling.

608. Add $\frac{3}{11}$ of a ton, $\frac{7}{9}$ of a ton, and $\frac{3}{7}$ of a cwt.

609. What is the sum of $\frac{2}{3}$ yd., and $\frac{4}{11}$ qr.?

610. What is the sum of $\frac{4}{11}$ mi. $\frac{4}{9}$ fur., $\frac{4}{9}$ mi. $\frac{4}{15}$ fur., and $\frac{3}{7}$ rds.?

611. What is the sum of $\frac{2}{11}$ A. $\frac{4}{5}$ R. $\frac{4}{7}$ sq. rd.?

612. Add 3 A. $4\frac{1}{7}$ R. $5\frac{4}{9}$ sq. rds.?

613. What is the sum of $3\frac{5}{7}$ cwt., 8 qr., and $9\frac{3}{7}$ lbs.?

614. What is the sum of $4\frac{3}{7}$°, $5\frac{3}{7}$ mi., and $4\frac{4}{7}$ rds.?

615. What is the sum of $3\frac{2}{5}$ deg., $5\frac{3}{5}$ mi., and $37\frac{3}{4}$ fur.?

616. What is the sum of $13\frac{4}{7}$ cu. ft., and $4\frac{2}{3}$ cu. in.?

Questions.—5 ounces is what part of 13 ounces? 13 ounces is how many times 5 ounces? What ratio has 5 to 11? **Ans.**—$\frac{5}{11}$. What is ratio? **Ans.**—*Ratio is the relation which two numbers of the same kind bear to each other in respect to magnitude, when compared.* Repeat the table of time. (97., *a.*) Name the number of days in each month. (97., *b.*) Repeat the table of circular motion. (98., *a.*)

LESSON LXIII.

617. From $\frac{4}{7}$ of a ton, take $\frac{6}{17}$ of a cwt.

618. From $\frac{7}{9}$ of a mile, take $\frac{7}{18}$ of a fur.

619. From $\frac{10}{11}$ of an A., take $\frac{2}{9}$ of a R.

620. A hogshead containing 103 gal. leaked $\frac{3}{11}$ gal., and I sold $\frac{2}{7}$ gal.; how much remained?

621. I burned $\frac{7}{13}$ of a load of wood containing 9 cd. 14 cu. ft.; how much remained?

622. 4 hhds. $3\frac{1}{4}$ gal. less $1\frac{1}{5}$ hhds. $\frac{2}{9}$ gal. $3\frac{1}{8}$ qts., equals what?

623. From 4 loads of hay, each containing $\frac{3}{4}$ T., I sold 12 cwt. $4\frac{3}{7}$ qrs. $18\frac{1}{2}$ lbs.; how much remained?

624. From 2 T. 4 qrs. 7 lbs. of hay, I sold $1\frac{2}{3}$ T. $8\frac{3}{4}$ cwt. $19\frac{1}{2}$ lbs.; how much is the remainder worth, at $\frac{1}{2}$ a cent per pound?

625. From 2 bbls. of cider, I sell $31\frac{2}{3}$ gal. $4\frac{1}{2}$ qts.; how much is the remainder worth, at $\frac{3}{5}$ of a cent per pint?

626. From $\frac{2}{3}$ of an acre, take $1\frac{1}{12}$ roods.

QUESTIONS.—By what is the difference of time between two places determined? (98.) Which has the earlier time, New York, or San Francisco? Why? When it is 12 o'clock M. at New York, will it be earlier, or later, in San Francisco? When it is 12 o'clock M. in San Francisco, is it earlier, or later, in New York? Why? What is the difference of time in a geographical mile? (98., *b*., 6.) In a degree?

LESSON LXIV.

I have three lots of land: the first contains $11\frac{3}{4}$ A. $2\frac{3}{4}$ sq. rds. 11 sq. ft.; the second, 13 A. $11\frac{2}{7}$ sq. rds. $85\frac{1}{2}$ sq. ft.; the third, $46\frac{3}{7}$ A. 47 sq. rds. $87\frac{1}{5}$ sq. ft.: how much land do the lots contain?

MODEL OPERATION.

$$
\begin{array}{llll|llllll}
\text{A.} & \text{sq. rds} & \text{sq. ft.} & & \text{A.} & \text{R.} & \text{sq. rds.} & \text{sq. yds.} & \text{sq. ft.} & \text{sq. in.} \\
11\frac{3}{4} & +\ 2\frac{3}{4} & +\ 11 & = & 11 & +\ 3 & +\ 2 & +\ 22 & +\ 17 & +\ 27 \\
13 & +\ 11\frac{2}{7} & +\ 85\frac{1}{2} & = & 13 & +\ 0 & +\ 11 & +\ 8 & +\ 91 & +\ 41\frac{1}{7} \\
46\frac{3}{7} & +\ 47 & +\ 87\frac{1}{5} & = & 46 & +\ 1 & +\ 75 & +\ 17 & +\ 89 & +\ 111\frac{3}{35} \\
\hline
 & & & & 71 & +\ 2 & +\ 10 & +\ 8(\frac{1}{2}) & +\ 0 & +\ 35\frac{8}{35} \\
 & & & & & & & & 4 & +\ 72 \\
\hline
 & & & & 71 & +\ 2 & +\ 10 & +\ 8 & +\ 4 & +\ 107\frac{8}{35}
\end{array}
$$

Analytical Steps.—1. Reduce $\frac{3}{4}$ A.$+\frac{3}{4}$ sq. rds. to equivalent integers of lower denominations.

2. Reduce $\frac{2}{7}$ sq. rds.$+\frac{1}{2}$ sq. ft. to equivalent integers of lower denominations.

3. Reduce $\frac{3}{7}$ A.$+\frac{1}{5}$ sq. ft. to equivalent integers of lower denominations.

4. Add, as in denominate numbers.

Questions.—How many dozen make a gross? (99., *a.*) Repeat the table of paper. (99., *b.*) How many words make a folio? (99., *d.*) What is a unit? (1.) What is a number? (2.) What is the difference between an abstract and a concrete number? (3.) (4.) Of what does Arithmetic treat? (5.) What is a quantity? (6.) What is a problem? (7.) What is the difference between a simple and a complex problem? (8.) (9.)

LESSON LXV.

627. A grocer bought 4 hhds. of sugar: the first weighed 11 cwt. $2\frac{1}{3}$ qrs. $22\frac{3}{5}$ lbs.; the second, 10 cwt. 1 qr. $16\frac{8}{10}$ lbs.; the third, $10\frac{3}{4}$ cwt. 22 lbs.; and the fourth, 9 cwt. 3 qrs.: how much did the whole weigh?

628. A man has his farm divided into two fields: the first contains 26 A. $2\frac{3}{4}$ R. $5\frac{2}{3}$ sq. rds.; the second, $48\frac{3}{4}$ A. $27\frac{5}{8}$ sq. rds.: how much land is there in the farm?

629. Bought 2 fat oxen: the first weighed $11\frac{3}{7}$ cwt. $2\frac{1}{4}$

qr. 12$\frac{2}{3}$ lbs. 11 oz.; the second, 9$\frac{3}{4}$ cwt. 23 lbs. 9$\frac{1}{3}$ oz.: how much did both weigh?

630. If I travel $\frac{8}{11}$ of a mile at one time, $\frac{4}{9}$ of a mile at another, and 2$\frac{3}{5}$ of a furlong at another; how far shall I travel in all?

631. Add together $\frac{2}{11}$ A. $\frac{4}{5}$ R. and $\frac{4}{7}$ sq. rds.

632. I had a load of oats containing 40$\frac{4}{7}$ bushels; I sold 11$\frac{2}{3}$ bu. 3$\frac{1}{2}$ pks. 4$\frac{1}{2}$ qt., how much had I remaining?

633. From a pile of wood containing 75$\frac{3}{5}$ cds. was sold, at one time, 16 cds. 51 cu. ft.; at another, 24 cds. 63 cu. ft.; at another, 27$\frac{3}{7}$ cds.: how much remained in the pile?

634. What is the difference of time between July 3d, 1860, and Apr. 13th, 1863?

635. A man dug a cellar 37$\frac{2}{3}$ ft. long, 16 ft. wide, 8$\frac{3}{7}$ ft. deep, at 16 cts. per cubic yard; how much should he receive for digging it?

QUESTIONS.—What is an analytical step? (10.) What is an analysis? (11.) What is a rule? (12.) What is a sign? (13.) What is an axiom? (14.) What are the fundamental rules? (15.) What is notation? (17.) What is the difference between the Roman and the Arabic notation? (18.)(19.) For what is the Roman notation chiefly used? How are numbers expressed in the Roman notation? (18., *a.*)

LESSON LXVI.

MISCELLANEOUS PRACTICAL EXAMPLES IN FRACTIONS.

636. If a horse eat $\frac{3}{7}$ of a bushel of oats in a day, in how many days will he eat 5$\frac{3}{7}$ bushels?

637. If a man spend 1$\frac{3}{5}$ dollars per month for tobacco, in what time will he spend 10$\frac{3}{7}$ dollars?

638. How many times will 4$\frac{3}{7}$ gallons of vinegar fill a vessel that holds $\frac{1}{2}$ of $\frac{5}{6}$ of a gallon?

639. If 14$\frac{1}{2}$ acres of meadow land produce 33$\frac{4}{7}$ tons of hay, how many tons will 5$\frac{3}{7}$ acres produce?

640. A man having $\$10\frac{3}{7}$, gave $\frac{2}{3}$ of it for clover seed at $\$3\frac{1}{3}$ per bushel; how many bushels did he buy?

641. Reduce $\frac{1}{2}$, $\frac{5}{6}$, $\frac{3}{8}$ and $\frac{1}{4}$, to equivalent fractions of the denomination 24ths.

642. Change $\frac{4}{7}$ to 91sts.

643. The less of two numbers is $4756\frac{4}{9}$; their difference is $128\frac{3}{4}$; what is the greater?

644. Simplify $\frac{4}{4\frac{1}{2}}$ and $\frac{3\frac{5}{7}}{4\frac{3}{11}}$.

645. What number multiplied by $\frac{3}{7}$ will produce $1837\frac{4}{5}$?

646. A farmer had $\frac{1}{5}$ of his sheep in one pasture, $\frac{1}{4}$ in another, and 77, the remainder, in another; how many sheep had he?

647. $14\frac{4}{7}$ less $\frac{\frac{1}{2} \text{ of } 8\frac{2}{5}}{14\frac{7}{10}}$ is $\frac{2}{3}$ of $\frac{7}{9}$ of what number?

648. A merchant bought $4\frac{3}{4}$ cords of wood, at $\$3\frac{1}{4}$ per cord, and paid for it in cloth, at $\$\frac{5}{7}$ per yard; how many yards were required to pay for the wood?

649. How much cloth, $\frac{3}{4}$ of a yard wide, will be required to line $20\frac{1}{2}$ yds. of cloth $1\frac{1}{4}$ yds. wide?

Questions.—What is the effect of repeating a letter in the Roman notation? (18., *b.*) What effect has placing a letter of less value before one of a greater? (18., *c.*) What is the effect of placing a letter of less after one of a greater value? (18., *d.*) What effect has a dash over a letter or combination of letters? (18., *e.*) What is the use of the Arabic notation? (19.) How many figures represent odd numbers in the Arabic notation? (107.)

LESSON LXVII.

650. If the dividend be $\frac{7}{9}$, and the quotient $\frac{4}{73}$, what is the divisor?

651. If the dividend be $\frac{3}{7}$, and the divisor 9, what is the quotient?

652. If the sum of two fractions be $\frac{5}{8}$, and one of them be $\frac{8}{21}$, what is the other?

653. If the smaller of two fractions be $\frac{22}{31}$, and the difference $\frac{7}{92}$, what is the greater?

654. A certain sum of money is to be divided among 5 persons; A is to have $\frac{1}{4}$, B $\frac{2}{5}$, C $\frac{1}{10}$, D $\frac{3}{20}$, and E the remainder, which is $24; what is the whole sum to be divided?

655. What number diminished by the difference between $\frac{3}{4}$ and $\frac{4}{5}$ of itself leaves a remainder of 50?

656. Bought 320 sheep, at 2\frac{3}{7}$ per head; afterwards I bought 435 sheep, at 2\frac{4}{7}$; finally, I sold all at 2\frac{1}{8}$. Did I gain or lose by the bargain, and how much?

657. If 7 be added to both the terms of the fraction $\frac{8}{9}$, will the value of the fraction be augmented or diminished, and how much?

658. If 7 be subtracted from both terms of the fraction $\frac{8}{9}$, will the value of the fraction be increased or diminished, and how much?

659. A merchant bought 672 lbs. of sugar, at 5$\frac{3}{7}$ cts. per pound; had the price been 2$\frac{3}{7}$ cts. less per pound, how many pounds could have been purchased?

660. What number divided by 1$\frac{3}{8}$ will give a quotient of 9$\frac{1}{2}$?

QUESTIONS.—How many figures represent even numbers in the Arabic notation? (108.) How many represent prime numbers? (109.) How many composite? (110.) Which figure has no value? (22.) How many values have figures? What is meant by the simple value? (20.) What by the local value? (21.) Can a figure have a simple and a local value at the same time? What is numeration? (23.)

LESSON LXVIII.

661. The product of two numbers is 8; one of the numbers is 1472; what is the other?

662. A carpenter worked $11\frac{2}{3}$ days, and after paying his board with $\frac{3}{7}$ of his earnings, he had \$20 left; how much did he receive a day?

663. A farmer having $272\frac{3}{5}$ acres of land, sold $\frac{1}{3}$ of it, and gave $\frac{3}{8}$ to his son; what was the value of the remainder, at \$$57\frac{5}{8}$ per acre?

664. A horse and chaise cost \$274. The horse cost $3\frac{1}{5}$ times as much as the chaise; what was the cost of the chaise?

665. A merchant bought a cargo of cotton for \$$2173\frac{1}{2}$, and sold it for $\frac{22}{23}$ of the cost, thereby losing \$$\frac{3}{4}$ on a bale; how many bales did he purchase?

666. A and B paid \$12 for building a wall; B paid twice as much as A; how much did each pay?

667. How many yards of cloth, $\frac{4}{5}$ of a yard wide, are equal to 13 yards $\frac{5}{8}$ of a yard wide?

668. How much will 115 lbs. of hay cost, at \$10 per ton?

669. What will $3\frac{1}{2}$ lbs. of sugar cost, at \$7.50 per cwt.?

670. What will 2 eggs cost, at $37\frac{1}{2}$ cts. per doz.?

671. What will 2 oz. of sugar cost, at \$10 per cwt.?

672. What will $3\frac{5}{7}$ oz. of lard cost, at \$$18\frac{5}{7}$ per cwt.?

673. What will $3\frac{1}{7}$ drams of opium cost, at \$8 per pound?

674. What will $7\frac{1}{2}$ lbs. of flour cost, at \$9 per barrel?

QUESTIONS.—What is reading numbers? (24.) Repeat the numeration table from right to left. (25.) From left to right? (25.) What is to be done with vacant places? (26.) What with vacant periods? (27.) How are periods usually separated? (27., *a.*) Repeat the rule for notation? (28.) What is Addition? (31.) Subtraction? (39.) Multiplication? (46.) Division? (54.)

LESSON LXIX.

675. A tobacconist bought $3\frac{4}{7}$ cwt. of tobacco, at $2\frac{5}{7}$ cts. per pound; how much did it cost him?

676. Sold $2\frac{3}{7}$ tons of cheese, at $\frac{3}{4}$ of a cent per pound; how much did I receive?

677. Sold $3\frac{1}{7}$ bbls. of salt, at $\frac{3}{4}$ of a cent per pound; how much did I receive?

678. How much will 37 A. 3 R. of land cost, at $29\frac{3}{8}$ cts. per square rod?

679. What will be the cost of $14\frac{3}{16}$ A. of land, at 9 cts. per sq. ft.?

680. How many square rods in a piece of land $47\frac{3}{7}$ rods long, and 36 rods wide?

681. How many acres in a piece of land 386 rods 14 li. long, and 875 rods 18 li. wide?

682. At $35 per acre, what will be the cost of a piece of land 37 rods 17 li. long, 29 rds. 13 li. wide?

683. How much will $9\frac{3}{8}$ sq. mi. of land be worth, at $37.50 per acre?

684. Bought a piece of land 2 mi. long, 91 ch. 3 rds. wide; how much did it cost, at $\$25\frac{3}{8}$ per acre?

685. What is a piece of land 7 fur. 13 rds. long, 15 rds. wide, worth, at $15 per acre?

686. What is the cost of plastering a ceiling $23\frac{3}{4}$ ft. long, and 17 ft. wide, at $37\frac{1}{2}$ cts. per sq. yd.?

687. At 25 cts. per sq. yd., what will it cost to plaster the four walls of a room $26\frac{3}{4}$ ft. long, $19\frac{3}{4}$ ft. wide, 9 ft. 3 in. high?

688. What will it cost to paint the floor of a room 47 ft. long, and $18\frac{3}{7}$ ft. wide, at 33 cts. per sq. yd.?

689. What will it cost to paper a room 87 ft. 3 in. long, 38 ft. wide, and 18 ft. 7 in. high, at $1.53 per sq. yard?

QUESTIONS.—What is the sum of two or more numbers? (30.) What is the answer called in Addition? (30.) Subtraction? (42.) Multiplication? (49.) Division? (57.) Illustrate the use of the sign of equality on the blackboard. (33.) The dollar sign. (34.) The sign of Subtraction. (43.) Addition. (32.) Multiplication. (51.) Division. (62.) Explain the fractional sign of Division. (62.)

LESSON LXX.

690. What will it cost to plaster the walls and ceiling of a room 17 ft. 2 in. long, 13 ft. wide, 9 ft. 8 in. high, at 89 cents per sq. yard?

691. What will it cost to lay 27 sq. ft. of roof, at $13.46 per hundred sq. ft.

692. What will it cost to make a walk $46\frac{3}{7}$ rds. long, and 8 ft. 6 in. wide, at 54 cts. per sq. yard?

693. What will one sq. ft. of land cost, at $3.27 per acre?

694. Bought $37\frac{1}{12}$ acres of land, at 2 mills per sq. in.; what was the cost?

695. How many blocks $1\frac{3}{4}$ ft. on each side, can be sawed from a cube 1 yd. on each side, allowing no waste for sawing?

696. How many cubic blocks $1\frac{1}{2}$ inches on each side, will it take to fill a mortise 1 foot deep, 1 foot wide, 1 foot long?

697. Bought $16\frac{3}{7}$ loads of hewn timber, at $4\frac{1}{7}$ cts. per cu. ft.; what was the cost?

698. Bought $17\frac{3}{4}$ loads of round timber, at $3\frac{3}{7}$ cts. per cu. ft.; what was the cost?

699. How many cubic feet in a block of marble $8\frac{1}{2}$ ft. long, $3\frac{2}{3}$ ft. wide, 2 ft. 6 in. thick?

700. How many cubic feet in a pile of earth $46\frac{3}{7}$ ft. long, 38 ft. 8 in. wide, 17 ft. 3 in. thick?

701. How many cu. ft. of masonry in a wall $67\frac{3}{8}$ ft. long, 12 ft. 3 in. high, 4 ft. 1 in. thick?

702. How many perches of masonry in a wall 49 ft. 3 in. long, 11 ft. 2 in. high, and $5\frac{3}{8}$ ft. thick?

QUESTIONS.—What is the difference between the minuend and subtrahend? (40.) (41.) Give the analysis of an example in subtraction, on the blackboard. (44.) Give the first twelve exercises in the addition of columns. (37.) Name the terms of multiplication. (47.) (48.) (49.) Which are called factors? (50.) Name the terms in subtraction. (40.) (41.) (42.)

LESSON LXXI.

703. How many perches of stone in the wall of a cellar $14\frac{3}{7}$ ft. long, 13 ft. 7 in. wide, $9\frac{3}{7}$ ft. deep, and $1\frac{3}{4}$ ft. thick, making allowance for the corners?

704. How many perches of stone in the wall of a cellar $37\frac{3}{8}$ ft. long, 26 ft. 3 in. wide, 11 ft. 2 in. deep, the wall being $2\frac{4}{5}$ ft. thick?

705. Making the proper allowance for the corners, how many perches of stone will it take to enclose a lot 38 rds. long, $14\frac{3}{7}$ rds. wide, the wall being 7 ft. 2 in. high, and $4\frac{3}{7}$ ft. thick?

706. How many perches of masonry in a rectangular fort, $138\frac{3}{7}$ ft. long, $97\frac{1}{2}$ ft. wide, the wall being 28 ft. 7 in. high, and 13 ft. 7 in. thick?

707. What will a pile of wood 3 ft. long, 7 ft. 8 in. wide, and 2 ft. 9 in. high, cost, at $5 per cord?

708. What will a pile of wood 47 ft. long, 3 ft. 8 in. wide, 7 ft. 9 in. high, cost, at $7.37 per cord?

709. Bought a load of wood 8 ft. 3 in. long, 2 ft. 7 in. high, 4 ft. 3 in. wide, at $9.37 per cord; what was the cost?

710. How many sq. ft. in 10 boards, each of which is 10 ft. 3 in. long, and 13 in. wide?

711. How many sq. ft. in 23 boards, each of which is $10\frac{3}{7}$ ft. long, and 17 in. wide?

712. What will 12 boards cost, each of which is 13 ft. 3 in. long, $12\frac{3}{7}$ in. wide, at \$30.75 per thousand feet?

713. What is the cost of a load consisting of 57 boards, each of which is 14 ft. long, and $1\frac{4}{7}$ ft. wide, at \$47.36 per thousand?

LESSON LXXII.

714. I engaged Mr. Smith to build the walls of a cellar $29\frac{1}{4}$ ft. long, 20 ft. 3 in. wide, $10\frac{1}{2}$ ft. deep, each wall being $3\frac{1}{2}$ ft. thick, at 4 cts. per cubic foot, making no allowance for the corners, and I paid him in wood at \$8.384 per cord; how many cords did he receive for his job?

715. Sold 43 loads of hewn timber, at $3\frac{1}{4}$ cts. per cubic foot, and agreed to take wood, at \$7.53 per cord; how many cords ought I to receive?

716. Sold $64\frac{3}{7}$ loads of round timber, at $2\frac{3}{8}$ cts. per cubic ft., and agreed to take wood, at \$5.37$\frac{1}{2}$ per cord; how many cords ought I to receive?

717. How much will it cost to transport a block of marble $17\frac{3}{4}$ feet long, 7 feet wide, $4\frac{1}{2}$ feet thick, at 7 mills per pound, allowing one cubic foot to weigh 178 pounds?

718. Bought $5\frac{2}{3}$ barrels of molasses, at 3 cts. per gill, and paid for it in wood, at \$5.50 per cord; how many cords did it take to balance the bill?

719. Bought $3\frac{4}{7}$ hhds. of molasses, at 75 cts. per gal., and paid for it with hewn timber, at $3\frac{5}{7}$ cts. per cubic foot; how many loads did it take to balance the account?

720. A liquor dealer sold $12\frac{3}{7}$ bbls. of brandy, at \$4.87$\frac{1}{2}$ per gal., and took in pay a gold tankard, at $97\frac{3}{7}$ cts. per pwt.; what was the weight of the tankard?

721. A man sold 2¼ hhds. of molasses, at 37 cts. per gallon, and took his pay in salt, at 5¾ mills per pound; how many barrels did he receive?

722. How many bushels of oats, at 1¾ cts. per pound, must be given for 487 bottles of champagne, at 18¾ cts. per gill, each bottle containing 1 qt.?

QUESTIONS.—Name the terms in Division. (55.) (56.) (57.) (61.) What is the difference between Long Division and Short Division? (59.) (60.) How is Addition proved? (36., *f.*) Subtraction? (44., *i.*) Multiplication? (53., *n.*) Division? (65., *i.*) Illustrate, on the blackboard, the contracted methods of multiplying by 10, 100, 1000, &c. (67., *a.*) The contracted methods of dividing by 10, 100, 1000, &c. (67., *f.*) Of multiplying by 25. (67., *b.*) Of dividing by 25. (67., *g.*) Of multiplying by 12½. (67., *c.*) Of dividing by 12½. (67., *h.*)

LESSON LXXIII.

723. Reduce 424 drams apothecaries' weight, to Troy weight.

724. Reduce 20 lbs. 8 oz. 12 pwt. Troy weight, to avoirdupois weight.

725. What will 37 lbs. 6 oz. 12 pwt. Troy weight of silver ware, be worth, at $12 per pound avoirdupois?

726. An apothecary bought 5 lbs. 10 oz. of rhubarb by avoirdupois weight, at 48 cts. per ounce, and retailed it at 13 cts. per dram, apothecaries' weight; how much did he gain?

727. Bought by avoirdupois weight 12¾ lbs. of opium, at 40 cts. per ounce, and sold the same by Troy weight, at 50 cts. per oz.; did I gain, or lose, and how much?

728. In 14 lbs. 13 oz. 8 dr., avoirdupois weight, how many drams, apothecaries' weight?

729. In 17 lbs. 14 oz. 6 dr., avoirdupois weight, how many pounds Troy?

730. In 371 gallons wine measure, how many bushels dry measure?

731. In 21 gal. 3 qts. 2 pts. 2 gi., how many bushels, pks. and qts., dry measure?

732. In 41 bushels dry measure, how many gal. wine measure?

733. In 347 gallons beer measure, how many gallons wine measure?

734. In 347 gal. 3 qts. 3 gills wine measure, how many gallons beer measure?

QUESTIONS.—What is a fraction? (123.) What is the difference between a common and a decimal fraction? (124.) (125.) What are the terms of a fraction? (126.) What does the denominator show? (127.) What does the numerator show? (128.) What is the difference between a proper and an improper fraction? (129.)(130.) What is a mixed number? (131.) What is a simple fraction? (132.) What is a complex fraction? (133.) What is a compound fraction? (134.) By what is the denomination of a fraction determined? (139.) By what is the value of a fraction of any denomination determined? (140.) What is the reduction of fractions? (141.) What is the difference between reduction ascending and reduction descending? (142.) (143.) When is a fraction said to be reduced to its lowest terms? (144.) In how many ways can fractions be reduced? (145.)(146.)(147.)(148.) When are fractions said to have a common denominator? (150.) In how many ways are fractions multiplied? (154.) (157.) (158.) In how many ways are fractions divided? (155.) (156.) (159.) (160.)

SECTION XI.

LESSON I.

DECIMAL FRACTIONS.

169. The complete decimal scale of notation consists of an indefinite number of places increasing in value from right to left in a tenfold ratio.

170. For convenience in expressing numbers, a place is selected which is called the UNITS' PLACE. All places on the left of units represent multiples of the unit. All places on the right of units represent fractional parts of the unit.

171. For the purpose of distinguishing the units' place a period (.) is placed at the right of it; thus, 3.4; 4.37; to be read 3 units and 4 *tenths;* 4 units and 37 *hundredths.*

(*a.*) NUMERATION OF THE DECIMAL SCALE.

NUMERATION TABLE.

INTEGERS.										DECIMALS.									
Billions.	Hundred of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	UNITS.	*Tenths.*	*Hundredths.*	*Thousandths.*	*Ten Thousandths.*	*Hundred Thousandths.*	*Millionths.*	*Ten Millionths.*	*Hundred Millionths.*	*Billionths.*	*Hundred Billionths.*
0	0	0	0	0	0	0	0	0	0.	0	0	0	0	0	0	0	0	0	0

172. When an integer and a decimal are written together, they form a MIXED NUMBER.

QUESTIONS.—What is a unit? (1.) What is a number? (2.) What is the difference between an abstract and a concrete number? (3.) (4.) Of what does Arithmetic treat? (5.) What is a quantity? (6.) What is a problem? (7.) What is the difference between a simple and a complex problem? (8.) (9.) What is an analytical step? (10.) What is an analysis? (11.) What is a rule? (12.) What is a sign? (13.) What is an axiom? (14.) What is the axiom for Addition? (16., *a.*) For Subtraction? (16., *b.*) For comparison? (16., *c.*) Illustrate each of the axioms. (16.)

LESSON II.

EXERCISES IN READING MIXED NUMBERS.

173. For convenience in reading, numbers are divided into periods of three figures each, commencing with the right hand figure in decimals, and the units' figure in integers; thus,—

47,864,134 . 4,178,642.

To be read,—Forty-seven millions eight hundred and sixty-four thousands one hundred and thirty-four, and four million one hundred and seventy-eight thousand six hundred and forty-two *ten millionths.*

NOTE.—The name, or denomination, of the decimal is determined by the name of the first significant figure at the right hand of the number; thus, in the above example, the figure, 2, is the right hand figure, and, as it stands in the *ten millionths'* place, the denomination of the decimal is *ten millionths.*

Read the following numbers:*

(1)	(2)	(3)	(4)	(5)	(6)
341.2	42.31	413.21	41.324	50.1345	2.01413
434.4	2.13	21.403	51.021	6.0201	41.3041
567.4	41.34	51.204	61.3024	3.004	31.40712
304.2	84.32	1.31	.4063	4.1304	2.40131
478.4	43.20	412.213	.2170	5.3014	4.30121

(7)	(8)	(9)	(10)	(11)	(12)
671.30145	413.4	.5102	5012.4107	40401.	.210134
2.041301	20.	.4002	401.4183	5010.34103	.00012
4.013	406.03	.0041	21.4671	210.4168	.001
2.141	30.0004	.04	502.4180	5120.413	.0412
.641	200.0003	.304	30.4072	3121.407	.3012
.382	41.602	.4034	507.4107	512.403	.0014
.0401	51.307	.2001	809.413072	91.60201	.0023
41.2071	401.2006	.0041	419.413072	4.401	.40241

*The following exercises should be read aloud by the pupils at recitation. They should be still further exercised, if necessary, by reading similar exercises from the blackboard, until they become quick and accurate.

The teacher should then require that the books be closed, and proceed to dictate the same exercise for the class to copy. When they have finished, let them exchange slates with each other, and let the teacher read the lesson from the book, requiring each pupil to note his neighbor's errors, by drawing a line under the numbers incorrectly written.

QUESTIONS.—What is notation? (17.) What is the difference between the Roman and the Arabic notation? (18.) (19.) How are numbers expressed in the Roman notation? (18., *a.*) What effect has the repetition of a letter? (18., *b.*) What effect has placing a letter of less value before one of greater value? (18., *c.*) What effect has the placing a letter of less value after one of a greater value? (18., *d.*) What effect has a dash placed over a letter or a combination of letters? (18., *e.*) What is the difference between the simple and the local value of a figure? (20.) (21.) Can a figure have both a local and a simple value?

LESSON III.

(1)	(2)	(3)	(4)	(5)
413.4021	403.413	5.40123	50000.001	34.41301
41.0341	4.4021	40.41371	5000.0001	4.00021
5000.001	30.204101	2.40101	500.00001	.140201
41.0213	3.012401	.4002	50.0001	.30402
4.4021	2.401	.40001	5.00001	4.340041
412.4134	3.1241	.401	.500001	2.41307
21.4012	4.3101	.001	.050001	.0024101
412.4302	400.4001	.0002	.00501	.3041

(6)	(7)	(8)	(9)	(10)
413.407012	4130.13021	4.3087041	300.0001	.3001001
4001.300010	40.41232	.301242	30.3030	.0041001
4.301	401.41237	.400001	400.400400	.0000120
400.300901	2.34234	.0400001	30.300300	.000004
30.410801	42.24137	.004001	89.800808	.0004001
.600001	.40001	.000401	47.891337	.0040014
4.300101	3.40121	.000041	4.467871	.0030013
5.800001	5.300132	.00004	9.600161	8003001

(11)	(12)	(13)	(14)
80.1020304	40030401.30	4130413.0004	214.03012
8.670123	4010213.403	400001.00040001	41.30120
4.410071	40040031.4301	30121.401301	4.0041
.000321	4401231.4301	501.241307	400.4004
.48753	412001.680301	4000.410203	3.40002
.00211	20121.413013	400.4021034	.408304
.080801	4.413071	40.491309	.040002
.601602	4000.413	.680001	.000802

QUESTIONS.—What is the use of a cipher? (22.) What is numeration? (23.) What is the reading of numbers? (24.) Repeat the period of the numeration table from right to left. (25.) From left to right. (25.) What is to be done with the vacant places and the vacant periods? (26.) (27.)

LESSON IV.

(1)	(2)	(3)	(4)
4003.041030	3.40130712	4.4030401	40101.3000101
4.0713024	41.400007001	.32004012	4000.4000301
.4013067	.413004012	.00300241	400.500042
.0401308	.500120413	.307102401	40000.3413070
.0091307	.00034102	.807034102	800.400003
.8070012	.00040001	.00830411	4.40003
.508704	.000004101	.60012013	500.4030404
.430704	.00004001	.41307103	6.0803041

(5)	(6)	(7)
3041.3071024	304030.043004	490.0000020001
3.0701101	200220.022002	4000.000004001
402104.00008	5000.0005002	.004000401
6120.41300001	400040.0040004	.008000401
401.040140014	30000.4000004	.4001004
40000.000004	4004.0004001	.4014041
30000.00003	40000.4000001	.4302410
40000.000040004	300000.400001	.2004001

(8)	(9)	(10)
41030.4000001	8304.00000301	4100.300010301
3.00040001	40.00004007	3.00040013004
.86040003	3.480602001	.4000130412
.0004003	.201786012	.6034500210
.4300001	.5010001301	.0003004034
.0030012	.000300001	.0000301201
.004302	.1000000001	.000008000001
004002101	.0001000001001	.00008111001201

Questions.—What is the use of the comma? (27., *a.*) What is the rule for notation? (28.) In what ratio do figures increase from the right to the left? (29.) How much greater are hundreds than units? (29.) Thousands than tens? (29.) Millions than thousands?

LESSON V.

174. ADDITION OF DECIMALS.

Add the following numbers:

Twelve, and five *tenths;*
Twenty-five, and three hundred and eight *thousandths;*
Two thousand, and 4637 *millionths;*
416, and 30042 *millionths;*
Thirty-seven thousand *ten thousandths;*
46 millions 402 thousand *hundred millionths.*

MODEL OPERATION.

12.5
25.308
2000.004637
416.030042
3.7000
.46402000

2458.006699

Analysis.—The analysis and proof are the same as in simple addition.

1. Add the following numbers:
   14, and five *tenths;*
   Thirty-six, and three hundred forty-eight *thousandths;*
   35, and 347 *ten thousandths;*
   Four hundred 16, and five hundred *thousandths;*
   437246, and seventy-nine *thousandths;*
   Four hundred and 64 *thousandths.*

2. Add
   54 thousand three hundred, and 22 *ten thousandths;*
   374 thousand 426, and two ten *thousandths;*
   461372, and 461 ten *thousandths;*

2 thousand 2, and 2 thousand and 2 *ten thousandths;*
4 thousand, and 4 hundred four *ten thousandths;*
472 thousand, and 204 *ten thousandths*

3. Add
437 thousand 349, and 467 *hundred-thousandths;*
420 thousand, and 2 hundred *hundred-thousandths;*
300 thousand, and 204 *hundred-thousandths;*
5003.87, and five hundred and five *thousandths;*
800 thousand, and five hundred sixty-five *hundred-thousandths;*
4000000, and five thousand and six *hundred-thousandths.*

4. Add
343 thousand 43, and six thousand *hundred-thousandths;*
241341, and four hundred and fourteen *hundred-thousandths;*
11, and eleven thousand one *hundred-thousandths;*
5, and five thousand six *millionths;*
303, and three thousand three *millionths;*
463 thousand 42, and 42 thousand eight *millionths;*
50 thousand, and 50 thousand fifty *millionths.*

5. Add
307 thousand 82, and 47 thousand and 43 *millionths;*
439 thousand 20, and 374 thousand and 40 *ten-millionths;*
4 hundred thousand, and 240 thousand and 301 *ten-millionths;*
204 million 20, and 3004 *ten-millionths;*
597 million 304, and forty thousand and thirty-six *ten-millionths;*
407 million 407 thousand, and four hundred and four *ten-millionths.*

6. Add

472 thousand 325, and 678 thousand 400 *ten-millionths;*

400734, and 4072401 *ten-millionths;*

25, and thirty-five thousand and 463 *hundred-millionths;*

3, and three thousand and 496 *hundred-millionths;*

540, and 437 thousand and fifty *hundred-millionths;*

40 millions, and forty-one *hundred-millionths.*

7. Add

437 thousand 25, and 463 *billionths;*

567 million, and 4374 *billionths;*

207 thousand, and 573402 *billionths;*

200, and three hundred and four *billionths;*

643 thousand, and thirteen *billionths;*

50 thousand, and 13 *ten-billionths.*

8. Add

seven thousand, and 43 *ten-billionths;*

347 thousand, 247 *ten-billionths;*

18 hundred *ten-billionths;*

347 thousand, and 14 *ten-billionths;*

437 thousand and 5, and one thousand and eight *billionths;*

3008, and four thousand and four *billionths.*

9. Add

five thousand 382, and five thousand five *ten-billionths;*

500 thousand, and five hundred thousand *ten-billionths;*

5374 *hundred-billionths;*

3469 *hundred-billionths;*

5000005 *hundred-billionths;*

five billion five million five thousand and five *hundred-billionths.*

10. Add

274 billions 38 thousand, and 14 *trillionths;*
274 billions 39 thousand and 15 *ten-trillionths;*
274 billions 39 thousand and 15 *hundred-trillionths;*
274 thousand and thirty-nine *hundred-trillionths;*
274 millions 39 thousand and 15 *hundred-trillionths;*
274 thousand five hundred and five *hundred-billionths.*

QUESTIONS.—What is the sum of two or more numbers? (30.) What is Addition? (31.) What is the sign of Addition? (32.) What is the sign of equality? (33.) What is the dollar sign? (34.) Illustrate the use of each of the foregoing signs. What is the order of solving a problem? (35.) Analyze an example in Addition, at the blackboard. (36.) How may addition be proved? (36., *f.*) What is the difference of two or more numbers? (38.) What is Subtraction? (39.) What is the minuend? (40.) What is the subtrahend? (41.) What is the remainder? (42.) What is the sign of Subtraction? (43.) Analyze an example in Subtraction, at the blackboard. (44.)

LESSON VI.

11. Add

374 thousand, and 83 *ten-billionths;*
374 thousand, and 83 *billionths;*
374 thousand, and 83 *ten-millionths;*
374 thousand, and 83 *hundred-billionths;*
374 thousand, and 83 *hundred-millionths.*

12. Add

nine thousand, and nine *billionths;*
9 thousand, and nine *trillionths;*
9 thousand, and nine *hundred-trillionths;*
9 thousand, and nine *hundred-thousandths;*
9 thousand, and nine *ten-trillionths;*
9 thousand, and nine *ten-thousandths.*

13. Add

9 thousand, and nine *ten-billionths;*
9 thousand, and nine *millionths;*
9 thousand, and nine *hundred-thousandths;*
9 thousand, and nine *hundredths;*
9 thousand, and nine *ten-thousandths;*
9 thousand, and nine *thousandths.*

14. Add

nine thousand, and nine *hundredths;*
nine thousand, and nine *tenths;*
90 thousand, and nine *hundred-thousandths;*
90 thousand, and nine *millionths;*
90 thousand, and nine *ten-millionths;*
90 thousand, and nine *hundred-billionths.*

15. Add

ninety millions, and nine *tenths;*
ninety millions, and ninety *hundredths;*
ninety millions, and nine *hundred-thousandths;*
ninety millions, and nine thousand *ten-thousandths;*
ninety millions, and ninety thousand *hundred-thousandths;*
ninety millions, and nine hundred thousand *millionths.*

16. Add

90 millions, and nine *ten-millionths;*
90 millions, and nine *hundred-millionths;*
90 millions, and ninety *hundred-millionths;*
90 millions, and ninety *billionths;*
90 millions, and nine *billionths;*
90 millions, and nine *ten-billionths.*

17. Add
37 thousand 40, and 15 *thousandths;*
42 millions 8, and 13 *millionths;*
247 thousand, 138 *thousandths;*
9 thousand 9, and 9 *tenths;*
9 thousand 9, and 9 *hundredths;*
9 thousand 900, and 9 *thousandths.*

18. Add
280 thousand 40, and 30 *ten-millionths;*
50 millions 37, and 30 *hundred-millionths;*
3005, and 730 *billionths;*
490 millions, and 7 *tenths;*
689743, and 371 *millionths;*
4703491, and four *billionths.*

19. Add
37 units,
467 hundreds,
36 *tenths,*
374 *tenths,*
467 *thousandths,*
467 thousands.

20. Add
372 hundreds,
372 *tenths,*
372 *thousandths,*
372 millions,
372 *millionths,*
372 *ten-thousandths.*

21. Add
417 *hundredths,*
417 hundreds,
417 thousands,
417 *millionths,*
417 hundred thousands,
417 *hundred-thousandths.*

22. Add
346347 *tenths,*
280643 *hundredths,*
571807 *thousandths,*
307602 *millionths,*
470864 *ten-thousandths,*
407684 *billionths.*

23. Add
307413 *billionths*,
307413 billions,
438764 ten-thousands,
507083 *ten-thousandths*,
6807021 *ten-millionths*,
417028 *hundred-thousandths*.

24. Add
30708604 *ten-thousandths*,
3041702 *tenths*,
417684 *thousandths*,
34166 *ten-thousandths*,
417068 hundreds,
487086 thousands.

LESSON VII.

175. SUBTRACTION OF DECIMALS.

From 68.417 take 63.4177.

MODEL OPERATION.

68.417 Minuend.
63.4177 Subtrahend.

4.9993 Difference.

ANALYSIS.—The analysis and proof are the same as in simple subtraction.

EXAMPLES FOR PRACTICE.

25. From ten thousand, take ten *thousandths*.
26. From ten millions, take ten *millionths*.
27. From five hundreds, take five *hundredths*.
28. From eight hundred thousands, take eight *hundred-thousandths*.
29. From five *tenths*, take five *hundredths*.
30. From five *hundred-thousandths*, take five *millionths*.
31. From eleven, take eleven *tenths*.
32. From 113, take 113 *tenths*.
33. From 417, take 1347 *millionths*.
34. From 8 *tenths*, take 436 *thousandths*.
35. From 297 take 4138 *thousandths*.
36. From 480 *thousandths*, take 483 *ten-thousandths*.

37. From one, take 3718643 *ten-millionths.*

38. From two, and 347 *thousandths*, take 4374 *millionths.*

39. From 341, and 389 *millionths*, take 39, and 507 *ten-millionths.*

40. From 4307.486, take 430, and 6708 *millionths.*

41. From 30700 4101, take 20007, and 89 *ten-millionths.*

Questions.—How may Subtraction be proved? (44., *i.*) What is Multiplication? (46.) What is the multiplicand? (47.) What is the multiplier? (48.) What is the product? (49.) Which are called factors? (50.) What is the sign of Multiplication? (51.) Illustrate its use. Analyze an example in Multiplication from the blackboard. (53.) How may Multiplication be proved? (53., *n.*)

LESSON VIII.

42. From 3701.2, take 483, and 674 *ten-millionths.*

43. From 2.071, take 5678 *ten-thousandths.*

44. From 46.21, take 896 *ten-millionths.*

45. From 2.6701, take 3472 *billionths.*

46. From .0000837, take 280 *billionths.*

47. From one, take .0470864.

48. From two, take 1.870864.

49. From 800, take .39040004.

50. From 300, take .000800801.

51. From 6001, take 40.683041.

52. From 8602, take 304.40702.

53. From 6801.40, take .0083042.

54. From 5607.0008, take .900087.

55. From 3000000, take 3 *millionths.*

56. From 50000, take 50 *thousandths.*

57. From 5786 *tenths*, take 5786 *hundredths.*

58. I owned a house worth $46.3074, and sold it for $30.40 less than it was worth; what did I receive for it?

59. I bought a book for $1.00401, and sold it for $1.8304; how much did I gain?

60. Bought a bushel of potatoes for $0.37468, and sold them for $0.803074; how much did I gain?

61. Bought 474.37 bushels of corn, and sold 372.43 bushels; how many bushels had I remaining?

62. Bought 347.246 lbs. of butter, and sold 241.3874 lbs. of it; how many pounds had I left?

63. Bought 2417.46 lbs. of cheese, and sold 43.253 lbs. of it; how many pounds had I left?

QUESTIONS.—What is Division? (54.) What is the dividend? (55.) What is the divisor? (56.) What is the quotient? (57.) What is the difference between Long Division and Short Division? (60.) (59.) What is the remainder? (61.) What are the signs of Division? (62.) Analyze an example in Short Division, at the blackboard. (64.) How may Division be proved? (64., *i.*) Illustrate each of the following principles of the fundamental rules on the blackboard? (66.)

LESSON IX.

EXAMPLES COMBINING ADDITION AND SUBTRACTION OF DECIMALS.

64. Three men bought a farm for $9407.: the first paid $2672.634; the second paid $3089.407; and the third the remainder: how much did the third pay?

65. A grocer bought 275.3 pounds of butter of one farmer, and 318.34 lbs. of another; he afterward sold 210.3 lbs. to one customer, and 97.567 to another; how many pounds had he remaining?

66. A farmer had 1864.3 bu. of wheat and 1129.47 bu. of corn; he sold 1340.24 bu. of wheat and 1000.32 bu. corn; how many bushels of each had he remaining?

67. A merchant buys 245.4 yds. of cloth of one person,

125.37 yds. of another, 1183. yds. of another, and then sells 803.40703 yds.; how many yards has he remaining?

68. The difference of two numbers is 9068.42; the less is .480702; what is the greater?

69. The difference between two numbers is .30702; and the greater number is 490.70; what is the smaller?

70. 9076.402 added to a certain number makes 47002.865; what is that number?

71. 40702.864 added to a certain number makes 40702.865; what is that number?

72. 4670.242 subtracted from a certain number leaves a remainder of 670.4012; what is that number?

73. The difference of two numbers is 30.21; the greater is 467.4; what is the sum of the numbers?

QUESTIONS.—Illustrate the contracted method of multiplying by 10, 100, &c. (67., *a.*) Of multiplying by 25. (67., *b.*) Of multiplying by 12½. (67., *c.*) By 33⅓. (67., *d.*) By 125. (67., *e.*) Illustrate the contracted method of dividing by 10, 100, 1000, &c. (67., *f.*) Of dividing by 25. (67., *g.*) By 12½. (67., *h.*) By 33⅓. (67., *i.*) By 125. (67., *j.*)

LESSON X.

176. MULTIPLICATION OF DECIMALS.

Multiply 304.32 by 24.4124.

MODEL OPERATION.

304.32			
24.4124			
.121	728	Product of	4 *ten-thousandths.*
.608	64	"	2 *thousandths.*
3.043	2	"	1 *hundredth.*
121.728		"	4 *tenths.*
1217.28		"	4 *units.*
6086.4		"	2 *tens.*
7429.181	568	Total product.	

NOTE.—The operation may be contracted by dropping the figures at the right of the perpendicular line. Their value will not materially affect the result.

ANALYSIS.—1. For convenience, the numbers are written so that units will stand in the column of units, tens in the column of tens, tenths in the column of tenths, &c.

2. For convenience, begin to multiply with the right hand figure.

3. 4 ten thousandths of* 2 hundredths are 8 millionths; which write in the place of millionths.

4. 4 ten thousandths of 3 tenths are 12 hundred thousandths, equal to 1 ten thousandth and 2 hundred thousandths; write the 2 hundred thousandths in the place of hundred thousandths, and add the 1 ten thousandth to the product of the ten thousandths.

5. 4 ten thousandths of 4 units are 16 ten thousandths, which with 1 ten thousandth are 17 ten thousandths, equal to 7 ten thousandths and 1 thousandth; write the 7 ten thousandths in the place of ten thousandths, and add the thousandths to the product of the thousandths.

6. 4 ten thousandths of no tens equals no thousandths, which with 1 thousandth equals 1 thousandth, which write in the place of thousandths.

7. 4 ten thousandths of 3 hundred equals 12 hundredths, &c.

N. B.—The teacher should require the analysis until it is thoroughly understood.

QUESTIONS.—What is United States money? (68.) What are the gold coins? (69., *a.*) The silver coins? (69., *b.*) The nickel coin? (69., *c.*) What is the use of the decimal point? (70.) What are the rules for the reduction of U. S. money? (71., *c.*) How are the numbers written in addition of U. S. money? (72., *a.*) In subtraction? (73., *a.*) In multiplication? (74., *a.*) In division? (75., *a.*)

177. PROPOSITIONS IN THE MULTIPLICATION OF DECIMALS.

(*a.*) Any *denomination* multiplied by *units* gives the

---

*NOTE.—*Of* signifies *times*, and is used in its stead, when the multiplier is a fraction.

same denomination; as, *hundredths* multiplied by units give *hundredths; thousandths* give thousandths, &c.

(*b.*) Any *denomination* multiplied by *tenths* gives a denomination *ten* times smaller than the denomination multiplied; as, *units* multiplied by *tenths* give *tenths;* hundreds give tens, &c.

(*c.*) Any *denomination* multiplied by *hundredths* gives a denomination *one hundred* times smaller than the denomination multiplied; as, hundreds multiplied by *hundredths* give units; tenths, give *thousandths*, &c.

(*d.*) Any *denomination* multiplied by *thousandths* gives a denomination *one thousand* times smaller than the denomination multiplied; as, hundreds multiplied by *thousandths* give tenths; tenths, give ten thousandths, &c.

(*e.*) GENERAL LAW.—Any *denomination* multiplied by any denomination *less than a unit*, will give for a product a denomination *as many times less* than the denomination multiplied, as the denomination of the multiplier is less than a *unit*.

N. B.—The teacher should question the pupils on these propositions, and also those in the division of decimals, until they can give the denomination of any product promptly and accurately.

QUESTIONS.—What is a bill of parcels? (76.) What is a simple number? (77.) What is a compound denominate number? (78.) What is the reduction of compound denominate numbers? (79.) What is reduction descending? (80.) What is reduction ascending? (81.) What is English money? (82., *a.*) Repeat the table. (82., *b.*) Name the gold coins. (82., *c.*) The silver coins. (82., *d.*) The copper coins. (82., *e.*)

LESSON XI.

Multiply 407.37 by 38.24.

MODEL OPERATION.

```
    407.37
     38.24
   -------
   1629 48
   8147 4
  325896
 122211
 ---------
 15577.8 288 Ans.
```

NOTE.—Those who prefer to use this model operation to the one before given, will find no difficulty in doing so. The former, however, is considered more simple and philosophical, and more convenient, especially when it is desirable to contract the operation by dropping the right hand decimal places. The reason for the following *rule* appears in the analysis of the former operation.

(*f.*) RULE I.—*Multiply as in simple numbers, and point off as many decimals in the product as the sum of the decimal places in the multiplicand and multiplier.*

II. *If there are not as many figures in the product as the sum of the decimal places in the multiplicand and multiplier, supply the deficiency by prefixing ciphers.*

	(74)	(75)	(76)	(77)	(78)
Multiply	434.64	43.41	53.04	41.23	3.41
by	3.49	41.2	2.11	.21	2.23

	(79)	(80)	(81)	(82)	(83)
Multiply	3417.2	6437.2	6710.24	464.24	412.301
by	3.8	4126.	413.2	3.02	4.121

	(84)	(85)	(86)	(87)	(88)
Multiply	416.71	434.02	831.24	51.24	41.23
by	3.21	4.12	43.63	3.41	2.13

QUESTIONS.—What is weight? (83.) What is the use of Troy weight? (84.) What is a carat? (84., note.) Repeat the table.

(84., *a.*) Repeat the Long-ton table. (86.) Repeat the Miscellaneous table. (87.) What is the use of Apothecaries' weight? (88.) How are medicines bought and sold? Repeat the table. (88., *a.*) Repeat the comparative table of weights. (88., *b.*) How many dimensions has the measure of extension? (89.)

LESSON XII.

	(89)	(90)	(91)	(92)	(93)
Multiply	341.42	32.42	413.43	512.3	41.837
by	6.48	831.21	86.431	413.423	4.4

	(94)	(95)	(96)	(97)	(98)
Multiply	384.21	41.32	4103.	46.71	43.042
by	6.431	4.137	3.041	4.112	23.

	(99)	(100)	(101)	(102)	(103)
Multiply	463.41	4134.	4.	421.	241.
by	21.301	3.4134	1.2	.321	.21

	(104)	(105)	(106)	(107)	(108)
Multiply	537.43	412.	671.2	83.	871.2
by	2.446	.4123	4.41	4.426	4.24

	(109)	(110)	(111)	(112)	(113)
Multiply	37.471	4123.	5.3074	413.42	37.4
by	2.021	1.0003	25.	3.4004	42.82

	(114)	(115)	(116)	(117)	(118)
Multiply	463.41	342.41	86.43	4918.3	4183.7
by	21.471	63.47	4.137	91.67	41.864

	(119)	(120)	(121)
Multiply	467.401	418.764	40003.
by	3.0701	3.0001	.00007

	(122)	(123)	(124)
Multiply	600000.	43742.	467.0483
by	.00007	.0006	41.86084

	(125)	(126)	(127)
Multiply	67134.	460000.	6000.
by	1.0000008	.80008	.00000008

QUESTIONS.—How many dimensions has a line? (89., *a.*) A surface? (89., *b.*) A solid? (89., *c.*) What is the use of Long measure? (90.) Repeat the table. (90., *a.*) Repeat the table of miscellaneous Long measure. (90., *b.*) What is the use of Gunter's Chain? (91.) Repeat the table. (91., *a.*) What is a square? (92.) What is the use of Square measure? (92., *c.*) How do artificers estimate their work? (92., *d.*) Repeat the table. (92., *f.*)

LESSON XIII.

128. Multiply 37.0804 by 4.604; by 4.06402; by .00003.

129. Multiply 430.704 by 37.4008; by 5.0007; by 600.0070001.

130. Multiply 380.412 by 2.00071; by 341.021; by 687.18302.

131. Multiply 437.401 by .3000002; by .00004; by 000008.

132. Multiply 3806.001 by .00007; by .000008; by 000007.

133. Multiply 40607.001 by .30404; by .30012; by .00070007.

134. Multiply 600000. by .00004; by 000008; by .0000007.

135. Multiply 900000. by .00009; by .0009; by .00000009.

136. Multiply 80000. by .00008; by .0008; by .000008.

137. Multiply 40000. by .0007; by .00006; by .000002.

QUESTIONS.—What is the use of Surveyors' square chain? (93.) Repeat the table. (93., *a.*) What is a cube? (94.) Give the analysis of the cubic yard. (94., *b.*) What is the use of cubic measure? (94., *c.*) Repeat the table. (94., *e.*) What are the dimensions of a cord of wood? (94., *d.* 3.) What is the difference between a cord foot, and a cubic foot of wood? What are measures of capacity? (95.) What is the use of Liquid measure? (95., *a.*) Repeat the table. (95., *b.*) Repeat the miscellaneous table. (95., *c.*)

LESSON XIV.

138. Multiply 468 thousand by 402 *ten-thousandths.*

139. Multiply 304 *thousandths* by 402 *hundred-thousandths.*

140. Multiply 8467 *millionths* by 4678 millions.

141. Multiply 2000 millions by 2000 *millionths.*

142. Multiply 4670 *ten-thousandths* by 4864 *millionths.*

143. Multiply 378 and 463 *thousandths* by 437 *hundred-thousandths.*

144. Multiply 467 and 437 *thousandths* by 600000 *ten-millionths.*

145. Multiply 384 and 304 *millionths* by 347 thousands.

146. Multiply 472 and 464 *thousandths* by 467 hundreds.

147. Multiply 386412 and 4 *ten-thousandths* by 8078.

148. Multiply 402 and 3 *hundred-thousandths* by 2 *hundredths.*

149. Multiply 407 and 4 *billionths* by 888 billions.

QUESTIONS.—What is the use of Dry measure? (96.) Repeat the table. (95., *a.*) Repeat the comparative table of measure. How can Dry measure be reduced to Liquid measure? ANS.—1*st. Reduce Dry measure to cubic inches.* 2*nd. Reduce the cubic inches to Liquid measure.* What is time? (97.) Repeat the table. (97., *a.*)

LESSON XV.

150. What will 467.3 bushels of wheat cost, at $1.123 per bushel? What, at $1.713? What, at $1.426?

151. What will 943.42 lbs. of tea cost, at $0.97½ per pound? What, at $1.43¼? What, at $0.506?

152. What will a firkin of butter, containing 92.3 lbs., cost, at $.25 per pound? At $.157? At $0.187?

153. Bought a bale of cloth containing 95 pieces. 14.75 pieces were damaged, and I was obliged to sell the whole at an average of $9.462; how much did I receive for it?

154. Bought 46783 oranges for $1000, and sold them for $0.02¼ apiece; did I make, or lose, and how much?

155. A man bought 4374 sheep for $16000. He sold one half of them at $3.75 a head, and the remainder at $4.12½ a head; did he make, or lose, and how much?

156. A man sold, at different times, 3876.4, 437.24, and 572.4, bushels of apples; what should he receive, at .25 cts. per bushel?

157. Bought 86.4 yds. of cloth at 68½ cts. a yard, 15.4 yards. of calico at 18 çts. a yard; how much did I give for both?

158. Bought 4.3 barrels of cider, each barrel containing 32.4 gal., at 14 cts. a gallon; how much did the cider cost?

159. How many sq. ft. of wall and ceiling in a room 24.2 ft. long, 13.4 ft. wide, and 11.3 ft. high?

160. How much would it cost to plaster it, at 2½ cts. per sq. ft.?

Questions.—Repeat the table of the calendar year? (97., *b.*) What is the use of circular motion? (98.) Repeat the table. (98., *a.*) Repeat the table of particulars. (99., *a.*) Repeat the table of paper. (99., *b.*) The table of books. (99., *c.*) How many words make a folio? (99., *d.*)

What is reduction descending? (80.) Analyze an example at the blackboard. (100., *a.*) Repeat the rule. (100., *b.*) What is reduction ascending? (81.) Analyze an example at the blackboard. (101., *a.*) Repeat the rule. (101., *b.*)

LESSON XVI.

161. A grocer bought five tubs of butter, each containing 64.37 lbs., at 18.5 cts. per pound; 7 cheeses, each weighing 46.34 lbs., at 9.3 cts. per pound: how much did the whole cost?

162 If a clerk receiving $4440.25 spends $1472.22 for board, and twice as much for clothes, books, and other expenses, how much will he have left?

163. B has 457.2 lbs. of sugar, C has three times as many as B, and D has as many as B and C together; how many pounds of sugar has D?

164. A has 43.2 tubs of butter, B has 2.4 times as many as A, and C has twice as many as A and B together; how many tubs have they in all?

165. A has 371.5 yards of cloth, B has .4 as many yards as A, and C has 3.7 as many yards as A and B together; what is the value of all the cloth, at $2.42 per yard?

166. There are 75.7 tubs of butter, each tub weighing 37.4 lbs.; the tubs which contain the butter weigh .14 as many lbs. as the butter they contain; how many pounds of butter do the tubs contain?

167. A farmer sold 19.3 tubs of butter weighing 78.3 lbs. each; the tubs which contain the butter weigh each .12 as many pounds as the butter they contain: how much is the butter worth, at 29.3 cts. per pound?

QUESTIONS.—Analyze at the blackboard an example in the addition of denominate numbers. (102., *a.*) Analyze at the blackboard an example in the subtraction of denominate numbers? (103., *a.*) Give the rule. (103., *g.*)

LESSON XVII.

178. DIVISION OF DECIMALS.

Divide 2.460 by 14.24.

MODEL OPERATION.

$$
\begin{array}{r}
14.24)2.460(.1727+\text{ Ans.}\\
1.424\phantom{0(.1727+\text{ Ans.}}\\
\hline
1.0360\phantom{(.1727+\text{ Ans.}}\\
9968\phantom{(.1727+\text{ Ans.}}\\
\hline
3920\phantom{.1727+\text{ Ans.}}\\
2848\phantom{.1727+\text{ Ans.}}\\
\hline
10720\phantom{727+\text{ Ans.}}\\
9968\phantom{727+\text{ Ans.}}\\
\hline
\end{array}
$$

Analysis.—1. For convenience write the divisor, dividend, and quotient, as in simple division.

2. 14.24 are equal to 1424 hundredths; and 2.460 are equal to 2460 thousandths.

3. 1424 hundredths are contained in 2460 thousandths 1 tenth of a time, with a remainder of 1036 thousandths, equal to 10360 ten thousandths. Write the 1 tenth in the place of tenths in the quotient.

4. 1424 hundredths are contained in 10360 ten-thousandths 7 hundredths of a time, with a remainder of 392 ten-thousandths, equal to 3920 hundred-thousandths. Write the 7 hundredths in the place of hundredths in the quotient.

5. 1424 hundredths are contained in 3920 hundred-thousandths 2 thousandths of a time, with a remainder of 1072 hundred-thousandths, equal to 10720 millionths. Write the 2 thousandths in the place of thousandths in the quotient.

1424 hundredths are contained in 10720 millionths 7 ten-thousandths of a time, with a remainder, &c.

NOTES.—1 The analysis should be carried as far as the teacher may deem advisable.
2. The sign + annexed to the quotient shows that the division is not exact.

179. PROPOSITIONS IN DIVISION OF DECIMALS.

(*a.*) *Any denomination* divided by *units* gives the same denomination for a quotient; as, hundredths divided by *units* give hundredths; thousandths give thousandths; hundreds give hundreds, &c.

(*b.*) *Any denomination* divided by *tenths* gives a denomination *ten* times larger than the denomination divided; as, units divided by tenths give tens; tenths give units; hundredths give tenths, &c.

(*c.*) *Any denomination* divided by *hundredths* gives a denomination *one hundred* times larger than the denomination divided; as, units divided by hundredths give hundreds; thousandths give tenths; millionths give ten thousandths, &c.

(*d.*) *Any denomination* divided by *thousandths* gives a denomination *one thousand* times larger than the denomination divided; as, thousandths divided by thousandths give units, &c.

(*e.*) GENERAL LAW.—Any denomination divided by any denomination *less than a unit*, gives for a quotient a denomination as many times *larger* than the denomination divided, as the denomination of the divisor is *less* than a *unit.*

NOTE.—For the use of teachers who prefer the old method of pointing, we give the following:

(*f.*) RULE.—*Divide as in simple division, making the number of decimal places in the dividend at least equal to those in the divisor; then subtract the number of decimal places in the divisor from the number of decimal places in the dividend, and the remainder will denote the number of* *decimal places in the quotient. Should there not be as many figures in the quotient, supply the deficiency by prefixing ciphers.*

QUESTIONS.—Analyze at the blackboard an example in the multiplication of denominate numbers. (104.) Analyze at the blackboard an example in the division of denominate numbers? (105.) What is an integer? (106.) What is an odd number? (107.) What is an even number? (108.) What is a prime number? (109.) What is a composite number? (110.) When are numbers said to be prime to each other? (111.) What is a prime factor? (112.) What is a composite factor? (113.)

LESSON XVIII.

EXAMPLES FOR PRACTICE.

168. Divide 463. by 38.
169. Divide 417. by 47.
170. Divide 347. by 59.
171. Divide 24.374 by 73.
*172. Divide 2.432 by 97.
173. Divide .6712 by 39.
174. Divide .672 by 58.
175. Divide .0743 by 69.
176. Divide .007 by 87.
177. Divide .43007 by 96.
178. Divide .00093 by 87.
179. Divide .00007 by 596.
180. Divide .00001 by 359.
181. Divide 4 by 72.
182. Divide 40 by 973.
183. Divide 63.1 by 25.
184. Divide 12 by 376.
185. Divide 864 by .3.
186. Divide 642 by .4.
187. Divide 572 by .5.
188. Divide 896 by .5.
189. Divide 305 by .5.
190. Divide 40.2 by .7.
191. Divide 4.3 by .9.
192. Divide 6.7 by .8.
193. Divide .4 by .9.

QUESTIONS.—Is a unit considered a factor? (113., *a.*) Separate a composite number into its prime factors. (114.) Give the rule. (114., *d.*) Give the proof. (114., *e.*) What is the use of a parenthesis or vinculum? (116.) What is cancellation? (117.) What is a common divisor of two or more numbers? (118.) What is the greatest common divisor? (119.) What is a common multiple of two or more numbers? (121.)

---

*NOTE.—Continue the division to five decimal places in the quotient.

LESSON XIX.

194. Divide .463 by .4.
195. Divide .437 by 4.6.
196. Divide 3.07 by 46.3.
197. Divide 4.07 by 916.3.
198. Divide .6703 by 571.2.
199. Divide 46.72 by 3712.3.
200. Divide 427.1 by .03.
201. Divide 467 by .07.
202. Divide 417 by .09.
203. Divide 387 by .07.
204. Divide 868 by .04.
205. Divide 407 by .23.
206. Divide 91.63 by .47.
207. Divide 46.72 by .32.
208. Divide 46.47 by .43.
209. Divide 96.876 by .43.
210. Divide 47.236 by .97.
211. Divide 37.26 by 4.38.
212. Divide 67.43 by 37.46.
213. Divide 21.3 by 467.23.
214. Divide 9 by 3724.86.
215. Divide 4 by 6712.43.
216. Divide 1 by 6738.41.
217. Divide .007 by 3672.49.
218. Divide .00006 by 7189.48.
219. Divide .000302 by 5763.43.

QUESTIONS.—What is a multiple of a number? (120.) What is the least common multiple? (122.) What is a fraction? (123.) What is a common fraction? (124.) What is a decimal fraction? (125.) What are the terms of a fraction? (126.) What is the denominator of a fraction? (127.)

LESSON XX.

220. Divide 307.464 by 34; by 3.4; by .34.
221. Divide 407.673 by 46.3; by 50.72; by .467.
222. Divide 876.4123 by 864.3; by 8.67; by 4.673.
223. Divide 896.437 by 967.3; by 5.703; by 6.934.
224. Divide 43.2 by 467.2; by 59.634; by 2.6734.
225. Divide 46.73 by 947.32; by 47.286; by 4.4834.
226. Divide 46.37 by 46.37; by 4.637; by .4637.
227. Divide 9 by 83; by 3.67; by 9876.
228 Divide 8.3 by 6.72; by 9.183; by 671.83.
229. Divide 4.7 by 9.1864; by 9.1372; by 8.6493.
230. Divide .87 by .89672; by .03496; by .0089672.

QUESTIONS.—What does the numerator of a fraction show? (128.) What is a proper fraction? (129.) What is an improper fraction?

(130.) What is a mixed number? (131.) What is a simple fraction? (132.) What is a complex fraction? (133.) What is a compound fraction? (134.) Of what may fractions be considered indications? (135.) To what does the numerator of a fraction correspond in division? (136.) To what does the denominator of a fraction correspond in division? (136.)

LESSON XXI.

231. Divide 4.07864 by .963; by 6.7863; by 8.6743.
232. Divide 4.83726 by .4867; by 4.8673; by 9.6724.
233. Divide 96.7387 by .46783; by 6.3732; by 67.834.
234. Divide 91.7837 by .6803; by .00786; by 44.6784.
235. Divide 4 by 96.783; by .000643; by 41867.
236. Divide .1 by 9.1867; by .000734; by 967863.
237. Divide .6 by .47136; by .96037; by 678397.
238. Divide 6 by 9673.42; by 9.0007; by 467321.
239. Divide .000006 by .000007; by 60000; by 6000000.
240. Divide 8000000 by .000009; by 6000.007; by 6000.00007.
241. Divide .0000009 by .0000007; by 700000; by 40000000.
242. Divide 8643.004 by .00000087; by 6000000; by 60000000.

QUESTIONS.—If both numerator and denominator of a fraction are increased in an equal ratio, is its value changed? (137., *a.*) If both are decreased in an equal ratio, is its value changed? (137., *b.*) What effect has multiplying the numerator, upon the value of a fraction? (137., *e.*) What effect has multiplying the denominator, upon the value of a fraction? (137., *c.*) What effect has dividing the numerator, upon the value of a fraction? (137., *f.*) What effect has dividing the denominator, upon the value of a fraction? (137., *d.*)

LESSON XXII.

What is the value of each of the following expressions?

243. $86 \div 9.13 \times 6.7 + 87.643 - 4.23$.

244. $43.6\times874.24\div8.74-6.43+86.743$.
245. $6.389\times471.23\div9.67-4.7+9.83$.
246. $5\div967\times41.3\div8.6-6.72+67.38$.
247. $46.783\times.4712\div967.4-.378+986.742$.
248. $507.683\div41.3\times86.4-21.3\times3.1+6.7183$.
249. $907.213\times8.643-21.32\times67.32+86.434$.
250. $864.307\div89.76\times34.7-6.72+8.38764$.

Questions.—What is a fraction? (138.) How is the denomination of a fraction determined? (139.) What is reduction of fractions? (141.) What is reduction ascending? (142.) What is reduction descending? (143.) When is a fraction said to be expressed in its lowest terms? (144.) What is the rule for reducing a fraction to its lowest terms? (145.)

LESSON XXIII.

PRACTICAL EXAMPLES.

A man sold 13.4 bushels of apples, at \$0.37 per bushel, and then divided the money among 14 poor men; how much did each man receive?

MODEL OPERATION.

$$\begin{array}{r}
13.4\text{ bu.}\\
.37\text{ cts.}\\
\hline
.938\phantom{\text{ s.}}\\
4.02\phantom{\text{ cts.}}\\
\hline
14)\$4.958\phantom{\text{ cts.}}\\
\hline
\$0.354+\text{Ans.}
\end{array}$$

Analytical Steps.—1. Find what was received for the apples.

2. Find what each man received.

Note.—The teacher should require the full explanation of each step in the analysis of the following examples.

QUESTIONS.—Analyze an example at the blackboard, in which a fraction is reduced to its lowest terms. (145.) Analyze an example in which an improper fraction is changed to a mixed number. (146.) In which a mixed number is changed to a given denomination. (148.) What is addition of fractions? (149.) When are fractions said to have a common denominator? (150.) Can fractions be added, when not of the same denomination? Why not? (16., *a.*)

LESSON XXIV.

251. What will 4.9 lbs. of sugar cost, at 13.42 cts. per pound?

252. If 23.7 lbs. of sugar cost 9.2 cts. per pound, and 41.3 lbs. of rice cost 4.2 cts. per pound, what do both cost?

253. If 14.6 lbs. of cotton cost 34.7 cts. per pound, and 91.3 gal. of molasses cost 43.2 cts. per gal., what do both cost?

254. If the interest on one dollar for one year is 7 cts., what is the interest on $39.42 for the same time?

255. If the interest on one dollar for one year is 8 cts., what is the interest on $93.41 for the same time?

256. If the interest on one dollar for one year is 4 cts., what is the interest on one dollar for one month?

257. If the interest on one dollar for one year is 9 cts., what is the interest on one dollar for nine months?

258. If the interest on one dollar for one year is 11 cts., what is the interest on $23.43 for one month?

259. If the interest on one dollar for one year is $0.125, what will be the interest on $16.37 for 27 months?

260. If the interest on one dollar for one year is $0.117, what is the interest on $87.34 for 37 months?

QUESTIONS.—What is the subtraction of fractions? (151.) How do you add and subtract fractions having a unit for the numerator? (152.) What is multiplication and division of fractions? (153.)

Analyze an example in which a fraction is multiplied by an integral number. (154.) Give the rule. Analyze an example in which a fraction is divided by an integral number. (155.) Give the rule.

LESSON XXV.

261. I invest $478.60 in business, and, at the end of one year, I find that my gains are $36.42; how much have I gained on one dollar?

262. If I invest $964.32 in business, and gain in one year $46.47, how much do I gain on $47.36?

263. The gain on one dollar is called *percentage;* now, if I invest $374.23 in sugar, and sell it so as to gain $37.46, how much per cent. do I gain?

264. A man bought some molasses for $374.65; he sold the molasses shortly after so as to gain $89.46: what was his gain per cent.?

265. A man bought a drove of sheep for $372.48, and sold them at a loss of $63; what was his loss on one dollar?

266. A merchant bought 478 pieces of cloth for $2413.60; he sold it at a loss of $967.40: how much did he lose on one dollar?

267. A grocer bought 43 sacks of rice for $564.30, but, it being damaged, he was obliged to sell it for $426.; how much did he lose? How much did he lose per cent.?

268. A speculator bought $3974. worth of gold on Saturday, and on Monday he was obliged to sell it for $3576.21. How much did he lose? How much did he lose per cent.?

QUESTIONS.—Analyze an example of the division of a mixed number by an integral number. (156.) Give the rule. Analyze an example of the multiplication of a fraction by a fraction. (157.) Give the rule. Analyze an example of the multiplication of a mixed number by a mixed number. (158.) Analyze an example of the division of an integral number by a fraction. (159.) Give the rule.

LESSON XXVI.

REDUCTION OF DECIMALS.

180. To change a common fraction to a decimal, and a decimal to a common fraction.

Change $\frac{3}{8}$ to an equivalent decimal.

MODEL OPERATION.

8.)3.

.375, Ans.

PROOF.

$$\frac{375}{1000}=\frac{3}{8}.$$

(*a.*) Analysis.—As the numerator is a dividend, and the denominator a divisor, reducing it to a decimal is simply an example of the division of decimals, and may be treated as such. (See Anal. of Divis. of Decimals, 178.)

(*b.*) Proof.—A decimal may be written in the form of a common fraction; thus, .375, equals $\frac{375}{1000}$, which reduced to its lowest terms equals $\frac{3}{8}$.

(*c.*) Notes.—1. When the denominator, or divisor, contains other prime factors than 2 or 5, the division must continue indefinitely, and some figure or figures will be continually repeated to form the quotient.

2. A decimal which is expressed by a continual repetition of a certain figure or figures, is called a *circulating decimal or repetend.*

3. A repetend is distinguished by a dot over the first and the last of the repeating figures thus, $.42\dot{3}$ shows that the 3 is a repetend; $.46\dot{3}7\dot{1}$ shows that 371 is a repetend. The repetend taken with the rest of the decimal is called a *mixed repetend.*

4. To change a repetend to an equivalent common fraction, write the repetend for a numerator, and as many nines for the denominator as the repetend has figures, thus, $.\dot{1}3\dot{5}=\frac{135}{999},=\frac{5}{37}$; the mixed repetend $.41\dot{6}=\frac{41\frac{6}{9}}{100}\times\frac{9}{9}=\frac{375}{900}=\frac{5}{12}$.

Questions.—Analyze an example of the division of a fraction by a fraction. (160.) Give the rule. Simplify an example of compound fractions. (161.) An example of complex fractions. (161.) Analyze an example of finding the greatest common divisor of fractions (162.) Analyze an example of finding the least common multiple of two or more fractions. (163.)

LESSON XXVII.

269. Change $\frac{5}{8}$ to a decimal.
270. Change $\frac{5}{17}$ to a decimal.
271. Change $3\frac{4}{8}$ to a decimal.
272. Change $43\frac{4}{38}$ to a decimal.
273. Change $2\frac{37}{961}$ to a decimal.
274. Change $\frac{8}{471}$ to a decimal.
275. Change $\frac{8.4}{262}$ to a decimal.
276. Change $\frac{5}{2.41}$ to a decimal.
277. Change $\frac{21.2}{.41672}$ to a decimal.
278. Change $\frac{374}{2.14}$ to a decimal.
279. Change $\frac{3.7}{4.42}$ to a decimal.
280. Change $\frac{.03}{41302}$ to a decimal.
281. Change .75 to a common fraction.
282. Change .875 to a common fraction.
283. Change .4375 to a common fraction.
284. Change .04 to a common fraction.
285. Change 3.067 to a common fraction.
286. Reduce 8.275 to a common fraction.
287. Reduce .00049 to a common fraction.
288. Reduce $.\dot{6}$ to a common fraction.
289. Reduce $.\dot{7}\dot{2}$ to a common fraction.
290. Reduce $.\dot{1}3\dot{5}$ to a common fraction.
291. Reduce $.09\dot{3}$ to a common fraction.
292. Reduce $.235\dot{6}\dot{3}$ to a common fraction.

QUESTIONS.—What is reduction ascending? (165.) What is reduction descending? (164.) What is the comparison of numbers? (166.) What axiom is used in the comparison of numbers? (16., *c.*) What are the rules for the addition and subtraction of fractional denominate numbers? (168., *a.*, *b.*)

LESSON XXVIII.

REDUCTION OF DENOMINATE DECIMALS.

181. Reduction Descending in denominate decimals is the same as in integral denominate numbers.

182. Reduction Ascending in denominate decimals is the same as in integral denominate numbers.

Change .34672£. to integral numbers of lower denominations.

MODEL OPERATION.

.34672£.
20s.

6.93440s.
12d.

11.21280d.
4 far.

.85120 far. $= \frac{85}{100} = \frac{17}{20}$ far.
6s. 11d. $\frac{17}{20}$ far. Ans.

PROOF.

6s.+11d.+0$\frac{17}{20}$+far.
20)17

---

4 far.).85 far.

12d.)11.2125d.

20s.)6.9343+s.

.34671+£.*

(*a.*) ANALYSIS.—1. Change pounds to shillings.

Since in one pound there are 20 shillings, in .34672£. there are 20 times as many shillings as pounds, which are 6s. and .93440 of a shilling.

2. Change .9344 of a shilling to pence.

Since in one shilling there are 12 pence, in .9344s. there are 12 times as many pence as shillings, which are 11d. and .2128 of a penny.

3. Change .2128 of a penny to farthings.

Since there are 4 farthings in one penny, in .2128d. there are

---

* NOTE.—When only tenths and hundredths are changed to common fractions, the example will not prove exactly correct, but nearly enough so for all practical purposes.

4 times as many farthings as pence, which are 0 far. with a remainder of .85* of a farthing, equal to $\frac{17}{20}$ far.

Therefore .34672£.=6s. 11d. $\frac{17}{20}$+far.

(*b.*) Analysis of Proof.—1. Change $\frac{17}{20}$ far. to farthings.

Since in twenty 20ths there is one far., in seventeen 20ths there are *one-twentieth* as many farthings as 20ths, which are .85 far.

2. Change farthings to pence.

Since in 4 far. there is one penny, in .85 far. there are *one-fourth* as many pence as farthings, which are .2125d., which with 11d. added equals 11.2125d.

3. Change pence to shillings.

Since in 12d. there is one shilling, in 11.2125d. there are *one-twelfth* as many shillings as pence, which are .9343+s., which with 6s. added equals 6.9343+s.

4. Change shillings to pounds.

Since in 20 shillings there is one pound, in 6.9343s. there are one *twentieth* as many pounds, &c.

Reduce .34£. .9s. .13d. .8$\frac{3}{4}$ far. to integers of lower denominations.

MODEL OPERATION.

$$\begin{array}{rl}
 & .34£.+.9s.+.13d.+.8\tfrac{3}{4}\text{ far.}\\
 & \underline{\quad 20s.}\\
 & 6.80s.\\
 & \underline{\quad .9\ s.}\\
 & 7.7s.\\
 & \underline{\quad 12d.}\\
 & 8.4d.\\
 & \underline{\quad .13d.}\\
 & 8.53d.\\
 & \underline{\quad\ 4\text{ far.}}\\
 & 2.12\text{ far.}\\
.8\tfrac{3}{4}\text{ far.}= & \underline{\ .875\text{ far.}}\\
 & 2.995\text{ far.}=2\tfrac{199}{200}\text{ far.}
\end{array}$$

Therefore .34£. .9s. .13d. .8$\frac{3}{4}$ far.=6s. 8d. 2$\frac{199}{200}$ far.

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*Note.—The remainder of the fraction may be rejected, because of its trifling value.

Analytical Steps.—1. Reduce pounds to shillings, and add the shillings.

2. Reduce shillings to pence, and add the pence.

3. Reduce pence to farthings, and add the farthings.

4. Reduce .995 to a common fraction.

Questions.—What is said of the decimal scale? (169.) What is said of units' place? (170.) Which are integral numbers, and which are decimals? (171., *d.*) Repeat the numeration table from right to left. (171., *a.*) From left to right. (171., *a.*) How is a mixed number formed? (172.) How are numbers usually divided for convenience in reading? (173.) Write a rule for the addition of decimals.

LESSON XXIX.

EXAMPLES FOR PRACTICE.

293. Reduce $.28125 to integers of lower denominations.

294. Reduce .3746 lbs. to integers of lower denominations.

295. Reduce .437 cwt. to integers of lower denominations.

296. Reduce .37 lb. apothecaries' weight, to lower denominations.

297. Reduce .34 lb. Troy weight, to lower denominations.

298. Reduce .374 A. to integers of lower denominations.

299. What is the value of .27 A. 62 R. in lower denomination?

300. How many cubic inches in .371 cd. of wood?

301. How many gills in .16 hhd. .43 gal.?

Questions.—Write a rule for the subtraction of decimals? (175.) Analyze an example in the multiplication of decimals. (176.) When any denomination is multiplied by units, what is the denomination of the product? (177., *a.*) If multiplied by a *tenth?* (177., *b.*) If multiplied by *hundredths?* (177., *c.*) By *thousandths?* (177., *d.*)

LESSON XXX.

302. How many ounces in 4.34 cwt.?
303. How many drams in 3.264 tons?
304. How many sq. in. in 4.3 A. 1.12 sq. rds.?
305. Reduce 3 rds. to the decimal of a mile.
306. Reduce 6 fur. 8 rds. 9 ft. to miles.
307. Reduce 9 A. 4 R. 2 sq. rds. to square miles.
308. Reduce 3 cwt. 9 qrs. 18 lbs. to hundred weight.
309. Reduce 9 cwt. 1 oz. to cwt.
310. Reduce 14 mi. $2\frac{3}{4}$ yds. 3 in. to miles.
311. Reduce 13 m. $14\frac{3}{7}$ fur. 9 yds. $8\frac{1}{8}$ in. to miles.

QUESTIONS.—What is the general law for the multiplication of decimals? (177., *e.*) Repeat the rule for multiplication of decimals. (177., *f.*) Analyze an example in the division of decimals. (178.) When any denomination is divided by units, what will be the denomination of the quotient? (179., *a.*) If divided by tenths? (179., *b.*) If divided by hundredths? (179., *c.*) By thousandths? (179., *d.*) What is the general law for division of decimals? (179., *e.*) Repeat the rule. (179., *f.*)

LESSON XXXI.

MISCELLANEOUS EXAMPLES.

312. What will 13 lbs. of hay cost, at $20 per ton?
313. What will $17\frac{3}{7}$ lbs. of hay cost, at $17 per ton?
314. What will 13 eggs cost, at 17 cts. per doz.?
315. What will 4 doz. eggs cost, at 19 cts. per doz.?

316. What is the value of 15 cwt. 3 qrs. 14 lbs. of coffee, at $9.50 per cwt.?

317. What cost 17 T. 18 cwt. 1 qr. 7 lbs. of pork, at $2.50 per cwt.?

318. What cost $15\frac{3}{7}$ yds. of cloth, at $3.75 per yard?

319. What cost 15 cd. $13\frac{5}{8}$ cu. ft. of wood, at \$4.63$\frac{1}{2}$ per cord?

320. What are the contents of a pile of wood 9 ft. $3\frac{3}{7}$ in. long, 4 ft. $3\frac{6}{7}$ in. wide, and 8 ft. 7 in. high?

321. At \$9.37 per cord, what will a pile of wood cost that is 8 ft. 2 in. long, 5 ft. $2\frac{3}{7}$ in. wide, 8 ft. $11\frac{5}{7}$ in. high?

Questions.—How are common fractions changed to decimals? (180., *a.*) What is the difference between changing common fractions to decimals and the division of decimals? (180., *a.*) How may the operation be proved? (180., *b.*) When the denominator of a fraction contains other factors than 2 or 5, will the division be exact? (180., *c.*, 1.) When a figure or figures are continually repeated, what are they called? (180., *c.*, 2.)

LESSON XXXII.

322. What are the contents of a board 12 ft. 6 in. long, and 2 ft. 9 in. wide?

323. What are the contents of 369 boards, each 12 ft. 8 in. long, 3 ft. 2 in. wide?

324. What would be the cost of a load containing 53 boards, each board 13 ft. 2 in. long, $7\frac{3}{8}$ in. wide, at $2\frac{3}{4}$ cts. per square foot?

325. Bought 13 casks of vinegar, each containing 25 gal. 3 qts. 1 pt., at \$0.375 per gallon; what was the cost?

326. Bought a farm containing 143 A. $30\frac{4}{7}$ sq. rds., at \$46.43 per acre; what was the cost?

327. Paid \$14.53 for 2.37 cwt. of nails; how much was that per pound?

328. Paid \$17.28 for $2\frac{5}{9}$ cwt. of nails; how much was that per pound?

329. Paid \$25.17 for 3.72 cwt. of sugar; how much was that per pound?

330. Paid $36.42 for $4\frac{5}{8}$ cwt. of sugar; how much was that per pound?

331. Bought 14.32 cwt. of hay at $0.05 per pound; how much did it cost?

QUESTIONS.—How are circulating decimals distinguished? (180., *c.*, 3.) How are repetends changed to common fractions? (180., *c.*, 4.) What is reduction descending in denominate decimals? (181.) What is reduction ascending? (182.) Give the analysis of an example of reduction descending. (182., *a.*) Give the analysis of an example of reduction ascending. (182., *b.*)

LESSON XXXIII.

332. Bought 2.42 cwt. of flour at $1.25 per qr., and sold it at 6.5 cts. per pound; how much did I gain?

333. Bought $9\frac{3}{7}$ qr. of rice at 7 cts. per pound, and sold it for 11 cts. per pound; how much did I make by the operation?

334. Bought 4.3 hhd. of wine at $2.14 per gallon, and sold it at 12.24 cts. per gill; how much did I make by the operation?

335. What will $\frac{1}{16}$ of an ounce of snuff cost, at $1.53 per pound?

336. What will $\frac{1}{2}$ oz. of hay cost, at $13.20 per ton?

337. What will .341 of a grain of gold be worth, at $16. per ounce?

338. What will $\frac{2}{11}$ of a grain of gold be worth, at $238 .46 per pound?

339. If $\frac{7}{8}$ of a cord of wood cost $6.50, what will one cord cost?

340. If $\frac{3}{7}$ of a cord of wood cost $8.72, what will 9.34 cords cost?

341. If $\frac{4}{7}$ of a cord of wood cost $9.37, what will be the

cost of a pile of wood 16 ft. 8 in. long, 3 ft. 7 in. wide, and 7 ft. 3 in. high?

QUESTIONS.—Write on the blackboard a rule for reducing a common fraction to a decimal. (180., *a.*) Write a rule in the same manner for reducing a decimal to a common fraction. (180., *b.*) What is the difference between a common and a decimal fraction? (124.) (125.) How many tenths make a unit? How much larger is a hundred than a tenth? A ten than a hundredth?

LESSON XXXIV.

342. A man bought a ship for $35000; $\frac{5}{8}$ of it was sold to Mr. Jones for $22314.23, and the remainder to another man at a gain of $416.82; what was made on the sale of the ship?

343. A man bought a ship for $25000; .75 of it was sold to Mr. Mead for $21037.42, and the remainder to Captain Read, at a gain of 33 cts. on each dollar of the cost of the remainder; how much was made on the sale of the ship?

344. A man had 2 bu. 2 pks. 4 qts. of oats; he sold 3 pks. 7 qts. 1 pt.; what decimal part of his oats did he sell?

345. Having 9.34 A. of land, I sold 3 A. 1 R. 18 sq. rds.; what decimal part of the whole did I have left?

346. Having 5 A. 2 R. 17 sq. rds. of land, I sold 3 A. 1 R. 14 sq. rds.; what decimal part of the land did I sell?

347. At $1.25 per yard, how many yards of cloth can I buy for $3.72?

348. At $1.21 per yard, how many yards of cloth can I buy for $0.003?

349. At 38.22 cts. per bushel, how many bushels of oats can I purchase for $0.02?

350. At 48$\frac{3}{8}$ cents per bushel, how many bushels of oats can I purchase for $4.37?

351. If you pay .375£. for 41 bbls. of cider, how much do you pay per gill?

352. At £.25 per yard, what will be the cost of 13 pieces of cloth, each piece containing 39⅜ yds. 1 qr.?

353. Paid £.⅞ for .625 of a yard of sarcenet; what was that per yard?

354. Paid ⅝ of a dollar for .125 bbls. of flour; how much was that per barrel?

355. Bought .37424 A. of land, at $.5 per foot; what did it cost?

356. Bought .673 ton of iron, and sold 1 qr. 13 lbs. to Mr. O. H. Gage; how much was the remainder worth, at 2s. 6d. per pound?

QUESTIONS.—How much larger is a unit than a thousandth? A ten than a ten thousandth? A thousand than a thousandth? If tens are multiplied by tenths, what will be the denomination of the product? (177.) If tens are multiplied by hundredths? By units? By ten thousandths? By millionths? By tens? If tenths are divided by hundredths, what will be the denomination of the quotient? (179.) What is the general law? (179., *e.*) Thousandths divided by tenths gives what denomination: By tens? By thousandths? By ten thousandths?

LESSON XXXV.

PER CENTAGE.

183. Per centum, or its abbreviation, **per cent.,** is a term which signifies *by the hundred;* hence, 3 pounds gain on 100 pounds is said to be a gain of 3 *per cent.*, or of 3 *hundredths* of a pound on *one* pound.

8 gallons loss on 100 gallons is said to be a loss of 8 *per cent.*, or of 8 *hundredths* of a gallon on *one* gallon, &c.

MENTAL EXERCISES.

1. At a gain of 3 pounds on 100 pounds, how much is gained on 7 pounds?

ANALYSIS.—(1.) If the profit on 100 pounds is 3 pounds, the profit on *one* pound will be 1 *hundredth* of 3 pounds, which is three hundredths (.03) of one pound.

(2.) If the profit on *one* pound is .03 of a pound, the profit on 7 pounds is 7 times .03 of a pound, which is .21 of a pound.

Therefore, 3 per cent of 7 pounds is .21 of a pound.

2. I lose 9 gallons on 100 gallons; how much is my loss on 12 gallons?

3. If I receive $4 interest on $100 for one year, how much do I receive on $12 for the same time?

4. What is 9 per cent. of 7 quarts?

ANALYSIS.—9 per cent. signifies 9 quarts on 100 quarts, or $\frac{9}{100}$ of a quart on *one* quart.

If 9 per cent. of one quart is .09 of a quart, then 9 per cent. of 7 quarts is 7 times .09 of a quart, which is .63 of a quart.

Therefore, 9 per cent. of 7 qts. is .63 of a quart.

5. What is 12 per cent. of 11 ounces? of 8 pks.? of $10? of 8 pounds?

6. What is 7 per cent. of 12 days? of 9 pks.? of 8 bushels? of 11 ounces?

7. If I lose 9 farthings on 100 farthings, what per cent. do I lose?

8. If I lose 17 ounces on 100 ounces, what per cent. do I lose?

9. If by investing $100 I gain $13, what per cent. do I gain?

10. I bought a horse for $100, and sold it for $15 more than I gave for it; how much did I gain per cent.?

11. I bought a house for $100, and sold it for $125; how much did I gain per cent.?

QUESTIONS.—What is a unit? (1.) What is the difference between a unit and a number? (1.) (2.) What is the difference between an

abstract and a concrete number? (3.) (4.) Of what does Arithmetic treat? (5.) What is a quantity? (6.) What is a problem? (7.) What is the difference between simple and complex problems? (8.) (9) What is an analytical step? (10.) What is an analysis? (11.) What is a rule? (12.) What is a sign? (13.) What is an axiom? (14.)

LESSON XXXVI.

184. Since any per cent. is some number of hundredths it may be expressed either in the form of a decimal, or of a common fraction.

TABLE.

Rate per cent		Decimal.			
1	per cent. equals	.01	or	$\frac{1}{100}$	of the quantity.
2	per cent. "	.02	or	$\frac{1}{50}$	" "
7	per cent. "	.07	or	$\frac{7}{100}$	" "
9	per cent. "	.09	or	$\frac{9}{100}$	" "
10	per cent. "	.10	or	$\frac{1}{10}$	" "
25	per cent. "	.25	or	$\frac{1}{4}$	" "
50	per cent. "	.50	or	$\frac{1}{2}$	" "
100	per cent. "	1.00	or	1	time the quantity.
125	per cent. "	1.25	or	$1\frac{1}{4}$	times the quantity.
225	per cent. "	2.25	or	$2\frac{1}{4}$	" "
$\frac{1}{2}$	per cent. "	.005	or	$\frac{1}{200}$	of the quantity.
$\frac{3}{4}$	per cent. "	.0075	or	$\frac{3}{400}$	" "
$12\frac{1}{2}$	per cent. "	.125	or	$\frac{1}{8}$	" "
$16\frac{1}{4}$	per cent. "	.1625	or	$\frac{13}{80}$	" "

Express the following rates per cent. on the blackboard, both in decimals, and in common fractions of the lowest terms, in the same manner as in the above table:

1. 6 per cent.
2. 11 "
3. 12 "
4. 24 per cent.
5. 25 "
6. 47 "
7. 36 per cent.
8. 9 "
9. 50 "

10. 99 per cent.
11. 125 “
12. 112 “
13. $6\frac{1}{3}$ “
14. $5\frac{3}{4}$ “
15. $9\frac{5}{8}$ “
16. $6\frac{3}{7}$ “
17. 225 “
18. $4\frac{3}{4}$ “
19. $10\frac{2}{5}$ per cent.
20. $103\frac{3}{4}$ “
21. $607\frac{2}{4}$ “
22. $20\frac{2}{4}$ “
23. $100\frac{7}{8}$ “
24. $100\frac{7}{100}$ “
25. $\frac{1}{4}$ “
26. $\frac{3}{4}$ “
27. $\frac{5}{9}$ “
28. $2\frac{4}{5}$ per cent.
29. $\frac{3}{9}$ “
30. $\frac{6}{8}$ “
31. $\frac{6}{36}$ “
32. $400\frac{3}{4}$ “
33. $\frac{2}{90}$ “
34. $\frac{6}{17}$ “
35. $\frac{5}{11}$ “
36. $37\frac{1}{100}$ “

QUESTIONS.—Illustrate each of the axioms on the blackboard. (16.) What is notation? (17.) What is the difference between the Arabic and the Roman notation? (19.) (18.) How are numbers expressed in the Roman notation? (18., *a.*) What effect has repeating letters? (18., *b.*) What effect has a letter of less value, when written before one of greater value? (18., *c.*) What effect has a letter of less value, when written after a greater? (18., *d.*) What effect has a dash, when placed over a letter or combination of letters? (18., *e.*)

LESSON XXXVII.

PER CENTAGE AND ITS APPLICATIONS.

185. To find any per centage of a quantity.

An agent collected \$400 for a merchant and received $4\frac{1}{2}$ per cent. for his services; what amount did he receive?

MODEL OPERATION.

(*a.*)

\$400, am't collected.
$.04\frac{1}{2}$, rate per cent.

\$16.00
2.00

\$18.00 am't received.

(*b.*)*

$$4\tfrac{1}{2}\text{ per ct.} = \frac{4\frac{1}{2}}{100}, = \frac{9}{200}\text{ of }\$1., = \$\frac{9}{200}$$

$$\$\tfrac{9}{200} \times 400 = \$18.,\text{ Ans.}$$

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*NOTE.—Those teachers who prefer can use operation (*b.*), but they should require the pupil to give a lucid analysis.

Analysis.—(*a.*) If he received \$0.04½ for collecting \$1, he must have received for collecting \$400, 400 times \$0.04½, which is \$18.

Therefore, the agent received \$18.

Note.—Require the pupil to write a rule from the analysis.

357. What is 2 per cent. of \$467?
358. What is 5 per cent. of \$834.72?
359. What is 7 per cent. of 463 gallons?
360. What is 13 per cent. of 34.71 lbs.?
361. What is 3⅕ per cent. of 67.31 cwt.?
362. What is 4½ per cent. of 83£.?
363. What is $\frac{7}{12}$ per cent. of 43.46£.?
364. What is $\frac{4}{12}$ per cent. of 12£. 8s.?
365. What is 3⅘ per cent. of 3 cwt. 16 lbs.?
366. What is 12 per cent. of 15 gal. 3 qts. 2 pts.?

Questions.—What is the difference between the simple and the local value of a figure? (20.) (21.) What is the use of the cipher? (22.) What is numeration? (23.) What is reading numbers? (24.) Repeat the numeration table from right to left. (25.) From left to right. (25.) What must be done with vacant places? (26.) What must be done with vacant periods? (27.) What is the sum of two or more numbers? (30.)

LESSON XXXVIII.

367. What is 33⅓ per cent. of 846 gallons?
368. What is 14 per cent. of 450 sheep?
369. What is 25 per cent. of ⅞?

MODEL OPERATION.

$$25 \text{ per cent.} = \tfrac{25}{100}, = \tfrac{1}{4} \times \tfrac{7}{8}, = \tfrac{7}{32}, \text{ Ans.}$$

370. What is 15 per cent. of ⅝?
371. What is 33⅓ per cent. of $\frac{12}{13}$?
372. Find 1⅔ per cent. of \$15.60.

373. A farmer having 760 sheep kept 25 per cent. of them; how many did he keep?

374. A farmer having 480 bushels of corn kept 35 per cent., and sold the remainder; how much did he sell?

375. A merchant had 1360 yds. of calico; he sold 14⅜ per cent. to one man, 18¾ per cent. to another; how many yards did he sell in all?

376. A speculator had 4789 barrels of flour; he sold 34 per cent. to one man, and 25 per cent. to another; how many barrels had he remaining?

377. A man has a capital of $24500; he invests 17 per cent. of it in bank stock, 15 per cent. in railroad stocks, and the remainder in bonds and mortgages; how much did he invest in bonds and mortgages?

378. A fruit merchant bought 1347 barrels of apples, and, upon opening them, he found 12½ per cent. spoiled; how many barrels did he lose?

QUESTIONS.—What is Addition? (31.) What is the sign of Addition? (32.) What is the sign of equality? (33.) What is the dollar sign? (34.) What is the order of solving problems? (35.) Analyze an example in Addition from the blackboard. (36.) What is the difference of two numbers? (38.) What is subtraction? (39.) What is the minuend? (40.) What is the subtrahend? (41.)

LESSON XXXIX.

379. A merchant having deposited $200.37 in a bank afterwards drew out 10½ per cent. of it; how much remained in the bank?

380. A young man had $1572.41, and lost 31¼ per cent. in gambling; how much had he left?

381. A merchant bought a cargo of salt for $1230, and paid 4½ per cent. for bringing it home; what was the whole cost of his salt?

382. What is the difference between 6 per cent. of $143.46, and 7 per cent. of the same?

383. What is the difference between 9 per cent. of $34.41, and 3¼ per cent. of $67.43?

384. I bought a house for $625.93, and sold it for 6 per cent. advance; what did I gain by the sale?

385. I bought a horse and carriage for $325.13, and sold them at 3½ per cent. advance; for how much did I sell them?

386. A man bought a farm for $3874, and sold it at a loss of 3¾ per cent.; how much did he lose?

387. A man bought a farm for $4173, and sold it at a loss of 4⅘ per cent.; how much did he receive for it?

388. How many gills in 3 per cent. of 437 gallons?

389. How many ounces in 11½ per cent. of 437 cwt.?

390. A merchant bought 46 gallons of wine; 8½ per cent. of it leaked out; how much is the remainder worth, at 6 cts. per gill?

QUESTIONS.—What is a remainder? (42.) What is the sign of Subtraction? Perform at the blackboard and analyze an example in Subtraction. (44.) Prove the same example. (44., *i.*) What is Multiplication? (46.) What is the multiplicand? (47.) What is the multiplier? (48.) What is the product? (49.) Which are called factors? (50.)

LESSON XL.

391. I sent to Paris 5000 bushels of wheat, which cost $1.12½ per bushel; 20½ per cent. of the wheat was thrown overboard in a storm, and the remainder was sold at $1.75 per bushel; did I make, or lose, by the operation, and how much?

392. Two men engaged in trade, each with $2760.14. One of them gained 27½ per cent. of his capital, and the

other gained 33 per cent.; how much more did one gain than the other?

393. A man bought $\frac{3}{7}$ of a certain mill, and afterwards sold 31½ per cent. of what he owned; what part of the whole mill did he still own?

394. A man owed $473.25; at one time he paid 35 per cent., at another time 25 per cent. of the remainder; how much did he still owe?

395. Mr. Jones owed $478.39. He paid 43 per cent. of it at one time; at another time he paid 25 per cent. of the remainder; at another time 12½ per cent. of what remained still unpaid; how much did he still owe?

396. A man bought a building lot in London for 37£. 17s. 8d., and sold it at 9¾ per cent. advance; how much did he receive for it?

397. A man bought a lot near London for 198£. 13s. 8d., and sold it at a loss, or discount, of 12½ per cent.; how much did he lose?

398. A man bought a house for 113£. 13s. 5d., and sold it at a discount of 8¾ per cent.; how much did he receive for it?

399. A man gave his two sons $5000 each; the elder added 12½ per cent. to his money during the first year; but the younger spent 12½ per cent. of his; what is the difference of their possessions at the end of the first year?

QUESTIONS.—What is the sign of Multiplication? (51.) Perform at the blackboard, and analyze an example in Multiplication. (53.) Prove the same example. (53., *n.*) What is Division? (54.) What is the dividend? (55.) What is the divisor? (56.) What is the quotient? (57.) What is the difference between Long Division and Short Division? (58.) (59.)

LESSON XLI.

186. To find what per cent. one number is of another.

A clerk having \$125 lost \$4; what per cent. of his money did he lose?

MODEL OPERATION.

Am't possess'd \$125) Am't lost \$4.

\$.03$\frac{1}{5}$ on one dollar or 3$\frac{1}{5}$ per cent.

Analysis.—If a clerk had \$125 and lost \$4, he must have lost on each dollar $\frac{1}{125}$ of \$4, which equals 3$\frac{1}{5}$ *hundredths* of a dollar, or 3$\frac{1}{5}$ per cent.

Therefore, the clerk lost 3$\frac{1}{5}$ per cent. of \$125.

Note.—Require the pupil to write a rule from the analysis.

2. A man bought a horse for \$97, and sold it so as to gain \$13; what was his gain per cent.?

MODEL OPERATION.

\$97.)13.

\$.13$\frac{39}{97}$ on one dollar, or 13$\frac{39}{97}$ per cent.

Analysis.—If a man gained \$13 on \$97, on *one* dollar he must have gained $\frac{1}{97}$ of \$13, which is \$0.13$\frac{39}{97}$, or 13$\frac{39}{97}$ per cent.

Therefore, the man gained 13$\frac{39}{97}$ per cent.

3. A gain of \$90 on \$450 is a gain of what part of a dollar on one dollar, or, \$90 is what per cent. of \$450?

Analysis.—A gain of \$90 on \$450 is a gain of $\frac{1}{450}$ of \$90 on *one* dollar, which is .20 of a dollar, or 20 per cent.

Therefore, \$90 is 20 per cent. of \$450.

Note.—Require the pupil to write a rule from the analysis.

400. A gain of \$175 on \$1400 is a gain of what per cent.?

401. $13.20 is what per cent. of $240?

402. $1.64 is what per cent. of $82?

403. 148 cts. is what per cent. of $37?

404. 6 lbs. loss on 72 lbs. is what part of a pound loss on *one* pound, or, 6 lbs. is what per cent. of 72 lbs.?

QUESTIONS.—What are the signs of Division? (62.) Give an analysis of an example in Short Division, from the blackboard. (64.) Give an analysis of an example in Long Division, from the blackboard (65.)

LESSON. XLII.

405. 25 cts. is what per cent. of 100 cts.?

406. 25 cts. is what per cent. of $100?

407. 9 gallons is what per cent. of 108 gallons?

408. 6 bushels 1 peck is what per cent. of 42 bushels 3 pecks?

409. What per cent. of 15 pounds is 5 pounds 10 ounces avoirdupois?

410. 8 pounds is what per cent. of 16 pounds?

411. 28 pounds is what per cent. of 28 pounds?

412. 48 pounds is what per cent. of 23 pounds?

ANALYSIS.—48 lbs. gain on 23 lbs. is $\frac{1}{23}$ of 48 lbs. gain on *one* pound, which is $208\frac{16}{23}$ per cent.

Therefore, 48 lbs. is $208\frac{16}{23}$ per cent. of 23 lbs.

413. 40 head of cattle are what per cent. of 250 head of cattle?

414. From a hogshead of sugar containing 864 lbs. there were sold 64 lbs., 317 lbs., and 347 lbs., what per cent. of the whole was sold?

415. A man has $550, and purchases goods to the amount of $38.62; what per cent. of his money does he expend?

416. A man having \$1864, spends 20 per cent. for a horse and carriage; he then buys a house for \$800; the value of his house is what per cent. of the money remaining after purchasing the horse and carriage?

417. The population of the village of Butternuts at a certain period was 1000; the increase in 5 years was 500: what was the per cent. of increase during the interval?

418. A man purchased 80 acres of land at \$45.50 per acre, and afterwards sold it for \$5500.25; what was his gain per cent. on the purchase money?

LESSON XLIII.

187. To find a number when a certain per cent. of it is given.

A clerk lost \$5, which was 4 per cent. of the money he had at first; how much had he?

MODEL OPERATION.

$$\$5 \div 4 = \$\tfrac{5}{4};\ \$\tfrac{5}{4} \times 100 = \$125.$$

ANALYSIS.—If \$5 is $\frac{4}{100}$, or 4 per cent., of all his money, $\frac{1}{100}$, or 1 per cent. of it must be $\frac{1}{4}$ of \$5, which is \$$\frac{5}{4}$, and $\frac{100}{100}$, or 100 per cent. of it is 100 times \$$\frac{5}{4}$, which equals \$125.

Therefore, the man must have had \$125.

NOTES.—1. Require the pupil to write a rule from the analysis.

NOTE.—2. For convenience, the multiplication may be made first by annexing two ciphers, as in operation (*b*.)

16 is 8 per cent. of what number?

MODEL OPERATIONS.

$$(a.)\ 16 \div 8 = 2;\ 2 \times 100 = 200.$$

$$(b.)\ 16 \times 100 = 1600;\ 1600 \div 8 = 200.$$

ANALYSIS.—(*a*.) If 16 is 8 per cent. of a certain number, 1 per cent. of it must be $\frac{1}{8}$ of 16, which is 2, and 100 per cent. of it is 100 times 2, which is 200.

Therefore, 16 is 8 per cent. of 200.

419. 33 is 2 per cent. of what number?

420. \$413.62 is $26\frac{2}{3}$ per cent. of what sum of money?

421. A farmer sold 500 sheep, which were 50 per cent. of his whole flock; how many had he at first?

422. A farmer sold 376 bushels of oats, which were 100 per cent. of what he had at first; how many had he at first?

423. A merchant had 947 yards of calico, which were 200 per cent. of what he sold; how many yards did he sell?

424. I loaned a man a certain sum of money. He paid me \$34.64, which was $13\frac{5}{9}$ per cent. of what I loaned him. How much did I loan him?

QUESTIONS.—What is United States money? (68.) Which are the gold coins? (69., *a.*) Which the silver coins? (69., *b.*) Which the nickel coins? (69., *c.*) What is the use of the decimal point? (70.) Repeat the rule for the reduction of United States money. (71., *c.*) Analyze an example in the addition of U. S. money. (72., *a.*) In the subtraction of U. S. money. (75., *a.*)

LESSON XLIV.

425. A man owning 25 per cent. of a cotton factory sold $12\frac{3}{5}$ per cent. of his share for \$1346.46; at that rate what is the value of the whole factory?

426. A merchant pays \$85 a month for rent, which is 18 per cent. of his entire profits for one year; how great are his profits for one year, after deducting the rent?

427. A merchant pays \$75 per month to his clerk, \$46 per month for rent, and \$63.25 per month for other expenses. His entire expenses are 50 per cent. of his entire profits for one year; how much are his profits for one year, after deducting his expenses?

428. The number of soldiers on the sick list of an army is 6400, which is $33\frac{1}{3}$ per cent. of the entire army; how many effective men are there in the army?

429. In a mixture of brandy and water there are 25 gallons of water, which is 75 per cent. of the whole; what is the number of gallons in the mixture?

430. Jones and Smith are in partnership. Jones received $423.42, which is $13\frac{5}{8}$ per cent. of the whole profit. What is Smith's share?

431. There were killed and wounded in a battle 1860 men, who were 10 per cent. of the armies engaged; how many men did the armies contain before the battle?

432. A dairyman agreed to make some cheese for one cent per pound; what per cent. did he charge for manufacturing, allowing that the cheese is worth 12 cts. per pound?

433. A man bought a yoke of oxen for $75.38, and sold them for $79.46; what per cent. did he gain?

QUESTIONS.—Analyze an example in the multiplication of U. S. money. (74., *a.*) In the division of U. S. money? (75., *a.*) What is a bill of parcels? (76.) What is the difference between simple and compound denominate numbers? (77.) What is the reduction of compound numbers? (79.) What is reduction descending? (80.) What is reduction ascending? (81.) Repeat the table of English money? (82., *b.*)

LESSON XLV.

COMMISSION, BROKERAGE, AND STOCKS.

188. An **Agent, Factor,** or **Broker,** is a person transacting business for another.

189. Commission is the per centage paid an agent, or factor, for buying or selling goods, making collections, or transacting other business.

190. Brokerage is the per centage paid to a broker, or dealer in money, stocks, bills of exchange, &c.

191. A **Firm** is an association of individuals for the purpose of transacting business.

192. A **Charter** is a legal act of incorporation, defining the powers and obligations of an incorporated body.

193. A **Corporation** is a number of persons authorized by a special charter to transact business.

194. Capital, or **Stock,** is the property of an individual, a corporation, or a firm, used to carry on the business of the individual, corporation, &c.

195. A **Share** is one of the equal parts into which the capital stock of a company or corporation is divided.

196. Stockholders are the individuals owning the shares.

197. Stock is said to be at *par* when it sells at its nominal value.

198. Stock is said to be *above par*, or at a premium, or in advance, when it sells for more than its nominal value.

199. Stock is said to be *below par*, or at a discount, when it sells for less than its nominal value.

200. The **Market Value** of stock is the price it will bring when sold.

201. A **Dividend** is the per centage paid to the stockholders from the profits of the company.

(*a.*) NOTES.—1. Premium, discount, dividends, and assessments, are computed at a certain per cent on the nominal value of the stock.

2. United States Stocks, or State Stocks, are the bonds, or "promises to pay," of the United States, or of a State, bearing a fixed interest.

3 The nominal, or par value, of a share varies in different companies, but is usually $100.

4. When the person transacting the commission business lives in a foreign country, he is frequently called a *correspondent*, or *consignee*.

5. The goods shipped or forwarded to be sold on commission are termed a *consignment*, and the person sending the same is called the *consignor*.

QUESTIONS.—Name the gold coins of English money. (82., *c.*) The silver. (82., *d.*) Copper. (82., *e.*) What is weight? (83.) What is the use of Troy weight? (84.) What is a carat? (84., note.) Repeat the table of Troy weight. (84., *a.*) Repeat the table of avoirdupois weight. (85., *a.*) What is the use of avoirdupois weight? (84.) Repeat the table of the Long, or Gross ton. (86.) Repeat the table of miscellaneous weights. (87.)

LESSON XLVI.

202. To find the commission, or brokerage on any sum.

A commission merchant sells $2323.24 worth of powder; what is his commission at 5¾ per cent.?

MODEL OPERATION.

$$\$2323.24 \times .05\tfrac{3}{4} = \$133.586+, \text{ Ans.}$$

ANALYSIS.—1. 5¾ per cent. is equal to 5¾ *hundredths* of a dollar on one dollar.

2. If the commission on *one* dollar is $0.05¾, on $2323.24 it is 2323.24 times $0.05¾, which is $133.586+.

Therefore, his commission is $133.586+.

NOTE.—Require the pupil to write a rule from the analysis.

434. A commission merchant sells goods to the amount of $9134.26; what is his commission at 2⅝ per cent.?

435. An agent in London purchased 1876 yards of English broadcloth at $1.75 per yard; what was his commission at 1¾ per cent.?

436. I request my correspondent in London to purchase 4374 yards of sheeting at 7 pence per yard; what will his commission amount to at 2¾ per cent.?

437 A broker in New York exchanged $43786.21 on the Citizens' Bank, New Orleans, at 4¼ per cent.; how much brokerage did he receive?

438. My agent in Chicago purchases 4786 bushels of

wheat, and is to receive $\frac{1}{2}$ per cent. of the wheat for his commission; how many bushels will he be entitled to?

439. An auctioneer sold a house for \$3244.40, and the furniture for \$3717.60; what did his fees amount to at $2\frac{3}{4}$ per cent.?

440. A man makes 2834 pounds of cheese, and is to receive $1\frac{5}{8}$ per cent. of the cheese for the trouble of making; how much will the remainder bring at $9\frac{5}{8}$ cts. per pound?

441. An agent sells goods to the amount of 450£. 11s. 6d.; what is his commission at 6 per cent.?

QUESTIONS.—What is the use of apothecaries' weight? (88.) Repeat the table.) (88., *a.*) Repeat the table of comparative weight. (88., *b.*) How many dimensions has a line? (89., *a.*) How many has a surface? (89., *b.*) How many a solid? (39., *c.*) What is the use of Long measure? (90.) Repeat the table. (90., *a.*)

LESSON XLVII.

203. To find the commission, when it is to be deducted from the remittance, and the balance invested.

I sent my agent in Paris \$4634.68 with which to purchase goods after deducting his commission of $2\frac{1}{4}$ per cent.; how much commission did he receive?

MODEL OPERATION.

1. $\$4634.68 \div \$1.02\frac{1}{4} = \$4532.69.$

2. $\$4634.68 - \$4532.69 = \$101.99$, Ans

ANALYSIS.—1. Find the amount invested.

Since the commission is $2\frac{1}{4}$ per cent., my agent must receive $\$1.02\frac{1}{4}$ for every dollar he expends; and he can expend as many dollars as $\$1.02\frac{1}{4}$ is contained times in \$4634.68, which are 4532.69.

2. Find the amount of commission.

If I send my agent \$4634.68, and he expends \$4532.69, his commission must amount to the difference between $4634.68 and $4582.69, which is $101.99.

Therefore, he received $101.99.

Note.—Require the pupil to write a rule from the analysis.

442. A man sends to his agent in Chicago $8364.87 with which to purchase wheat after deducting his commission of 1¾ per cent.; how many dollars worth of wheat did he purchase?

443. What amount of stock can be bought for $9764.50, allowing 3 per cent. brokerage?

444. An agent receives a remittance of $6783.50 with which to purchase shoes at a commission of 2 per cent.; how much money should he expend for shoes, and what will be the amount of his commission?

Questions.—Repeat the table of miscellaneous long-measure. (90., *b.*) What is Gunter's chain? (91.) Repeat the table. (91., *a.*) What is a square? (92.) Give the analysis of the square yard. (92., *a.*) What is the use of square measure? (92., *c.*) How do artificers estimate their work? (92., *d.*) Repeat the table of square measure. (92., *f.*)

LESSON XLVIII.

204. To find the market value of stock, when at a premium or discount.

(*a.*) What will railroad stock to the amount of $347.13 cost, at 12 per cent. advance?

(*b.*) What will be the cost of railroad stock amounting to $34.713, at 12 per cent. discount?

MODEL OPERATION.

(*a.*) $\$347.13 \times \$1.12 = \$388.785+$.

(*b.*) $\$347.13 \times \$0.88 = \$305.474+$.

Analysis.—(*a.*) Since $1. worth of stock at 12 per cent. premium costs one dollar *plus* 12 *hundredths* of a dollar ($1.12), $347.13 will cost 347.13 times $1.12, which is $388.785+.

(*b.*) Since $1. worth of stock at 12 per cent. discount costs one dollar *minus* 12 *hundredths* of a dollar, ($1.—$0.12=$0.88), $347.13 will cost 347.13 times $0.88, which is $305.474+.

RULE.—I. *When stock is at a premium, add the per cent. to* $1., *and multiply the sum by the amount of stock.*

II. *When stock is at a discount, subtract the per cent. from* $1., *and multiply the difference by the amount of the stock.*

445. What is the market value of 36 shares of New York and Erie R. R. stock at 37⅝ per cent. discount?

446. What must be paid for 15 shares of Providence railroad stock at a premium of 9¾ per cent.?

447. What must be paid for 13 shares of U. S. stock at 2⅝ per cent. premium, the par value being $1000. per share?

448. I purchased 37 shares of Hudson River R. R. stock at 15 per cent. discount, and sold them at an advance of 13⅞ per cent.; how much did I gain?

QUESTIONS.—What is the use of surveyor's square-measure? (93.) Repeat the table. (93., *a.*) What is a cube? (94.) Give the analysis of a cubic yard. (94., *b.*) What is the use of cubic measure? (94., *c.*) Repeat the table. (136., *a.*) What are measures of capacity? (95.) What is the use of Liquid measure? (95., *a.*) Repeat the table. (95., *b.*) Repeat the table of miscellaneous measure. (95., *c.*)

LESSON XLIX.

205. To find the quantity of stock that may be purchased for a given sum.

How many shares of railroad stock may be purchased for $3885. at 5 per cent. advance?

MODEL OPERATION.

1. \$3885.÷\$1.05=\$3700.
2. \$3700÷\$100=37 shares.

ANALYSIS.—1. Since \$1 stock at 5 per cent. advance is worth \$1.05, as many dollars' worth of stock can be purchased for \$3885. as \$1.05 is contained times in \$3885. which are 3700.

2. If \$100 will buy one share, \$3700 will buy as many shares as \$100 are contained times in \$3700, which are 37.

Therefore, at 5 per cent. advance, 37 shares of R. R. stock can be purchased for \$3885.

NOTE.—Require the pupil to write a rule from the analysis.

449. How many shares of bank stock, at 5 per cent. advance, can be purchased for \$6300?

450. I invested \$4678.73 in the Home Insurance Company, at 11¾ per cent. discount; how much stock did I purchase?

451. I sent my agent \$8964.39, with which to purchase Illinois Railroad stock after deducting his commission of 2¼ per cent., the stock being at a premium of 13⅝ per cent. How much stock did he purchase?

452. I sold 50 shares of the Providence Railroad stock at a discount of 8 per cent., and received \$4600.; what is the par value of a share?

QUESTIONS.—What is the use of dry-measure? (96.) Repeat the table. (96., *a.*) Repeat the table of comparative measures. (96., *c.*) What is the measure of time? (97.) Repeat the table. (97., *a.*) How is the calendar year divided? (97., *b.*) What is circular measure? (98.) Repeat the table. (98., *a.*) Repeat the table of particulars. (99., *a.*) Of paper. (99., *b.*) Of books. (99., *c.*) Of copying. (99., *d.*)

LESSON L.

MISCELLANEOUS EXAMPLES.

453. An agent buys 26896 pounds of wool at 67 cts. per pound, and receives a commission of 2⅜ per cent.; what amount does he receive?

454. My agent purchases, in New Orleans, 400 bales of cotton, each bale weighing 570 lbs., for which he gives 9 cts. per pound; how much must I remit to him, including his commission of 2½ per cent.?

455 How much must I send a broker to purchase 38 shares of Hudson River R. R. stock at 3½ per cent. discount, including the brokerage of ¼ per cent.?

456. A broker purchased for me 128 shares of Virginia State Bonds, at 46¾ per cent. discount. He sold them, on my account, at an advance of ⅝ per cent., allowing ¼ per cent. brokerage for buying and selling; did I make or lose by the operation, and how much?

457 A commission merchant in New York sells 437 barrels of flour, at $7.50 per barrel; 43 firkins of butter, each containing 97 pounds, at 23½ cts. per pound; 136 cheeses, each weighing 103 pounds, at 8¾ cts. per pound: how much is his commission for selling, at 5 per cent.?

458. A man purchased 73 shares of stock in the Newark City Bank, at 7 per cent. discount, and sold it for 3½ per cent. premium; what was his gain?

QUESTIONS.—What is reduction descending? (80.) Analyze at the blackboard an example in reduction descending. (100., *a.*) Give the rule. (100., *b*) What is reduction ascending? (81.) Analyze (at the blackboard) an example of reduction ascending. (101., *a.*) Give the rule. (101, *b*) Analyze an example of compound addition, at the blackboard. (102., *a.*) Give the rule. (102, *b.*) Prove the same example. (102., *i.*)

LESSON LI.

459. Fred. Curtis exchanged 13 shares railroad stock at 11 per cent. premium, for 26 shares of bank stock at 8½ per cent. discount. He paid the difference in cash; how much cash did he pay?

460. S. J. Brundige sends $6384. to a broker to be invested in government funds after deducting his commission of 2 per cent.; how much was invested?

461. I sent to J. H. Johnston $600 with which to purchase a silver tea set after deducting his commission of 2½ per cent.; how much did it cost?

462. An agent sold a quantity of oil for $375.85, and charged 2¼ per cent. commission; how much did the owner receive?

463. I sent my agent $375.75 with which to purchase oil at $1. per gallon, after deducting his commission of 2¼ per cent.; how much oil did I receive?

464. A widow invested $3968 in Beverly Bank stock at par. She afterward sold it at 98 per cent. discount; how much did she lose?

465. A man bought $3784.40 worth of cotton, and afterward sold it at a gain of 50 per cent.; what did he receive for it?

466. I sold 6 pieces of cloth, each containing 31¼ yards, at $2.87½ per yard; I charged 2½ per cent. commission, and 3 per cent. for guaranteeing payment; how much should I pay over?

467. I bought 13 shares of bank stock at 4¾ per cent. premium; how much did it cost me?

QUESTIONS.—Analyze an example of the subtraction of denominate numbers. (103., *a*.) Prove the same example. (103., *h*.) Give the rule. (103., *g.*) Analyze an example of the multiplication of denominate numbers. (104., *a.*) Analyze an example of the division of denominate numbers. (105., *a.*)

LESSON LII.

PROFIT AND LOSS.

206. Profit and **Loss** are commercial terms indicating gain or loss in business transactions, and are always estimated from the cost price.

207. To find the amount of gain or loss, the cost and the gain or loss per cent. being given.

A man bought a cow for $39.25, and afterward sold her for 15 per cent. more than her cost; how much did he gain?

MODEL OPERATION.

$39.25×$0.15=$5.887+.

ANALYSIS.—Since on $1. he gained $0.15, on $39.25 he must have gained 39.25 times $0.15, which are $5.887+.

Therefore, the man must have gained $5.887+.

NOTE.—Require the pupil to write a rule from the analysis.

468. A grocer bought a hogshead of sugar for $96.37, and sold it at 15 per cent. profit; what was his gain?

469. A miller bought 346 bushels of rye at 87½ cts. per bushel. He sold the flour made from it for 29 per cent. more than the cost; what was his gain?

470. A wool merchant bought 43874 pounds of wool at 57 cts. per pound; he paid 1½ per cent. for storage and insurance, and then sold it at an advance of 33⅓ per cent.; what was his gain?

471. If I buy a hogshead of sugar containing 1280 pounds, at 6¼ cts. per pound, and wish to sell it at an advance of 18 per cent., for how much must I sell it per pound?

472. I purchased charts at \$3.42 apiece, and wish to sell them so as to gain 25 per cent.; for how much must I sell them apiece?

QUESTIONS.—What is an integer? (106.) What is the difference between odd and even numbers? (107.) (108.) What is a prime number? (109.) What is a composite number? (110.) When are numbers said to be prime to each other? (111.) What is the difference between a prime and a composite factor? (112.) (113.) Is unity ever considered a factor? (113., *a*.)

LESSON LIII.

208. To find the gain or loss per cent., the cost and the selling price being given.

I bought sugar for 9 cts., and sold it for $10\frac{1}{2}$ cts.; what per cent. did I gain?

MODEL OPERATION.

1. $\$0.10\frac{1}{2} - \$0.09 = \$0.01\frac{1}{2}$.
2. $\$0.01\frac{1}{2} \div 9 = \$0.0016\frac{2}{3}$.
3. $\$0.0016\frac{2}{3} \times 100 = \$0.16\frac{2}{3} = 16\frac{2}{3}$ per cent.

ANALYSIS.—1. Find the amount gained.

If I bought sugar for 9 cents, and sold it for $10\frac{1}{2}$ cts., I must have gained the difference between 9 cts. and $10\frac{1}{2}$ cts., which is $1\frac{1}{2}$ cts.

2. [illegible]d the gain on 1 hundredth of a dollar.

If the gain on 9 *hundredths* of a dollar is $1\frac{1}{2}$ *hundredths* of a dollar, the gain on *one* hundredth of a dollar is 1 ninth of $1\frac{1}{2}$ hundredths of a dollar, which is $16\frac{2}{3}$ ten thousandths of a dollar.

3. Find the gain on one dollar, or the gain per cent.

If the gain on 1 hundredth of a dollar is $\$0.0016\frac{2}{3}$, the gain on one dollar is one hundred times $\$0.0016\frac{2}{3}$, which is $\$0.16\frac{2}{3}$ $=16\frac{2}{3}$ per cent.

Therefore, if I buy sugar for 9 cts., and sell it for $10\frac{1}{2}$ cts., I will gain $16\frac{2}{3}$ per cent.

NOTE.—Require the pupil to write a rule from the analysis.

473. I bought wool at 60 cts., and sold it at 70 cts. per pound; what was the gain per cent.?

474. A man bought a horse for $343; he paid $14.50 for keeping it one month, and then sold it for $364; what did he gain per cent.?

475. A man bought a span of horses for $187; it cost him $50.87 for keeping them one year; he received for their work during the year $102.13; he then sold them for $150; did he make, or lose, and what per cent.?

476. I bought a yoke of oxen for $78; their keeping cost me $20.38; I worked them 110 days at a value of 50 cts. per day; I then sold them for $50: did I make, or lose, and what per cent.?

QUESTIONS.—Illustrate each of the facts having relation to composite numbers, and draw from each the inference. (113., *b.*) Give the analysis of a composite number. (114.) Give the rule. (114., *d.*) Give the proof. (114., *e.*) Illustrate on the blackboard the method of multiplying by factors of composite numbers. (115.) By dividing by the factors of a composite number, and finding the true remainder. (115., *a.*)

LESSON LIV.

209. To find the selling price, the cost, and gain or loss per cent. being given.

A farmer bought a cow for $33.22; for how much must he sell her to gain 20 per cent.?

MODEL OPERATION.

$$\$33.22 \times 1.20 = \$39.864.$$

ANALYSIS.—Since the farmer to gain 20 per cent. must sell $1 of the cost for $1.20, $33.22 must be sold for 33.22 times $1.20, which are $39.864.

Therefore, if a farmer bought a cow for $33.22, to gain 20 per cent. he must sell her for $39.864.

NOTE.—Require the pupil to write a rule from the analysis.

477. If 12¾ cwt. of sugar cost $139.47, for how much must it be sold to gain 23 per cent.?

478. If I buy 1478 pounds of butter for $448, for how much must I sell it per pound to gain 18¾ per cent.?

479. For how much must I sell coffee which cost me 13 cts. so as to lose 11 per cent.?

480. A man bought goods to the amount of $3874, but, being desirous of raising money, sold them at a sacrifice of 12 per cent.; what did he receive for his goods?

481. A man bought a farm for $1347; he paid $374.20 for draining, $78.83 for fencing, and then sold it for 12½ per cent. advance upon the whole cost; what was his gain?

482. A man purchased 13 casks of rice, each containing 624 pounds, at 6¼ cts. per pound. He sold it at an advance of 28 per cent.; what was the selling price?

483. Bought 8 boxes of tea, each box containing 78 pounds, at 58 cts. per pound; 10 per cent. of the tea being spoiled, for how much must he sell the remainder per pound, to gain 20 per cent. on the whole cost?

QUESTIONS.—What is the use of the vinculum and of the parenthesis? (116.) What is cancellation? (117.) What is a common divisor? (118.) What is the greatest common divisor? (119.) What is a multiple of a number? (120.) What is a common multiple of two or more numbers? (121.) What is the least common multiple of two or more numbers? (122.)

LESSON LV.

210. To find the cost, the selling price and the gain or loss per cent. being given.

A grocer sold sugar for 13⅓ cts. per pound, and thereby lost 8 per cent.; what did it cost?

MODEL OPERATION.

1. \$1—\$0.08=\$0.92.
2. \$0.13⅓÷\$0.92=\$0.144+.

ANALYSIS.—1. If a grocer lost \$0.08 on \$1, he must have received for \$1 the difference between \$1 and \$0.08, which is \$0.92.

2. If the sugar sold for \$0.92 cost \$1, one pound or the quantity which he sold for \$0.13⅓, cents must have cost as many dollars as \$0.92 is contained times in \$0.13⅓, which are \$0.144+.

Therefore, the sugar must have cost him \$0.144+ per lb.

NOTE.—Require the pupil to write a rule from the analysis.

484. By selling rice at 8 cts. per pound a grocer lost 25 per cent.; how much did it cost him?

485. A merchant sold flour for \$7.87½ per barrel, and by so doing gained 23 per cent.; what did it cost him?

486. A grocer by selling sugar for \$1.25, gained 37½ per cent.; what did it cost him?

487. I sold 9⅔ cwt. of sugar at \$11.50 per cwt., thereby losing 12½ per cent.; what was the cost of my sugar?

488. A farmer sold 27 barrels of pears for \$69.75, and made 25 per cent.; how much did they cost per barrel?

489. Sold 268 barrels of flour for \$1881.50, which was 20¾ per cent. more than it cost; how much did it cost per barrel?

490. I bought 36 shares of N. Y. & E. R. R. stock at 17 per cent. discount; I sold it for 27 per cent. in advance of its cost; how much did I receive for it?

491. A broker bought for me 36 shares of the Harlem Railroad stock at a premium of 2½ per cent., and sold it at a discount of 5⅝ per cent.; how much did I lose, the brokerage being 2¼ per cent. for each transaction?

QUESTIONS.—What is a fraction? (123.) What is a common fraction? (124.) What is a decimal fraction? (125.) Are decimal fractions ever written and used in the same way as common fractions? (125.*) What are the terms of a fraction? (126.) What is the use of the denominator? (127.)

LESSON LVI.

MISCELLANEOUS EXAMPLES.

492. A man has a capital of $38746; he invests 13 per cent. of it in railroad stock; 14 per cent. in a manufacturing company; and the remainder in bank stock; how much bank stock does he purchase?

493. Two men, A and B, went into business with equal capital; A gained 25 per cent. of his capital; B lost 25 per cent. of his capital; B had left after his loss $3876; how much had A after his gain?

494. A man owning $\frac{3}{5}$ of a cotton factory sold 63 per cent. of his share; what part of the whole did he still own?

495. A man having 437 A. of land sold at one time $\frac{1}{4}$ of it, and at another time $\frac{1}{3}$ of the remainder; what per cent. remained unsold?

496. A vessel started from New York with 6783 bushels of wheat. In a storm 2894 bushels were thrown overboard. What per cent. was lost?

497. A man bought a quantity of cotton; 347 bales, which were $23\frac{1}{2}$ per cent. of it, were burned by a party of soldiers; how much did he buy?

498. A commission merchant sold a quantity of iron for $38740, gaining $31\frac{1}{4}$ per cent.; allowing the merchant 5 per cent. commission on the selling price, what was the net gain?

499. If a hogshead of vinegar cost $5.30, for what must

it be sold per pint that there may be a gain of 10 cts. per gallon?

500. A farm cost $4750.37; for what must it be sold that the owner may realize a profit of $673, after paying his agent $39?

QUESTIONS.—What is the use of the numerator of a fraction? (128.) What is the difference between a proper and an improper fraction? (129.) (130.) What is a mixed number? (131.) What is a simple fraction? (132.) What are complex fractions? (133.) What is a compound fraction? (134.) Of what may fractions be considered as indications? (135.)

LESSON LVII.

501. What amount of Government stocks at 3½ per cent. premium, can I purchase for $8741?

502. I send my agent $4374 with which to purchase R. R. stock at 5½ per cent. discount; how much can be purchased after deducting his commission of 2 per cent.?

503. From a cask of wine containing 270 gallons, 30 gal. 2 qts. are drawn; what per cent. is this?

504. A man purchased a farm of 37 A. for $3000; he sold 29 A. 36 sq. rds. for $2300; what per cent. of his farm did he have remaining?

505. A man owes $4738.03, and his property is worth only $2783.14; what per cent. can he pay of his debts?

506. I used a carriage six months, and then sold it for $100, which was 18 per cent. below cost; what would I have received had I sold it for 12 per cent. above cost?

507. A sells a yoke of oxen to B, and gains 10 per cent.; B sells them to C for $85, thereby gaining 12½ per cent.; how much did the oxen cost A?

508. A grocer purchased eight hogsheads of molasses

for $50 per hogshead. On four he gained 25 per cent., and on the others he lost 25 per cent.; how much did he lose or gain on the whole?

509. A grocer sold six barrels of sugar at $20 per barrel. On two of them he gained 30 per cent.; on the other four he lost 30 per cent.; what per cent. did he lose on the whole?

QUESTIONS.—To what is the numerator equivalent? (136.) To what the denominator? (136.) How is the value of a fraction affected by multiplying both numerator and denominator by the same number? (137., *a.*) How, by dividing both numerator and denominator by the same number? (137., *b.*)

LESSON LVIII.

INSURANCE.*

211. **Insurance** is an obligation by which individuals or companies bind themselves for stipulated sums to indemnify owners of property for loss by fire, shipwreck, or any other casualty.

212. The **Insurer,** or **Underwriter,** is the party who takes the risk of insurance.

213. The **Insured** is the party protected by the insurance.

214. The **Policy** is a written contract made by the parties.

215. The **Premium** is the amount paid by the owner of the property to those who insure it and is estimated at a certain rate per cent. on the amount insured.

216. To find the premium, the rate of insurance and the amount insured being given.

What is the premium on a policy of $3784 at 4 per cent.?

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*NOTE.—When property is only partially injured insurance companies usually pay the actual loss not exceeding the amount insured. When insured in different companies, the actual loss is apportioned between them in proportion to the amount insured by each.

MODEL OPERATION.

$3784.×$0.04=$151.36.

Analysis.—If the premium on $1 is ($0.04) 4 *hundredths* of a dollar, on $3784 it is 3784 times 4 *hundredths* of a dollar, which is 15136 hundredths of a dollar, equal to $151.36.

Therefore, at 4 per cent., the premium on $3784 is $151.36.

Note.—Require the pupil to write a rule from the analysis.

510. What must I pay annually for insuring my house to the amount of $3874.46 at 2½ per cent. premium?

511. A vessel and cargo valued at $29347.46 are insured at 3⅔ per cent.; what is the premium?

512. What must I pay for insuring a cargo of wheat valued at $15378, at a premium of 2½ per cent.?

513. My house and furniture are valued at $4378.50. I have them insured at $3⅘ per cent. If they are destroyed by fire, what will be the actual loss to the company?

514. A cotton factory and its contents valued at $38746 is insured at 3½ per cent. on ⅔ its value; what is the premium?

515. A merchant wishes to insure a vessel and a cargo of corn from New York to Liverpool, both valued at $20374; what will be the premium on ¾ its value at ¾ per cent.?

516. I purchased 4964 bales of cotton, each bale containing 641 pounds, at 13 cts. per pound. I effected an insurance at 3½ per cent. on the selling price, 18 cts. per pound. The vessel containing it foundered at sea; did I make, or lose?

517. My house and furniture, valued at $4374, are damaged to the amount of 25 per cent.; and there is an insur-

ance of 75 per cent. on the loss; how much shall I receive?

518. A merchant owns $\frac{3}{4}$ of a ship valued at $39748, and insures his interest at $3\frac{1}{3}$ per cent.; what does his policy cost him?

QUESTIONS.—How is the value of the fraction affected by multiplying the denominator alone? (137., *c.*) How by dividing the denominator only? (137., *d.*) How by multiplying the numerator only? (137., *e.*) How by dividing the numerator only? (137., *f.*)

LESSON LIX.

TAXES.

217. A **Tax** is a sum of money assessed for public purposes by government, on the person or property of an individual.

218. Taxes are commonly assessed at a certain per cent. on the estimated value of property.

219. A **Poll Tax** is apportioned equally among the male citizens liable to assessment, and is estimated at a certain amount per poll.

220. **Real Estate** consists of property that is not movable, as lands, houses, &c.

221. **Personal Property** consists of property that is movable, as money, furniture, tools, &c.

222. An **Inventory** is a list of articles with their value.

223. Before assessing taxes, make a complete inventory of all the taxable property in the township, together with a full list of taxable polls, if there is to be a poll-tax assessed.

In a certain town $16377.70 are to be raised by tax; the valuation of the taxable property as shown by the assessment roll is $800000, and there are 225 polls to be assessed 90 cts. each; what will be Mr. Jones's tax whose property is valued at $3426, and who pays for 3 polls?

MODEL OPERATION.

1. $0.90×225=$202.50, am't raised by poll-tax.
2. $16377.70—$202.50=$16175.20, am't to be assessed on the property.
3. $16175.20÷800000=$0.020219 on $1, or $2\frac{219}{10000}$ per cent.
4. $0.020219×3426=$69.37, am't of Mr. Jones's tax on property.
5. $0.90×3=$2.70, am't of Mr. Jones's poll-tax.
6. $69.27+$2.70=$71.97, am't of Mr. Jones's entire tax.

Analysis.—1. Find the amount raised by poll-tax.

If 1 poll is taxed 90 cts., 225 polls will be taxed 225 times 90 cts., which are 20250 cts., equal to $202.50.

2. Find the amount to be raised on the property.

If $16377.70 are to raised by tax, and $202.50 are raised by poll-tax, the amount to be assessed on the property must be the difference of these quantities, which is $16175.20.

3. Find the amount to be assessed on $1.

If $16175.20 are to be assessed on $800000, on *one* dollar there is to be assessed *one eight hundred thousandth* of $16175.20, which is $0.020219, or $2\frac{219}{10000}$ per cent.

4. Find the amount of the tax on Mr. Jones's property.

If *one* dollar is taxed $0.020219, $3426 will be taxed 3426 times $0.020219, which are $69.27.

5. Find the amount of Mr. Jones's poll-tax.

If one poll is taxed 90 cts., 3 polls are taxed 3 times 90 cts., which are 270 cts., equal to $2.70.

6. Find the entire amount of Mr. Jones's tax.

If the property of Mr. Jones is taxed $69.27, and his poll-tax amounts to $2.70, his entire tax will amount to the sum of those quantities, which is $71.97.

(*a*) NOTE.—Having found the tax on $1, or the per cent., assessors facilitate their business by making out a tax table, as follows:—

TABLE.

$1	gives	$0.020219	tax.	$60	gives	$1.21314	tax.
2	"	0.040438	"	70	"	1.41533	"
3	"	0.060657	"	80	"	1.61752	"
4	"	0.080876	"	90	"	1.81971	"
5	"	0.101095	"	100	"	2.0219	"
6	"	0.121314	"	200	"	4.0438	"
7	"	0.141533	"	300	"	6.0657	"
8	"	0.161752	"	400	"	8.0876	"
9	"	0.181971	"	500	"	10.1095	"
10	"	0.20219	"	600	"	12.1314	"
20	"	0.40438	"	700	"	14.1533	"
30	"	0.60657	"	800	"	16.1752	"
40	"	0.80876	"	900	"	18.1971	"
50	"	1.01095	"	1000	"	20.219	"

By the table, Mr. Jones's tax is as follows:—

on $3000,	$60.657
" 400,	8.0876
" 20,	0.4043
" 6,	0.1213
" 3 polls,	2.70

$71.9702, amount of Jones's tax.

NOTE—Require the pupil to write a rule for constructing a tax table, and for finding the amount of tax by it.

519. The people of a certain city vote to raise $25000 to build a public hall. The property of the city is valued at $1496826. Construct a tax table, and from it calculate

the amount of A's tax, whose property is valued at $8374; of B's tax, whose property is valued at $4374; of C's tax, whose property is valued at $9874.

520. The property in a certain school district is valued at $217386. A teacher is employed for ten months at a salary of $45 per month, and the contingent expenses are $100.50. Construct a tax table, and from it calculate the tax of Mr. Osborn, whose property is valued at $3568; of Mr. Bryant, whose property is valued at $4000; of Mr. Griggs, whose property is valued at $3888; of Mr. Morgan, whose property is valued at $500; of Mr. Blore, whose property is valued at $4500.

QUESTIONS.—How may fractions be considered? (138.) What determines the denomination of a fraction? (139.) On what does the value of a fraction depend? (140.) What is the reduction of fractions? (141.) What is reduction ascending? (142.) What is reduction descending? (143.) When are fractions said to be reduced to their lowest terms? (144.)

LESSON LX.

CUSTOM HOUSE BUSINESS.

224. Duties, or **Customs,** are taxes levied by government on goods imported or exported.

225. A **Custom House** is an office established by government for the purpose of collecting duties on goods imported or exported.

226. A **Port of Entry** is a port designated by law where duties may be received on imported or exported goods.

227. Tonnage is a tax levied upon vessels according to their size, for the privilege of coming into a port of entry.

228. Revenue is the income of government from duties, tonnage, taxes, and other sources.

229. An **Ad Valorem Duty** is estimated at a certain per cent. on the cost of the goods in the country from which they are imported.

230. A **Specific Duty** is estimated on the weight or measure of goods, independent of their cost.

231. An **Invoice** is a list of imported goods, giving the quantity and price.

232. Tare is an allowance for the weight of packages that contain goods.

233. Leakage is an allowance for the waste by leaking of liquors imported in casks or barrels.

234. Breakage is an allowance for the loss by breaking of liquors imported in bottles.

235. Gross Weight is the weight of goods, including the packages.

236. Net Weight, or **Value,** is the weight or value of goods after all allowances of tare, leakage, &c., have been made.

Notes.—1. Tare is ascertained by actually weighing one or more of the empty boxes, casks, or other packages It is sometimes computed at a certain per cent.

2. The customary breakage for porter, ale, and beer, in bottles, is 10 per cent ; but 5 per cent on all other liquors in bottles. At the Custom House 1 doz bottles of the common size are estimated to contain 2¾ gallons.

3. The customary leakage for liquors in casks is 5 per cent.

Questions.—Reduce a fraction to its lowest terms (at the blackboard) and give the analysis. (145.) Reduce an improper fraction to a mixed number, and give the analysis. (146.) Reduce a mixed number to an improper fraction, and give the analysis. (147.) Reduce a mixed number to a fraction of a given denomination, and give the analysis. (148.)

LESSON LXI.

237. To find the amount of duty on an invoice of merchandise.

What is the duty at 25 per cent. on 30 gross of champagnes, invoiced at $1.50 per doz., $2\frac{3}{4}$ per cent. being allowed for breakage?

MODEL OPERATION.

1. $1.50×360=$540, gross value.
2. $540×.02$\frac{3}{4}$=$14.85, breakage.
3. $540—$14.85=$525.15, net value.
4. $525.15×.25=$131.28, amount of duty.

Analysis.—1. Find the gross value.

2. Find the deduction for breakage.

3. Find the net value.

4. Find the amount of duty.

Note.—Require the pupil to write a rule from the analysis.

521. What is the duty at $18\frac{3}{4}$ per cent. on 237 yds. of black silk invoiced at $78\frac{1}{2}$ cts. per yard?

522. What is the duty at 25 per cent. on 125 barrels of whale oil invoiced at $87\frac{1}{2}$ cts. per gallon, allowing $2\frac{1}{2}$ per cent. leakage?

523. What is the duty at 20 per cent. on 250 bags of coffee, each weighing 160 lbs., invoiced at $6\frac{3}{4}$ cts. per pound, an allowance of 2 per cent. being made for tare?

524. A merchant imports 137 cases of indigo, each case weighing 197 lbs.; how much will the duty amount to at 5 cts. per pound, allowing 15 per cent. tare?

Questions.—What is the addition of fractions? (149.) When are fractions said to have a common denominator? (150.) What is the subtraction of fractions? (151.) Give the analysis of the addition of fractions. (150.) Of the subtraction of fractions. (151.) What is the multiplication or division of fractions? (153.)

LESSON LXII.

INTEREST.

238. Interest is an amount paid for the use of money.

239. The **Principal** is the sum borrowed, on which interest is paid.

240. Rate per cent. is the per cent. paid annually.

241. Amount is the sum of the principal and the interest.

242. Simple Interest is interest paid for the use of the principal only.

243. Compound Interest is interest on both principal and interest.

244. Legal Interest is the rate per cent. fixed by law. It varies in the different states, according to the following:

(*a.*) TABLE.

Louisana,	5 per cent.	New Hampshire,	6 per cent.
Arkansas,	6 "	New Jersey,	6 "
Connecticut,	6 "	North Carolina,	6 "
Delaware,	6 "	Ohio,	6 "
Illinois,	6 "	Pennsylvania,	6 "
Indiana,	6 "	Rhode Island,	6 "
Kentucky,	6 "	Tennessee,	6 "
Maine,	6 "	Vermont,	6 "
Maryland,	6 "	Virginia,	6 "
Massachusetts,	6 "	Georgia,	7 "
Missouri,	6 "	Iowa,	7 "

Michigan,	7 per cent.	Alabama,	8 per cent.
New York,	7 "	Florida,	8 "
South Carolina,	7 ".	Mississippi,	8 "
Wisconsin,	7 "	Texas,	8 "

District of Columbia, 6 per cent.

NOTES.—1. In some of the other states any rate per cent. that is agreed upon by the parties is recognized as legal.

2. When the rate per cent. is not specified in notes, contracts, &c., the legal rate is always understood.

3. When more than the legal rate is charged it is called *usury*, and those charging it are punishable by law.

QUESTIONS.—Multiply a fraction by an integral number, and give the analysis. (154.) Divide a fraction by an integral number, and give the analysis. (155.) Divide a mixed number by an integral, and give the analysis. (156.) Multiply a fraction by a fraction, and give the analysis. (157.)

LESSON LXIII.

245. To find the interest on any sum at any rate per cent., for years, months, and days by taking aliquot parts.

What is the interest on $132.41 for 3 yrs. 7 mo. 16 da., at 4 per cent.?

MODEL OPERATION.

$132.41
.04

5.2964, interest for 1 year.
3

1. 15.8892, interest for 3 years.
2. $5.296÷2= 2.648 " " 6 mo., or ½ year.
3. $2.648÷6= .4413 " " 1 mo., or $\frac{1}{6}$ of 6 months.
4. $0.4413÷2= .2206 " " 15 da., or ½ of 1 month.
5. $0.2206÷15= .0147 " " 1 da., or $\frac{1}{15}$ of 15 days.
6. $19.2138, interest for 3 yrs. 7 mo. (6 mo.+1 mo.)
[16 da. (15 da.+1 da.)

* **ANALYSIS.**—1. Find the interest for 3 years.

If the interest on $1 is $0.04, on $132.41 it is 132.41 tim $0.04, which are $5.2964; and for 3 years it is 3 times $5.296 which are $15.8892.

2. Find the interest for 6 months.

If the interest for 1 year, or 12 months, is $5.2964, for months, or *one half* a year, it is *one half* of $5.2964, which $2.648.

3. Find the interest for 1 month.

If the interest for 6 months is $2.648, for 1 month it is *o* *sixth* of $2.648, which is $0.4413.

4. Find the interest for 15 days.

If the interest for 1 month is $0.4413, for 15 days, or ½ mont it is *one half* of $0.4413, which is $0.2206.

5. Find the interest for 1 day.

If the interest for 15 days is $0.2206, for 1 day it is *one f* *teenth* of $0.2206, which is $0.0147.

6. Find the interest for 3 years 7 months 16 days.

If the interest for 3 years is $15.8892, for 6 months is $2.64 for 1 month is $0.4413, for 15 days is $0.2206, for 1 day $0.0147, the interest for 3 years 7 months 16 days is the su of these quantities, which is $19.21.

Therefore, the interest on $132.41 for 3 years 7 months days, at 4 per cent., is $19.21.

NOTE.†—Require the pupil to write a rule from the analysis.

QUESTIONS.—Multiply a mixed number by a mixed number, a give the analysis. (158.) Divide an integral number by a fracti and give the analysis. (159.) Divide a fraction by a fraction, a give the analysis. (160.) Simplify a complex fraction, and give t analysis. (161.)

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* **NOTE.**—This method of analysis in calculating interest, while often shorter t the other methods, is besides such an admirable discipline for the mind that give it a prominent place.

†This rule like the 6 per cent. method makes the year consist of 360 days inst of 365. In some states this is considered legal; but in New York one day is c sidered *one*-365th part of a year. Deduct $\frac{1}{37}$ of the interest at 360 days per y from itself, and the remainder will be the interest at 365 days per year.

LESSON LXIV.

525. What is the interest on $374 for 7 yrs. 9 mo. 17 da. at 9 per cent.?

526. What is the interest on $325 for 2 yrs. 9 mo. 12 da. at 7 per cent.?

527. What is the interest on $397.25 for 8 yrs. 7 mo. 13 da. at 8 per cent.?

528. What is the interest on $848.88 for 4 yrs, 6 mo. 17 da. at 4 per cent.?

529. What is the amount of $413 for 12 yrs. 3 mo. 2 da. at 11 per cent.?

530. What is the amount of $384.24 for 3 yrs. 9 mo. 27 da. at 9¾ per cent.?

531. What is the amount of $243 for 11 yrs. 8 mo. 29 da. at 8 per cent.?

532. What is the interest on $471 for 5 yrs. 7 mo. 25 da. at 4 per cent.?

533. What is the amount of $3.86 for 3 yrs. 2 mo. 11 da. at 9 per cent.?

534. What is the amount of $0.88 for 5 yrs. 9 mo. 2 da. at 6½ per cent.?

535. What is the amount of $0.06 for 11 yrs. 7 mo. 19 da. at 12 per cent.?

536. What is the interest on 43 cts. for 3 yrs. 2 mo. 1 da. at 6 per cent.?

537. If a person borrows $375.75 at 7 per cent., how much will be due at the end of 2 yrs. 7 mo. 19 da.?

538. What is the interest paid on a loan of $1374.74 at 7 per cent., made Jan. 1st, 1858, and paid July 7th, 1863?

539. A note of $384.25 was given May 23d, 1860, bear-

ing interest at 8 per cent.; what will be the amount due Aug. 3d, 1864, if no interest has been paid?

540. I gave a note of $397.28, Jan. 30th, 1860; what was the amount due Sept. 8th, 1863?

QUESTIONS.—Give the analysis of finding the least common multiple of fractions. (163.) What is reduction descending of fractional compound numbers? (164.) Reduction ascending? (165.) What is the comparison of numbers? (166.) What axiom is used in the comparison of numbers? (16., *c.*)

LESSON LXV.

541. What is the interest on $3874.63 for 9 yrs. 11 mo. 29 da., at 5¾ per cent.?

542. What is the interest on $173.40, from Sept. 25th, 1850, to July 9th, 1861, at 8½ per cent.?

543. What is the interest on $169.73, from Dec. 10th, 1849, to July 7th, 1863, at 5¾ per cent.?

544. What is the interest on $1, from July 29th, 1847, to Oct. 8th, 1862, at 6⅜ per cent.?

545. What is the interest on $1, from Sept. 18th, 1853, to Aug. 11th, 1860, at 3⅜ per cent.?

546. What is the interest on $23.50, from Apr. 9th, 1857, to Jan. 13th, 1864, at 11 per cent.?

547. What is the interest on $3478.41, from Aug. 13th, 1860, to Dec. 15th, 1860, at 9 per cent.?

548. What is the amount of $413.67, from Feb. 29th, 1853, to Jan. 18th, 1863, at 7 per cent.?

549. I gave Luther Bryant my note for $132.25, dated July 16th, 1858, and bearing 7 per cent. interest; what is due March 11th, 1864?

550. I gave my note for $111.37, March 13th, 1860; what shall I owe Apr. 17th, 1864, at 7¼ per cent. interest?

QUESTIONS.—Analyze an example of the addition of denominate fractions. (168.) Analyze an example of the subtraction of denominate fractions. (168.) What is said about decimal fractions? (169.) What determines units' place? (170.) How are integers separated from decimals? (171.) Repeat the table. (171., *a.*)

LESSON LXVI.

246. To find the interest on any sum, for any time, at any rate per cent., by the 6 per cent. method.

247. TABLE.

The interest on $1 for one year, or 12 months, is $0.06.
" " $1 " 2 months ($\frac{1}{6}$ of 12 mo.) is .01,
" " $1 " 1 month, or 30 da. ($\frac{1}{12}$ of 12 mo.) is .005.
" " $1 " 6 da. ($\frac{1}{5}$ of 30 da.) is .001.
" " $1 " 1 da. ($\frac{1}{6}$ of 6 da.) is .000$\frac{1}{6}$.

Hence the interest on $1 for every year is 6 cts.
" " $1 " 2 months is 1 ct.
" " $1 " 6 days is 1 mill.
" " $1 " 1 day is $\frac{1}{6}$ of a mill.

What is the interest on $1 for 2 yrs. 3 mo. 7 da. at 6 per cent.?

MODEL OPERATION.

1. Interest for 2 yrs. = $0.12
2. " " 3 mo. = .015
3. " " 7 da. = .001$\frac{1}{6}$

4. 0.136\frac{1}{6}$ Ans.

RULE.—*Calculate the interest for every year at 6 cts., for every 2 months at 1 ct., for every 6 days at 1 mill, and for every day at $\frac{1}{6}$ of a mill; and the sum will be the interest on one dollar for the given time.*

QUESTIONS.—What is a mixed number? (172.) How are numbers divided, for convenience in reading? (173.) Analyze an example of the addition of decimals. (174.) Analyze an example of the subtraction of decimals. (175.) Analyze an example of the multiplication of decimals. (176.) Give the rule for pointing off decimal places. (176., *f.*) Repeat each of the propositions relating to the multiplication of decimals. (177.) Give the general law. (177., *e.*)

LESSON LXVII.

EXAMPLES FOR PRACTICE.

Note.—When no rate per cent. is mentioned, 6 per cent. is always understood.

What is the interest of $1 for the following periods of time?

551. 1 year 7 mo. 12 da.
552. 3 yrs. 3 mo. 11 da.
553. 9 yrs. 8 mo. 25 da.
554. 8 yrs. 3 mo. 24 da.
555. 9 yrs. 8 mo. 27 da.
556. 14 yrs. 9 mo. 6 da.
557. 9 yrs. 11 mo. 8 da.
558. 6 yrs. 11 mo. 9 da.
559. 8 yrs. 3 mo. 3 da.
560. 4 yrs. 4 mo. 4 da.
561. 9 yrs. 3 mo. 28 da.
562. 4 yrs. 8 mo. 16 da.
563. 1 yr. 1 mo. 1 da.
564. 0 yrs. 0 mo. 27 da.
565. 0 yrs. 8 mo. 8 da.
566. 0 yrs. 9 mo. 0 da.
567. 14 yrs. 0 mo. 0 da.
568. 27 yrs. 7 mo. 7 da.
569. 10 yrs. 10 mo. 10 da.
570. 3 yrs. 3 mo. 3 da.

Questions.—Give the analysis of an example of the division of decimals. (178.) Give each of the propositions relating to the division of decimals. (179.) Give the general law. (179., *e.*) Give the rule. (179., *f.*) Change a common fraction to a decimal, and give the analysis. (180., *a.*) Give the proof. (180., *b.*) When are decimals said to be repetends? (180., *c.*, 2.)

LESSON LXVIII.

What is the interest on $372.12 for 8 yrs. 9 mo. 13 da.?

MODEL OPERATION.

(*a.*)

Interest on $1 for 8 yrs. = $0.48.
" " $1 " 9 mo. = .045.
" " $1 " 13 da. = $.002\frac{1}{8}$.

$\$0.527\frac{1}{8}$

(*b.*)

$$\begin{array}{r} \$372.12 \\ .527\frac{1}{8} \\ \hline 6202 \\ 260484 \\ 74424 \\ 186060 \\ \hline \$196.16926, \text{ Ans.} \end{array}$$

Analysis.—(*a.*) Find the interest on $1 for the given time. By 247. the interest on $1 for 8 yrs. 9 mo. 13 da. is $\$0.527\frac{1}{8}$.

(*b.*) Find the interest of $372.12 for the same time.

If the interest on $1 for the given time is $\$0.527\frac{1}{8}$, on $372.12 it is 372.12 times $\$0.527\frac{1}{8}$, which is $196.169.

Therefore, the interest on $372.12 for 8 yrs. 9 mo. 13 da. is $196.169.

Note.—Require the pupil to write a rule from the analysis.

571. What is the interest on $397.46 for 8 yrs. 3 mo. 9 da.?

572. What is the interest on $371.46 for 9 yrs. 3 mo. 19 da.?

573. What is the interest on $572.86 for 18 yrs. 11 mo. 29 da.?

574. What is the amount of $37.26 for 13 yrs. 2 mo. 11 da.?

575. What is the amount of $33.46 for 2 yrs. 2 mo. 23 da.?

576. What is the interest on $6.47 for 3 yrs. 9 mo. 11 da.?

577. What is the interest on $62.35 for 2 yrs. 8 mo. 29 da.?

578. What is the interest on $12.42 for 2 yrs. 8 mo. 13 da.?

579. What is the interest on $37.85 for 1 yr. 9 mo. 28 da.?

580. What is the interest on $12.80 for 3 yrs. 3 mo. 3 da.?

581. What is the interest on $37.40 for 2 yrs. 6 mo. 2 da.?

582. What is the interest on $8.30 for 3 yrs. 8 mo. 7 da.?

583. What is the interest on $9.20 for 9 yrs. 9 mo. 9 da.?

584. What is the interest on $11 for 8 yrs. 8 mo. 8 da.?

QUESTIONS.—What is reduction descending in denominate decimals? (181.) What is reduction ascending? (182.) Give the analysis of an example of reduction descending in denominate decimals. (182., *a.*) Of reduction ascending. (182., *b.*) What is per cent.? (183.)

LESSON LXIX.

What is the interest on $34 for 8 yrs. 7 mo. 27 da., at 8 per cent.?

MODEL OPERATION.

At 6 per cent. the interest on $1 for 8 yrs. is $0.48
" 6 " " " $1 " 7 mo. " .035
" 6 " " " $1 " 27 da. " .004½

" 6 " for 8 yrs. 7 mo. 27 da. the int. is $0.519½
34

17
2076
1557

1.

$17.663
1⅓

2.

5887
17663

$23.550, Ans.

ANALYSIS.—1. Find the interest on the given amount, for the given time, at 6 per cent.

The interest on $34 for 8 yrs. 7 mo. 27 da., at 6 per cent., is $17.663.

2. Find the interest at 8 per cent.

If the interest at 6 per cent. is $17.663, the interest at 8 per cent. is $1\frac{1}{3}$ ($\frac{8}{6}$ of 6 per cent.) of the interest at 6 per cent., which is $23.55.

Therefore, the interest on $34 for 8 yrs. 7 mo. 27 da., at 8 per cent., is $23.55.

NOTE.—Require the pupil to write a rule from the analysis.

NOTES.—1. The interest at 1 per cent. is $\frac{1}{6}$ of the interest at 6 per cent.
“ “ 2 “ “ $\frac{1}{3}$ ($\frac{2}{6}$) “ “ “ 6 “
“ “ 3 “ “ $\frac{1}{2}$ ($\frac{3}{6}$) “ “ “ 6 “
“ “ 4 “ “ $\frac{2}{3}$ ($\frac{4}{6}$) “ “ “ 6 “
“ “ 5 “ “ $\frac{5}{6}$ “ “ “ 6 “
“ “ 7 “ “ $1\frac{1}{6}$ “ “ “ 6 “
“ “ 8 “ “ $1\frac{1}{3}$ “ “ “ 6 “
“ “ 9 “ “ $1\frac{1}{2}$ “ “ “ 6 “
“ “ 10 “ “ $1\frac{2}{3}$ “ “ “ 6 “
“ “ 11 “ “ $1\frac{5}{6}$ “ “ “ 6 “
“ “ 12 “ “ 2 times “ “ 6 “

2. Since the interest of $1 at 6 per cent. is $0.01 for 60 days, the interest for any number of days will be as many cents as 60 days is contained times in the number of days Hence the following rule.

RULE.—*Multiply the principal by the number of days in the given time, divide the product by* 60, *and the quotient will be the interest at* 6 *per cent. in cents and decimals of a cent.*

585. What is the interest on $34.26 for 3 yrs. 9 mo. 17 da., at 3 per cent.?

586. What is the interest on $413 for 8 yrs. 8 mo. 17 da., at 9 per cent.?

587. What is the interest on $837 for 1 yr. 3 mo. 12 da., at 7 per cent.?

588. What is the interest on $387.24 for 1 yr. 1 mo. 1 da., at 9 per cent.?

589. What is the interest on $341.20 for 3 yrs. 11 mo. 24 da., at 3 per cent.?

590. What is the interest on $486 for 4 yrs. 3 mo. 11 da., at ½ per cent.?

591. What is the interest on $34.20 for 2 yrs. 9 mo. 12 da., at ¾ per cent.?

592. What is the interest on $873.10 for 1 yr. 1 mo. 1 da., at 6½ per cent.?

593. What is the interest on $637 for 1 yr. 3 mo. 11 da., at 8¾ per cent.?

594. What is the amount of $371 for 1 yr. 8 mo. 11 da., at 7 per cent.?

595. What is the amount of $386 for 2 yrs. 6 mo. 21 da., at 9 per cent.?

596. What is the amount of $3847.20 for 1 yr. 8 mo. 24 da.?

597. What is the amount of $42.86 for 3 yrs. 3 mo. 27 da., at 3½ per cent.?

598. What is the amount of $32.47 for 2 yrs. 8 mo. 11 da., at 5¾ per cent.?

599. What is the amount of $37.41 for 8 yrs. 9 mo. 21 da., at ⅜ per cent.?

600. What is the amount of $38.88 for 9 yrs. 9 mo. 29 da.?

QUESTIONS.—How is per cent. expressed? (184.) Find any per cent. of a number, and give the analysis. (185.) Give the rule. Give the analysis of finding what per cent. one number is of another. (186.) Write a rule from the analysis. (186.) Give the analysis of finding a number when a certain per cent. of it is given. (187.) Write a rule.

LESSON LXX.

601. A man bought property for $2873.25, and agreed to pay for it in 2 yrs. 7 mo. 9 da., with 3⅓ per cent. interest; at the expiration of that time what amount did he owe?

602. I settled with a creditor and gave my note for $317.42, to be paid in 1 yr. 7 mo. 9 da., with 5 per cent. interest; what must I pay at the end of that time?

603. How much interest will accrue on a note of $3874.20, in 3 yrs. 8 mo. 19 da., at 8½ per cent.?

604. What is the interest of $983.27 from Apr. 9th, 1860, to Jan. 13th, 1863, at 8 per cent.?

605. Find the interest on $371.48 from July 11th, 1849, to Oct. 18th, 1860, at 12 per cent.

606. A man bought a farm and paid $384 down; after 8 months he paid $583, and 13 months after that time he paid the balance; what was the whole amount paid, allowing 7 per cent. interest?

607. A man sold a store and the goods in it for $80000; he received $25300 on delivering; 80 days after $37400, and the remainder at the end of 6 mo.; what was the entire amount, allowing 6 per cent. interest?

608. A merchant bought the following bill of goods at interest at 6 per cent.:

1859,	Jan.	1,	30 yds.	Broadcloth,	@	$3.
"	"	17,	257 "	Calico,	@	.10.
"	Mar.	19,	125 "	Carpeting,	@	1.50.
"	Jan.	8,	375 "	Oilcloth,	@	.50.

How much will his bill amount to December 6th, 1859?

609. OTIS MERWIN,

To OTIS LYON, Dr.

1862,	Jan.	3,	59 yds. Sheeting	@ $0.13,	
"	Apr.	9,	Cash,		$347.
"	June	11,	20 Stoves	@ $20,	
"	July	18,	1 gross Scythes,		200.25
"	"	19,	1 box Hoes,		100.50
"	Aug.	12,	100 Axes,		100.00

This account is at interest at 7 per cent.; what will be the amount at the time of settlement, Sept. 13th, 1862?

§10. I commenced building a house in the year 1860. My expenditures were as follows: Jan. 11th, $374.63; Feb. 19th, $1860.97; Apr. 3d, $94.68; Aug. 13th, $47.98. On the 8th of December I sold it at 12 per cent. advance on the cost, including interest; what did I receive for it?

QUESTIONS.—What is an agent or factor? (188.) What is commission? (189.) What is brokerage? (190.) What is a corporation? (191.) What is a charter? (192.) What is a firm? (193.) What is capital or stock? (194.) What is a share? (195.) What are stockholders? (196.) When are stocks said to be at par? (197.)

LESSON LXXI.

PARTIAL PAYMENTS.

248. A **Partial Payment** is a payment of a part of a note, or bond, &c.

249. When the amount paid, the date of the payment, and the name of the person receiving the money, are written on the back of the obligation, the writing is called an *Indorsement.*

250. The following rule for computing interest on bonds, notes, &c., when partial payments have been made, has been adopted by the Supreme Court of the United States, and by all the states of the Union, with the exception of Connecticut, and New Hampshire.

RULE.—I. "*Compute the interest on the principal to the time of the first payment, and, if the payment exceed this interest, add the interest to the principal, and from the sum subtract the payment: the remainder forms a new principal.*

II. "*If the payment is less than the interest, take no notice of it until other payments are made, which in all shall exceed the interest computed to the time of the last payment; then add the interest so computed to the principal, and from* *the sum subtract the sum of the payments: the remainder will form a new principal, on which interest is to be computed as before."*

NOTES.—1. The Connecticut Rule is as follows:

I. "*Payments made one year or more from the time at which the interest commenced, or from another payment; and payments less than the interest due are treated according to the U. S. rule.*

II. "*Payments exceeding the interest due, and made within one year from the time interest commenced, or from a former payment, shall draw interest for the balance of the year, provided the interval does not extend beyond the settlement, and the amount must be subtracted from the amount of the principal for one year; the remainder will be a new principal.*

III. "*If the year extend beyond the settlement, find the amount of the principal to that day; the remainder will be the sum due."*

† **Vermont has the following**

RULE.—I. "*Find the amount of the principal from the time interest commenced, to the time of settlement.*

II. "*Find the amount of each payment from the time it was made to the time of settlement.*

III. "*Subtract the sum of the amounts from the amount of the principal and the remainder will be the sum due."*

‡ **New Hampshire has the following**

RULE.—I. "*Find the amount of the principal for one year, and deduct from it the amount of each payment of that year from the time it was made, up to the end of the year; the remainder will be a new principal with which to proceed as before.*

II. "*If the settlement occur less than a year from the last annual term of interest, make the last term of interest a part of a year accordingly.*"

QUESTIONS.—When is stock said to be above par? (198.) When are stocks said to be below par? (199.) What is the real, or market value, of stocks? (200.) What is a dividend? (201.) Give the analysis of finding the commission on any amount of stock bought or sold. (202.) Give the analysis of finding the amount of commission, when deducted in advance from the remittance. (203.) Write a rule from the analysis.

LESSON LXXII.

EXAMPLES FOR PRACTICE.[*]

$369.84. NEWARK, Jan. 1, 1858.

For value received, I promise to pay to Jacob Wilson, or order, on demand, three hundred sixty-nine dollars eighty-four cents, with interest.

JOHN F. HUNTSMAN.

On this note are the following indorsements:

June 13, 1858,	$50.	July 9, 1860,	$120.
Dec. 11, 1858,	25.	Aug. 13, 1862,	15.
Sept. 13, 1859,	60.	Dec. 29, 1862,	89.

What will remain due Jan. 9, 1863.

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[*] NOTES.—1. When a note is signed by two or more persons, they are together holden for its payment.

2. A *joint* and *several* note is one signed by two or more persons who are together and separately holden for its payment.

3. A negotiable note is one which is written payable to *order* or to *bearer*, and which can be transferred from one to another.

4. When a note is written payable to *order*, the holder of the note must write his name on its back, when he transfers it to another person, and by so doing he guarantees its payment.

5. The *maker*, or *drawer*, of a note is the person who signs it. The holder is the person to whom it is to be paid.

6 The *indorser* is the person who writes his name on the back to transfer it, or to guarantee its payment. The *face* of a note is the amount for which it is given.

MODEL OPERATION.

	Principal, or face of the note, Jan. 1, 1858,	$369.84
1.	Interest on $369.84, from Jan. 1, 1858, to June 13, 1858, Time of first payment (5 mo. 12 da.)	9.985
2.	Amount,	$379.825
	First payment, June 13, 1858, (greater than the interest, and therefore to be deducted)	50.
	Balance, for a new principal,	$329.825
3.	Inter'st on $329.825, from June 13, 1858, to Dec. 11, 1858, Time of second payment (5 mo. 28 da.)	9.762
4.	Amount,	$339.587
	2nd payment, Dec. 11, 1858, (greater than the interest, and therefore to be deducted)	25.
	Balance, for a new principal,	$314.587
5.	Interest on $314.587 from Dec. 11, 1858, to Sept. 13, 1859, Time of third payment (9 mo. 2 da.)	14.25
6.	Amount,	$328.837
	3rd payment, Sept. 13, 1859, (greater than the interest, and therefore to be deducted,)	60.
	Balance, for a new principal,	$268.837
7.	Interest on $268.837, from Sept. 13, 1859, to July 9, 1860, Time of fourth payment (9 mo. 26 da.)	13.253
8.	Amount,	$282.09
	4th payment, July 9, 1860, (greater than the interest, and therefore to be deducted,)	120.
	Balance, for a new principal,	$162.09
9.	Interest on $162.09, from July 9, 1860, to Ang. 13, 1862, Time of fifth payment (2 yrs. 1 mo. 4 da.)	20.358
10.	Interest on $162.09, from Aug. 13, 1862, to Dec. 29, 1862, Time of sixth payment (4 mo. 16 da.)	3.663
11.	Amount,	$186.111
	5th payment is less than the interest. The sum of the 5th and the 6th payments, made respectively Aug. 13, 1862, and Dec. 29, 1862, (greater than the interest, and therefore to be deducted,)	104.00
	Balance, for a new principal,	$82.111
12.	Interest on $82.111, from Dec. 29, 1862, to Jan. 9, 1863, (10 da.)	.130
13.	Amount due Jan. 9, 1863,	$82.241

ANALYSIS.

ANALYTICAL STEPS.—1. Find the interest on the face of the note, to the time of the 1st payment.

2. Find the new principal, June 13, 1858.

3. Find the interest on this principal, to the time of the 2nd payment.

4. Find the new principal, Dec. 11, 1858.

5. Find the interest on this principal, to the time of the 3rd payment.

6. Find the new principal, Sept. 13, 1859.

7. Find the interest on the principal, to the time of the 4th payment.

8. Find the new principal, July 9, 1860.

9. Find the interest on this principal, to the time of the 5th payment.

10. Find the interest on this principal, to the time of the 6th payment.

11. Find the new principal, Dec. 29, 1862.

12. Find the interest on this principal, to Jan. 9, 1863.

13. Find the amount due, Jan. 9, 1863.

LESSON LXXIII.

611. $489\frac{39}{100}$. CINCINNATUS, N. Y., Oct. 9, 1860.

Sixteen months from date, we promise to pay Oscar Osgood, or order, four hundred eighty-nine dollars thirty-nine cents, with interest from date, value received.*

RICHARD F. WESTON.
JOHN C. LEE.

INDORSEMENTS.

Nov. 3, 1860,	$29.13.	Mar. 11, 1861,	$5.37.
Jan. 11, 1861,	125.	May 13, 1861,	91.61.

What was the amount when the note became due?

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*NOTE.—Notes on time do not draw interest until they are due, unless it is so stated in the body of the note. Demand notes draw interest from date, and notes on time, after they become due, although there be nothing said in them about interest.

612. $\$984\frac{00}{100}$. UNADILLA, N. Y., Aug. 6, 1860.

Three years from date, we jointly and severally promise to pay Israel Putnam, or bearer, nine hundred eighty-four dollars, with interest after four months, value received.

LUTHER BRYANT.
HENRY BRYANT.
CHARLES BRYANT.

Received on the within note:

Dec. 29, 1860, $125.37.
Mar. 13, 1861, $327.90.
Apr. 11, 1861, $57.
July 5, 1861, $3.12.
Oct. 11, 1861, $2.41.
Jan. 11, 1862, $150.

How much is to be paid when the note becomes due?

613. $500. BROOKLYN, Sept. 9, 1859.

One year from date, I promise to pay O. H. Hall, five hundred dollars, value received.*

WILLIAM H. WILLIAMS.

Received on the within note:

Jan. 12, 1861, $300.50.
June 13, 1861, $21.63.
Apr. 25, 1863, $2.23.
May 27, 1864, $25.50.

How much was due when the note was paid, Oct. 18, 1864?

614. $\$1345.\frac{38}{100}$. HARTFORD, Jan. 13, 1862.

Thirty days from date, I promise to pay Ayres Jennings & Co., one thousand three hundred forty-five dollars thirty-eight cents, at 8 per cent., value received.

THEODORE LYON.

What was due April 8, 1864?

QUESTIONS.—Give the analysis of finding the value of stock, when at a premium. (204.) Write a rule from the analysis. Give the analysis of finding the quantity of stock that can be purchased for a certain sum. (205.) Write a rule from the analysis. What is profit and loss? (206.)

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* NOTE.—This note is not negotiable, and it draws interest only after it becomes due.

LESSON LXXIV.

PROBLEMS IN INTEREST.

251. In **Problems in Interest** there are five parts to be considered: 1st. The Principal. 2d. The Rate per cent. 3d. The Time. 4th. The Interest. 5th. The Amount. These may be symbolized by the following abbreviations:—

P. = Principal. T. = Time.
R. = Rate per cent. I. = Interest.
A. = Amount.

These parts are so related to each other, that, any three of them being known, the other may be found.

252. Prob. I.—Given the principal, rate per cent., and time, to find the *interest.*

What is the interest of $57 for 3 years, at 7 per cent.?

MODEL OPERATION.

1. $\$57 \times .07 = \$3.99$.
2. $\$3.99 \times 3 = \$11.97$.

ANALYSIS.—1. If the interest on $1 for one year is 7 *hundredths* of a dollar, on $57 it is 57 times 7 hundredths of a dollar, which are 399 hundredths dollars, or $3.99.

2. If the interest on $57 for one year is $3.99, for 3 years it is 3 times $3.99, which are $11.97.

Therefore, the interest on $57 for 3 years, at 7 per cent., is $11.97.

FORMULA: $P. \times R. \times T. = I.$

NOTE.—Require the pupil to write a rule from the formula.

615. What is the interest on $37.86 for 9 yrs. 3 mo. 11 da., at 8 per cent.?

616. What is the interest on \$27.41, for 2 yrs. 3 mo. 27 da., at $7\frac{1}{2}$ per cent.?

QUESTIONS.—Give the analysis of finding the amount of gain, when the cost and gain per cent. are given. (207.) Write a rule from the analysis. Give the analysis of finding the gain or loss per cent., when the cost and selling price are given. (208.) Write a rule from the analysis. Give the analysis of finding the selling price, when the cost and gain or loss per cent. are given. (209.) Write a rule from the analysis.

LESSON LXXV.

253. Prob. II.—Given the time, rate per cent., and interest, to find the *principal.*

What principal in 2 yrs. 6 mo., at 6 per cent., will gain \$25.36 interest?

MODEL OPERATION.

1. $\$1\times.06\times2\frac{1}{2}=\$0.15$, int. on \$1.
2. $\$25.36\div.15=\$169.066$.

ANALYSIS.—1. The interest on \$1 for 2 yrs. 6 mo., at 6 per cent., is \$0.15.

2. If it requires \$1 to gain \$0.15 in 2 yrs. 6 mo., at 6 per cent., to gain \$25.36 will require as many dollars as \$0.15 is contained times in \$25.36, which are 169.066.

Therefore, \$169.066 are required to gain \$25.36 in 2 yrs. 6 mo., at 6 per cent.

FORMULA: $\frac{\text{I.}}{\$1\times\text{R.}\times\text{T.}}=\text{P.}$

NOTE.—Require the pupil to write a rule from the formula.

617. A gentleman loaned money at 7 per cent., and received \$700 interest a year; how much money did he loan?

618. What principal will, in 4 yrs. 8 mo. 24 da., at 6 per cent., give \$18.88 interest?

619. What principal will, in 4 mo. 6 da., at 7 per cent., give \$25.50 interest?

620. A gentleman bequeathes his wife $600 a year, and his son $400 a year; what sum must be invested, at 7 per cent. interest, to produce these amounts?

621. What principal will, in 1 yr. 11 mo. 26 da., at 5 per cent., produce $188.88?

622. A widow is receiving $1888 per annum; what is her property, supposing it invested at 7 per cent.?

623. What principal will, in 8 yrs., at 7½ per cent., produce $4000 interest?

624. What principal will, in 3 yrs. 3 mo. 3 da., at 5½ per cent., produce $28 75?

625. What principal will, in 6 yrs., at 7 per cent., produce $800.50 interest?

626. An estate was divided among three children: the first loans his money at 8 per cent. interest, and receives $400 a year; the second loans his at 7½ per cent., and receives $450.42 interest per annum; the third loans his at 5 per cent., and receives $400.75 interest per annum; what was the value of the estate?

QUESTIONS.—Give the analysis of finding the cost when the selling price and gain or loss per cent. are given. (210.) Write a rule from the analysis. What is insurance? (211.) Who is the insurer? (212.) Who is the insured? (213.) What is the policy? (214.) What is the premium? (215.) Give the analysis of finding the amount of the premium, when the rate per cent. is given. (216.) Write a rule from the analysis.

LESSON LXXVI.

254. Prob. III.—Given the time, rate per cent., and amount, to find the *principal.*

What principal, in 3 years 4 months, at 8 per cent., will produce an amount of $73.86?

MODEL OPERATION.

1. $0.08 \times 3\frac{1}{3} = \$0.266$; $\$0.266 + \$1 = \$1.266$.
2. $\$73.86 \div \$1.266 = \$58.341$.

ANALYSIS.—1. If the interest of $1 for one year is $0.08, for $3\frac{1}{3}$ years it will be $3\frac{1}{3}$ times $0.08, which is $0.266; and the amount is the sum of $0.266 and $1, which is $1.266.

2. If $1.266, the amount of $1 for the given time and rate, require $1 principal, $73.86 will require as many dollars principal as $1.266 is contained times in $73.86, which are 58.341.

Therefore, the amount $73.86 for 3 yrs. 4 mo., at 8 per cent., require $58.341 principal.

FORMULA: $\frac{A.}{\$1 + R. \times T.} = P.$

NOTE.—Require the pupil to write a rule from the formula.

627. What principal, in 8 years, at 8 per cent., will amount to $800.80?

628. What principal, in 4 years 6 months, at 6 per cent. interest, will amount to $80.50?

629. What principal, in 2 years 4 months 16 days, at $5\frac{1}{2}$ per cent. interest, will amount to $89.25?

630. What principal, in 5 years 9 months 7 days, at 7 per cent. interest, will amount to $148.69?

QUESTIONS.—What is a tax? (217.) How are taxes assessed? What is a poll tax? (219.) What is real estate? (220.) What is personal property? (221.) What is an inventory? (222.) What is the method of assessing taxes? (223.) For what purpose are tax tables constructed? What are duties? (224.) What is a custom house? (225.) What is a port of entry? (226.)

LESSON LXXVII.

255. Prob. IV.—Given the principal, time and interest, to find the rate per cent.?

I receive $25 interest for the use of $325 for 2 years 9 months; what is the rate per cent.?

MODEL OPERATION.

1. $\$325 \times .01 \times 2\frac{3}{4} = \$8.937.$
2. $\$25 \div 8.937 = 2.79+$ per cent.

Analysis.—1. If the interest of $325, at 1 per cent., for 1 year, is $3.25, for 2¾ years it will be 2¾ times $3.25, which is $8.937.

2. If $8.937, the interest on $325 for the given time, requires the rate of 1 per cent., $25, the interest on $325 for the same time will require as many times the 1 per cent. as $8.937 is contained times in $25, which is 2.79+.

Therefore, if I receive $25 interest for $325, for 2 years 9 months, the rate per cent. is 2.79+.

Formula: $\frac{I.}{P. \times .01 \times T.} = R.$

Note.—Require the pupil to write a rule from the formula.

631. A man pays $14 for the use of $40 for 5 years; what is the rate per cent.?

632. A man paid $10 for the use of $100 for 1 year 8 months; what is the rate per cent.?

633. At what per cent. must $875 be loaned, to gain $125 in 2 years?

634. At what rate per cent. will $4820, in 5 months, gain $18.50 interest?

635. At what per cent. will $25.48, in 3 years 3 months 12 days, give $2 interest?

636. If $246.44 are paid for the use of $840 for 5 years and 5 months, what is the rate per cent.?

637. If $28.41 are paid for the use of $120 for 2 years 1 month and 27 days, what is the rate per cent.?

638. A gentleman deposited $625 in a savings bank, from

which he receives $8.52 every 3 months; what per cent. interest does he receive?

639. If I borrow $1020 for 1 year 6 months and 15 days, and pay $85.75 interest, what is the rate per cent.?

640. A man built a vessel at the expense of $450000, and it brought him in $38,000 per year; what per cent. interest did his money yield him?

641. If a capitalist invests $18048 in railroad stock, and draws a quarterly dividend of $525.94, at what rate per cent. does he receive interest?

QUESTIONS.—What is tonnage? (227.) What is revenue? (228.) What are ad valorem duties? (229.) What are specific duties? (230.) What is an invoice? (231.) What is tare? (232.) What is leakage? (233.) What is breakage? (234.) What is gross weight? (235.) What is net weight? (236.) Give the analysis of finding the amount of duty on an invoice of goods. (237.) What is interest? (238.) What is the principal? (239.)

LESSON LXXVIII.

256. Prob. V.—Given the principal, interest and rate per cent., to find the time.

In how long a time will $343, at 7 per cent. interest, gain $90?

MODEL OPERATION.

1. $\$343 \times .07 = \$24.01$.
2. $\$90 \div 24.01 = 3.748$ yrs.+, = 3 yrs. 8 mo. 29 da.

ANALYSIS.—1. If the interest of $1 for 1 year is $0.07, for $343 it is 343 times $0.07, which is $24.01.

2. If, to gain $24.01, at 7 per cent., $343 requires 1 year to gain $90, it will require as many years as $24.01 are contained times in $90, which is 3.748+, equal to 3 years 8 months 29 days.

Therefore, it will require 3 years 8 months 29 days for $343 to gain $90, at 7 per cent.

FORMULA: $\frac{I.}{P.\times R.}=T.$

NOTE.—Require the pupil to write a rule from the formula.

642. How long will it take $450, at 6 per cent., to gain $12.48 interest?

643. How long will it take $5040, at 7 per cent., to gain $180.44 interest?

644. How long will it take $9000, at 5 per cent., to gain $430.88 interest?

645. A man loaned $848.89 at 3½ per cent., and received $98.56 interest; how long was it loaned?

646. In what time will a given principal double itself at 6 per cent. interest? at 7 per cent.? at 8 per cent.? at 9 per cent.? at 5 per cent.? at 4 per cent.? at 5½ per cent.? at 10 per cent.? at 6½ per cent.?

647. In what time will $8648.50 amount to $9988, at 7 per cent.?

648. In what time, at 7 per cent., will $1000 gain $100 interest?

649. A man loaned $8999.86 at 8½ per cent., and received $168.50 interest; how long was it loaned?

650. In what time will $21000, at 6 per cent., gain $8460.43 interest?

QUESTIONS.—What is rate per cent.? (240.) What is the amount? (241.) What is simple interest? (242.) What is compound interest? (243.) What is legal interest? (244.) Give the analysis of finding the interest on a given principal for a given time. (245.) Write a rule from the analysis. Give the analysis of finding the interest on a given principal by the 6 per cent. method. (246.) Write a rule from the analysis. Repeat the table. (247.) What are partial payments? (248.) What is an indorsement? (249.) Give the

Supreme Court rule for casting interest, when partial payments have been made. (250.)

LESSON LXXIX.

257. COMPOUND INTEREST.

258. Interest is added to the principal *annually*, *semi-annually*, or *quarterly*, as the parties agree.*

What is the compound interest on $250 for 2 years 3 months and 12 days, at 6 per cent.?

MODEL OPERATION.

1. $250×.06=$15; $250+$15=$265, am't 1st year.
2. $265×.06=$15.90; $15.90+$265=$280.90, am't 2nd year.
3. $280.90×.017=$4.775; $4.775+$280.90=$285.675, am't for 3 mo. 12 da.
4. $285.675—$250=$35.675, compound interest for 2 years 3 months 12 days.

ANALYSIS.—1. If the interest on $1 for one year is $0.06, on $250 it is 250 times $0.06, which is $15; $15 added to the principal equals $265, the amount.

2. If the interest on $1 for one year is $0.06, on $265 it is 265 times $0.06, which is $15.90; this added to the principal equals $280.90, the amount.

3. If the interest on $1 for 3 months and 12 days is $0.017, for $280.90 it is 280.90 times $0.017, which is $4.775, which added to the principal gives $285.675.

4. If the amount for 2 yrs. 3 mo. 12 da. is $285.675, and the principal is $250, the interest is the difference of these numbers, which is $35.675.

Therefore, the compound interest on $250 for 2 yrs. 3 mo. 12 da., at 6 per cent., is $35.675.

FORMULA: A.—P.=I.

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* NOTE.—It is not legal to receive compound interest, but if annual, semi-annual, or quarterly interest is mentioned in the note, it can be collected when due.

(*a*.) Rule.—I. *Find the amount of the given principal for one year, and make this amount the principal for the second year.*

II. *Find the amount of this principal, for one year, and make it the principal for the third year; and so continue for the given number of years.*

III. *Subtract the given principal from the final amount, and the remainder will be the required compound interest.*

Questions.—How many elements in the problems in interest, and what are their symbols? (251.) Write a rule for problem I. from the formula. (252.) For problem II. (253.) For problem III. (254.) For problem IV. (255.) For problem V. (256.) What is compound interest? (257.) Give the analysis. (258.) Give the rule. (258., *a*.)

LESSON LXXX.

EXAMPLES FOR PRACTICE.

651. What is the compound interest on $100, at 6 per cent., for 3 years 2 months and 18 days?

652. What is the compound interest on $800.63, at 7 per cent., for 4 years 2 months and 28 days?

653. What is the compound interest on $2642.49, at 5½ per cent., for 5 years?

654. What is the compound interest on $100 for 4 yrs., payable semi-annually?

655. What is the amount of $8064.21, at 9 per cent., for 3 yrs. 9 mo. 8 days?

656. What is the amount of $6840.29 for 4 yrs. 3 mo., payable quarterly?

657. What is the amount of $963.87, at 8½ per cent., for 4 yrs. 3 mo. 27 da., payable semi-annually?

658. What is the compound interest on $428.75 for 5 years 11 months and 25 days, at 4½ per cent.?

659. What is the compound interest on $2496.94, at 7 per cent., for 5 years 7 months and 11 days?

660. What is the compound interest on $962.48, at 6 per cent., for 4 years 8 months and 28 days?

661. What is the compound interest on $962.48, at 6 per cent., for 4 years 8 months and 28 days, payable quarterly?

LESSON LXXXI.

259. Discount is a deduction or allowance made for the payment of a debt before it is due. Its symbol is D.

260. The **Present Worth** of a sum due at a future time without interest, is such a sum as, being placed at interest, will equal the given debt when it becomes due. The symbol is P–W. The terms *present worth*, *discount*, and *debt*, are equivalent to *principal*, *interest*, and *amount*.

I hold a note of $320 against Mr. Jones: it is made payable in 2 years 3 months and 15 days, without interest; what is its present worth, the rate of interest being 8 per cent.? What allowance must I make for its immediate payment?

MODEL OPERATION.

1. $\$320 \div \$1.18\frac{1}{3} = \$270.422$, P.–W.
2. $\$320 - \$270.422 = \$49.578$, D.

ANALYSIS.—If the amount of $1 for 2 years 3 months 15 days is $1.18⅓, the present worth of $1.18⅓ is $1.

1. If the present worth of $1.18⅓ is $1, the present worth of $320 is as many dollars as $1.18⅓ is contained times in $320, which are 270.422.

2. If the present worth of $320 is $270.422, the discount will be the difference of these quantities, which is $49.578.

I. FORMULA:* $\dfrac{\text{A.}}{\text{R.} \times \text{T.} + \$1} = \text{P–W.}$

II. FORMULA:* A.—P–W.=D.

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*Formulas I., and II., correspond with the formulas of articles 254 and 258 respectively.

(*a.*) RULE.—I. *Divide the debt by the amount of* $1 *for the given rate and time, and the quotient will be the present worth.*

II. *Deduct the present worth from the given debt, and the remainder will be the discount.*

QUESTIONS.—Deduce a rule from the formula for Prob. I. (252.) From the formula for Prob. II. (253.) From the formula for Prob. III. (254.) From the formula for Prob. IV. (255.) From the formula for Prob. V. (256.) Give the analysis of an example of compound interest. (258.)

LESSON LXXXII.

662. What is the present worth of $92.60, payable in 40 days?

663. What is the present worth of $62.22, payable in 2 months and 15 days?

664. What is the present worth of $124.45, payable in 2 years 2 months and 9 days, the rate of interest being 8 per cent.?

665. What is the discount of $420.25, due 4 years and 6 months hence?

666. What is the discount of $678.45, due 2 years 4 months and 18 days hence?

667. I bought $520 worth of goods on 1 year and 4 months credit; what sum will pay the debt now?

668. A man was offered for a farm $2244 in cash, or $2442 payable in 1 year and 8 months without interest; he chose the latter; how much did he lose, supposing the note to be discounted at the rate of 12 per cent. per annum?

669. What is the discount on a draft for $1250, payable in 1 month and 15 days, at 4½ per cent.?

670. What is the difference between the discount on

$250 for 1 year, and the interest on $250 for the same time?

671. I bought a bill of goods on 3 months' credit, amounting to $486.75. If the cash is paid at the time of receiving the goods, how much ought to be deducted, the rate of interest being 7 per cent.?

672. What is the difference between the discount on $15000 for 3 years 4 months and 9 days, and the interest on $15000 for the same time, both at 9 per cent.?

673. What is the difference between the discount on $8000 for 2 years 9 months and 21 days, and the interest on the same amount for the same time?

674. A young lady has a legacy of $4844, to be paid to her at 21 years of age; what should be the discount, the interest being 8 per cent., if the legacy is paid her when she becomes 18 years of age?

675. I bought a bill of goods of $2800, one half on a credit of 6 months, and the other half on a credit of 8 months. If payment is made at the time of the purchase, how much ought to be deducted, the interest being at 7 per cent.?

QUESTIONS.—What is per cent.? (183.) How may per cent. be expressed? (184.) Give the analysis of finding any per cent. of a number. (185.) Give the analysis of finding what per cent. one number is of another. (186.) Give the analysis of finding a number when a certain per cent. of it is given. (187.) What is an agent, or factor? (188.) What is commission? (189.) What is brokerage? (190.) What is a corporation? (193.) What is a charter? (192.)

LESSON LXXXIII.

BANKING.

261. A **Bank** is a joint stock company organized for the purpose of receiving on deposit, and loaning money, and issuing notes for currency.

262. Bank Discount is the amount charged by a bank for the use of money, and it is always paid in advance.

263. A **Promissory Note** is a written engagement to pay a certain amount, either on demand, or at some future specified time, and in banking business generally draws no interest until due.

264. Bank Notes, or **Bank Bills,** are promissory notes without interest, issued by a bank and made payable to the bearer, on demand, in specie, at the bank which issued them. They are circulated as currency, or money.

265. Days of Grace are the three days allowed by law for the payment of a note after it becomes due.

266. The **Maturity** of a note is the expiration of the time specified for its payment, together with the days of grace allowed by law.

267. The **Proceeds** of a note is the amount received for it when discounted at a bank, and this amount is equal to the face of the note after the discount is deducted.

(*a*) NOTES.—1. When a note is *on interest*, due at a future time, the *amount* at the time due, including 3 days of grace, is considered the face of the note.

2. Banks are organized somewhat on the following plan: A and B propose to organize a bank; they first obtain a charter or privilege from the legislature, and then deposit bonds, notes, or other securities, to a certain amount, in the hands of a person appointed by government to receive them. They then issue notes to a certain amount less than that of the securities deposited for their redemption, payable to the bearer on demand in specie at the bank. These notes circulate as money.

QUESTIONS.—What is a firm? (191.) What is capital, or stock? (194.) What is a share? (195.) What are stockholders? (196.) When is stock said to be at par? (197.) When is stock said to be above par? (198.) When is stock said to be below par? (199.)

LESSON LXXXIV

268. To find the bank discount and the proceeds of a note.

What is the bank discount of $350, at 90 days? what are the proceeds?

MODEL OPERATION.

$\$350 \times \$0.015\frac{1}{2} = \$5.425$, the discount for 93 days.

$\$350 - \$5.425 = \$344.575$, the proceeds.

NOTE.—It is the custom of banks to make the discount of a note equal to the simple interest for three days more than the time specified in the note. Hence the following

(*a*.) RULE.—I. *Compute the interest on the face of the note to maturity; the result will be the bank discount.*

II. *From the face of the note deduct the bank discount, and the remainder will be the proceeds.*

676. What is the discount of a note of $375, at 90 days, discounted at a bank at 7 per cent.?

677. What are the proceeds of a note of $437, at 60 days, discounting at a bank, at 8 per cent.?

678. What shall I receive from a note due in 30 days, discounted at 6½ per cent.?

679. What are the proceeds of a note due in 90 days, discounted at a bank at 5 per cent.?

680. I sold a quantity of merchandise worth $240, and in payment took a note for $275, at 90 days, which I had discounted at a bank; how much did I make by the operation?

681. A man paid $375 cash for a span of horses, and sold them immediately for a note of $400; he took this note to a bank and had it discounted at 90 days; how much did he gain by the sale of his horses?

682. A man bought a house for $8741, and gave his note at 90 days for that amount; how much ready money did his house cost him?

683. A man sold a farm for $8471, and took a note at 90 days for the amount; he had the note discounted at a bank; how much ready money did he realize from his farm?

684. $300. NEW ORLEANS, LA., Oct. 8, 1863.

Ninety days from date I promise to pay to the order of Luther Bryant, at the Farmers' Bank, three hundred dollars, value received.

JOHN BROWN.

This note was discounted Oct. 15, 1863; what were the proceeds?

685. $530. BOSTON, MASS., Jan. 1, 1864.

Nine months after date I promise to pay Allen Wild, or order, five hundred and thirty dollars, with interest, value received.

H. H. WILLIAMSON.

This note was discounted March 13, 1864, at 8 per cent.; what were the proceeds? (See 267. 1.)

LESSON LXXXV.

269. To find the face of a note, when the proceeds, time and rate per cent. are given.

For what amount must I draw my note, to realize $500 at a bank, discounting at 90 days?

MODEL OPERATION.

1. $\$1 \times 0.015\frac{1}{2} = \$0.015\frac{1}{2}$, the discount on $1 for the giver time.
2. $\$1 - \$0.015\frac{1}{2} = \$0.984\frac{1}{2}$, the proceeds of $1.
3. $\$500 \div 0.984\frac{1}{2} = \$507.872$, the face of the required not

Analysis.—1. Find the bank discount of $1 for 93 days.

2. Find the proceeds of $1 for 93 days.

3. Find the amount of the required note.

If $0.984\frac{1}{2}$ proceeds require $1 of the required note, $500 proceeds will require as many dollars as $0.9845 are contained times in $500, which are 507.872.

Therefore, to realize $500 at a bank, discounting at 90 days, my note must be drawn for $507.872.

Note.—Require the pupil to write a rule from the analysis.

686. What should be the face of a note at 30 days, to realize $574, when discounted at 10 per cent.?

687. What must be the face of a note at 90 days, to realize $870, when discounted at 8 per cent.?

688. What must be the face of a bankable note that, when discounted for 5 months, at 7 per cent., will have a present worth of $94.50?

689. I sold a span of horses for $389 cash; should I take a note for 2 months at 6 per cent., for how much must the note be drawn that by having it discounted I may realize my price?

690. What must be the face of a bankable note which, when discounted at 3 months and 8 days, gives a present worth of $528, the interest being 6 per cent.?

691. For what sum must I draw my note, so that the bank proceeds, at 7 per cent. at 60 days, may be $400?

692. I bought, in New York, merchandise to the amount of $5387 cash. Not having the money, I gave my note, payable in 60 days; what must be the face of it, the discount being at 6 per cent.?

693. A merchant bought merchandise to the amount of $587 cash; what must be the face of a note on interest for 3 months, that will meet the demand, the discount being at 7 per cent.?

QUESTIONS.—What is the market value of stock? (200.) What is a dividend? (201.) Give the analysis of the operation of finding the commission on any amount or quantity. (202.) Give the analysis of finding the commission, when deducted from the remittance. (203.) Give the analysis of finding the value of stock when at a premium or discount. (204.) Give the analysis of finding what quantity of stock can be purchased for a certain sum. (205.)

LESSON LXXXVI.

EXCHANGE.

270. **Exchange** is the process of remitting value from one place to another by means of written orders.

271. A **Bill of Exchange** is a written order addressed to a person residing at a distance, directing him to pay to the person in whose favor the bill is drawn or his order, a certain sum of money, at a given time.

272. The **Drawer,** or **Maker,** is the person who draws the bill, or orders the money to be paid.

273. The **Drawee** is the person who is ordered to pay the bill.

274. The **Payee** is the person to whom the bill is ordered to be paid.

275. The **Buyer,** or **Remitter,** is the person who purchases a bill of exchange.

276. An **Acceptance** is the promise of the drawee to pay the bill when due, and it is usually made by writing *accepted* and the signature of the drawee, on its face.

277. An **Indorsement** of a bill is a writing upon

(*a.*) NOTE.—When the payee simply writes his name on the back of the bill it is transferred to *bearer*, but if he prefixes to his signature "Pay to the order of John Edmonds" then the bill is transferred to Mr. Edmonds, who, when he collects it, must in turn indorse it. Indorsers are separately responsible for the amount of the bill in case the *drawee* fails to make payment. A bill made payable to *bearer* may be transferred without indorsement.

its back, by which the payee transfers the bill to another person.

278. A **Domestic Bill** is one whose drawer and drawee reside in the same country.

279. A **Foreign Bill** is one whose drawer and drawee reside in different countries.

280. The **Course of Exchange** is the variation between the face of a bill and its cost. It may be either at a premium or at a discount.

281. Exchange is said to be at PAR when the cost equals the face of the bill.

282. Exchange is said to be at a PREMIUM, or ABOVE PAR, when the cost is greater than the face of the bill.

283. Exchange is said to be at a DISCOUNT, or BELOW PAR, when the cost is less than the face of the bill.

QUESTIONS.—What is discount? (259.) What is present worth? (260.) What is a bank? (261.) What is bank discount? (262.) What is a promissory note? (263.) What are bank notes? (264.) What are days of grace? (265.) What is the maturity of a note? (266.) What are the proceeds of a note? (267.) What is considered the face of a note when on interest? (267., *a.* 1.) How are banks organized? (267., *a.* 2.)

LESSON LXXXVII.

284. To find the cost of a draft in domestic exchange when at a premium or at discount.

ROCHESTER, N. Y., Mar. 13, 1864.

At sight pay to John Clark or order five hundred and fifty dollars, value received, and charge the same to our account.

F. SHAW & CO.

To Messrs. DANIEL PARKS & CO.,
Washington, D. C.

What is the cost of the above draft, the rate of exchange being 3 per cent. discount?

MODEL OPERATION.

1. \$1—\$0.03=\$0.97, cost of \$1.
2. \$550×0.97=\$533.50, cost of draft.

ANALYSIS.—1. If the exchange is at 3 per cent. discount, \$1 will cost the difference between \$1 and \$0.03, which is \$0.97.

2. If \$1 costs \$0.97, \$550 will cost 550 times \$0.97, which are \$533.50.

Therefore, at 3 per cent. discount, a draft of \$550 will cost \$533.50.

(*a.*) RULE.—I. *Find the cost of one dollar by adding the per centage to one dollar, when at a premium; or subtracting it from one dollar, when at a discount.*

II *Multiply the cost of one dollar by the face of the draft, and the product will be the cost required.*

CHARLESTON, S. C., Aug. 9, 1864.

Sixty days after sight, pay to J. H. Zelie or bearer, four hundred eighty dollars, value received, and charge the same to the account of

L. C. G. KSHINKE.

To WM. H. ALBRO,
New York City.

What is the cost of the above draft, at 5 per cent. premium?

MODEL OPERATION.

\$1.—\$0.01225=\$0.98775, the proceeds of \$1 for the given rate and time.

\$0.98775+.05=\$1.03775, the cost of \$1 of the draft.

\$1.03775×480=\$498.12, the cost of the draft.

ANALYSIS.*—1. If the bank discount of $1. at the given rate and time is $0.01225, the proceeds of $1 will be the difference between $1. and $0.01225, which is $0.98775.

2. If the proceeds of $1. is $0.98775, and the rate of exchange is 5 per cent. premium, the cost of $1. will be the sum of $0.98775 and $0.05, which is $1.03775.

3. If the cost of $1. of the draft is $1.03775, the cost of $480. will be 480 times $1.03775, which is $498.12.

Therefore, the cost of a draft of $480. 60 days after sight, exchange being at 5 per cent. premium, is $498.12.

When the draft is on time after sight we have the following

(*b*.) RULE.—I. *Find the proceeds of* $1. *at the given rate and time, including* 3 *days of grace.*

II. *When the rate of exchange is at a premium, add the rate per cent. to the proceeds; if at a discount, subtract the rate per cent. from the proceeds, which will give the cost of* $1.

III. *Multiply the cost of* $1 *by the face of the draft, and the result will be the cost of the draft.*

QUESTIONS.—What is profit and loss? (206.) Give the analysis of finding the amount of gain or loss, the cost and gain or loss per cent. being given. (207.) Deduce a rule from the analysis of finding the gain or loss per cent., when the cost and selling price are given. (208.) Deduce a rule from the analysis of finding the selling price, when the cost and gain or loss per cent. are given. (209.) Deduce a rule from the analysis of finding the cost, when the selling price and gain or loss per cent. are given. (210.) What is insurance? (211.) What is the insurer? (212.) Who are the insured? (213.) What is a policy? (214.) What is the premium? (215.)

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* NOTES—1 Since time is allowed on the draft after it is presented for payment, it must suffer discount from sale in a bank · consequently bank discount must be deducted

2 In practice drafts are usually drawn at sight, 30, 60, or 90 days, and each are quoted at a certain per cent premium or discount

LESSON LXXXVIII.

EXAMPLES FOR PRACTICE.

694. NEW ORLEANS, LA., Jan. 8, 1864.

At sight pay to J. S. Herbert or order eight thousand forty-eight dollars, value received, and charge the same to our account.

A. B. COHU & Co.

To Messrs. CHARLES F. MAWBEY & Co.,
New York City.

What is the cost of the above draft, the rate of exchange being 4 per cent. discount?

695. PHILADELPHIA, PA., Nov. 8, 1864.

Thirty days from sight pay to Sanford Landt or order nine hundred eighty dollars, value received, and charge the same to our account.

LYDMEN A. PARKS & Co.

To Messrs. OTIS T. HALL & Co.,
New Orleans.

What is the cost of the above draft, at 4 per cent. premium?

696. A merchant in Chicago wishes to remit a draft of $2000 to his agent in New York; what will it cost, exchange being at 3 per cent. premium?

697. What will be the cost of a Charleston draft on New York for $550, exchange being at 10 per cent. premium?

698. What is the cost of a New York draft on Savannah for $8646, exchange being 15 per cent. discount?

699. A merchant in Richmond, Va. orders goods from New York to the amount of $5000, which amount he re-

mits by draft, exchange being at 7 per cent. premium; what will the goods cost him in Richmond?

700. A merchant in Mobile wishes to remit $387.88 to Charleston; what will a draft cost, at 2⅗ per cent. discount?

701. What is the amount of a bill on New York that $3674 will purchase, if exchange is at 6 per cent. discount?

702. What is the amount of a bill on Montgomery that $387.43 will purchase, if exchange is at 6 per cent. premium?

LESSON LXXXIX.

285. To find the cost of a Foreign bill of exchange, when at a premium or at a discount.

286. In all bills of exchange on England, the pound sterling is reckoned at its former, or nominal value of $4⁴⁄₉, instead of its present value of $4.84. The real value is 9 per cent. above the nominal value.

287. In all bills of exchange on France, the franc is reckoned at the value of $0.186.

TABLE OF EXCHANGE.

£1 = $4.444+, nominal value.
£1 = 4.84, true value.
1 Franc = .186, value of exchange.

(*a.*) NOTES.—1. Bills of exchange in England are drawn in pounds, shillings and pence, and exchange is quoted at a certain per cent. upon the nominal value; hence, 9 per cent. *premium* is really *par* value, and 9¾ per cent. premium is really ¾ per cent. premium.

2. Bills of exchange on France are drawn in francs, and exchange is quoted at so many francs and centimes to the dollar.

3. Foreign bills are usually drawn in sets of the same tenor, and date, each containing a condition that it shall be payable only while the others remain unpaid.

QUESTIONS.—Deduce a rule from the analysis of finding the premium, when the amount and rate per cent. are given. (216.) What is a tax? (217.) How are taxes assessed? (218.) How is a poll tax estimated? (219.) What is real estate? (220.) What is personal property? (221.) What is an inventory? (222.) What is the method of assessing taxes? (223.) What are duties? (224.) What is a custom house? (225.) What is a port of entry? (226.) What is tonnage? (227.) What is revenue? (228.) How are ad valorem duties estimated? (229.) How are specific duties estimated? (230.) What is an invoice? (231.)

LESSON XC.

288. To find the cost of a bill of exchange on England, when at a premium, or at a discount.

£347. 7s. 8d. NEW YORK, Oct. 13, 1864.

At sight of this my first of exchange (the second and third of the same date and tenor unpaid) pay to L. K. Dennis or order three hundred forty-seven pounds seven shillings eight pence, value received, and place the same to my account.

JONAS J. JONES.

To Messrs. ROTHSCHILD & Co.,
London, England.

What is the cost of the above bill, when exchange is quoted at 43 per cent. premium?

MODEL OPERATION.

1. 347£. 7s. 8d.=347.388£.
2. 1£.+.43£=1.43£., the cost of 1£. of the bill.
3. 1.43£.×347.388=496.764£., the cost of the bill.
4. 496.764£.×4$\frac{4}{9}$=$2207.84, the cost in U. S. currency.

ANALYSIS.—1. 347£. 7s. 8d. equals 347.388£.

2. If the rate of exchange is 43 per cent. premium, £1. will cost the sum of 1£. and .43£., which is 1.43£.

3. If 1£. costs 1.43£., 347.388£. will cost 347.388 times 1.43£., which are 496.764£., the cost of the bill.

4. If, 1£. costs $\$4\frac{4}{9}$, 496.764£. will cost 496.764 times $\$4\frac{4}{9}$, which are $2207.84, the cost in U. S. currency.

Therefore, the bill of exchange on England, when exchange is at 43 per cent. premium, will cost $2207.84.

(*a.*) RULE.—I. *Multiply the amount of the bill expressed in pounds and decimals of a pound by the cost of one pound at the given rate of exchange, and the product will be the cost of the bill.*

II. *Multiply the cost of the bill by* $\$4\frac{4}{9}$, *the nominal value of a pound, and the result will be the cost of the bill in dollars.*

703. What will be the cost in U. S. currency of a bill of exchange on Liverpool of 371£. 8s. 9d., the rate of exchange being 25 per cent. premium?

704. What will be the cost of a bill of exchange on London of 321£. 2s. 3d., the rate of exchange being 6 per cent. premium?

705. I wish to remit 347£. to my agent in Manchester; what will a bill of exchange cost in New York, the rate of exchange being 20 per cent. discount?

LESSON XCI.

289. To find the amount of a bill of exchange on England, which can be purchased for a given sum of U. S. currency.

What is the amount of a bill of exchange on Liverpool, which I can purchase for $1000, the rate of exchange being 11 per cent. premium?

MODEL OPERATION.

1. 1£.+.11=1.11£., the cost of one pound.
2. $1.11 \times \$4\frac{4}{9} = \$4.933$, the cost of 1£. in U. S. currency.
3. $1000 \div \$4.933 = £\,202.7163, = £\,202 + 14s. + 3d.$, the amount of the required bill.

ANALYSIS.—1. Find the cost of 1£.

2. Find the cost of 1£. in U. S. currency.

3. Find the amount of the bill.

If $4.933 at the given rate of exchange are required to purchase 1£.; as many pounds can be purchased for $1000 as $4.933 are contained times in $1000, which are 202.7163, equal to 202£. 14s. 3d.

Therefore, $1000 will purchase a bill of exchange in Liverpool to the amount of 202£. 14s. 3d., if the rate of exchange is 11 per cent. premium.

(*a*.) RULE.—*Divide the given sum by the value of one pound at the given rate of exchange, and the quotient will be the amount in pounds and decimals of a pound.*

706. I wish to make a remittance to London of $4673, exchange being at 8¾ per cent. premium; what is the amount of the bill that I can remit for that sum?

707. A merchant wishes to send $4834 to his agent in Liverpool, with which to purchase goods; what is the amount of the bill which he can remit for that sum, the exchange being 37½ per cent. premium?

708. What is the amount of a bill of exchange on London, that I can purchase for $3783, exchange being at 13 per cent. premium?

QUESTIONS.—What is tare? (232.) What is leakage? (233.) What is breakage? (234.) What is gross weight? (235.) What is net weight? (236.) Deduce a rule from the analysis for finding the amount of duty on an invoice of merchandise. (237.) What is interest? (238.) What is the principal? (239.) What is the rate per cent.? (240.) What is the amount? (241.) What is simple interest? (242.) What is compound interest? (243.) What is legal interest? (244.) Deduce a rule for finding the interest on a given principal at a given time at a given per cent. (245.)

LESSON XCII.

290. To find the cost of a bill of exchange on France.

What must be paid for a bill on Paris of 234.31 francs, exchange being 5.15 francs per dollar?

MODEL OPERATION.

234.31 fr.÷5.15=$45.497, the cost of the bill.

(*a*.) Analysis.—If 5.15 fr. cost $1, at the given rate of exchange, 234.31 fr. will cost as many dollars as 5.15 fr. are contained times in 234.31 fr., which are 45.497.

Therefore, a bill of exchange on Paris, of 234.31 fr., at 5.15 fr. per dollar, will cost $45.497.

709. What must be paid for a bill on Paris of 34087 francs, the exchange being 51⅓ francs per dollar?

710. What is the value of a bill on Paris of 83090 fr., exchange being 11.31 francs per dollar?

291. To find the amount of a bill on France, that can be purchased for a given sum of U. S. currency.

I wish to remit $500 to Paris; what will be the amount of the bill in francs, the exchange being 7.5 fr. per dollar?

MODEL OPERATION.

500×7.5 fr.=3750 fr., the amount of the bill.

(*a*.) Analysis.—If $1 at the given rate of exchange will purchase 7.5 fr., $500 will purchase 500 times 7.5 fr., which are 3750 fr.

Therefore, $500 will purchase a bill to the amount of 3750 fr., when the exchange is 7.5 fr. per dollar.

711. A merchant has $1374 with which to purchase a bill of exchange on Paris; what will be the amount, the exchange being 5.14 fr. per dollar?

712. What will be the amount of the bill of exchange on Paris that can be purchased with $5000, exchange being 13.14 fr. per dollar?

QUESTIONS.—Deduce a rule from the analysis for finding the interest on a given principal, for a given time, at a given per cent., by the 6 per cent. method. (247.) What are the established rates of interest in the several states? (244., *a.*) What is a partial payment? (248.) What is an indorsement? (249.) What is the Supreme Court rule for finding the amount of interest on notes, when partial payments have been made? (250.)

LESSON XCIII.

EQUATION OF PAYMENTS.

292. Equation of Payments is the process of finding the *time* in which several amounts due at different times without interest may be paid without loss to either party.

293. The **Term of Credit** is the time from the incurring of the debt to the date at which it becomes due.

294. The **Equated Time** is the date at which the several debts may be paid without loss to either debtor or creditor.

295. An **Account** is a statement of items, or a record of mercantile transactions.

296. The **Balance** of an account is the difference between the debits and the credits.

297. To find the equated time of payment of an account, when the items are reckoned from the same date.

On Mar. 1, 1864, I owe Mr. Blore 3 dollars to be paid in 3 months, 5 dollars to be paid in 8 months, and 12 dollars to be paid in 9 months; at what time may I pay the whole amount without loss to either party?

MODEL OPERATION.

$\$\ 3 \times 3\ \text{(mo.)} = \$1 \times 9\ \text{(mo.)}$

$\$\ 5 \times 8\ \text{(mo.)} = \$1 \times 40\ \text{(mo.)}$

$\$12 \times 9\ \text{(mo.)} = \$1 \times 108\ \text{(mo.)}$

$\$20 \qquad 157\ \text{mo.} \div 20 = 7.85 = 7\ \text{mo. } 25\ \text{da.}$

Mar. 1, 1863+7 mo. 25 da.=Oct. 26, 1868, Ans.

Analysis.*—1. The interest of $3 for 3 mo. is equal to the interest of $1 for 3 times 3 mo., or 9 mo.

2. The interest of $5 for 8 mo. is equal to the interest of $1 for 5 times 8 mo., or 40 mo.

3. The interest of $12 for 9 mo. is equal to the interest of $1 for 12 times 9 mo., or 108 mo.

4. The interest of $3, $5, and $12, to the times of their payment is equal to the interest of $1 for the sum of 9 mo., 40 mo., and 108 mo., (157 mo.)

5. If $1 requires 157 mo. to gain a certain amount of interest, $20 will require $\frac{1}{20}$ of 157 mo., which is 7.85 mo., equal to 7 mo. 25 da., which is the average term of credit.

6. 7 mo. and 25 da. after Mar. 1, 1863, falls on Oct. 26, 1863, which is the equated time of payment.

(*a.*) Rule.—I. *Multiply each payment by the time to the date at which it becomes due, divide the sum of the products by the sum of the payments, and the result will be the average term of credit.*

II. *Add the average term of credit to the date at which the items become due and the sum will be the equated time.*

Questions.—What is the rule for computing bank discount? (268., *a.*) What is the rule for finding the proceeds? (268., *a.*) Deduce a rule from analysis for finding the face of a note, when the proceeds, time, and rate per cent. are given. (269.) What is exchange? (270.) What is a bill of exchange? (271.) What is the drawer? (272.)

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*Notes.—This method of averaging accounts is used on the principle of bank discount; that is, that the discount equals the interest.

2. When a payment is to be paid down it has no product, but it must be added to the other payments.

LESSON XCIV.

713. HARTFORD, CONN., July 17, 1864.

J. H. HEADY,

Bought of H. TILESON & Co.

9 hhds. of molasses,	@ $0.57	per gal.,	on 2 months;
31 bbls. of sugar,	@ $25		on 3 months;
25 bbls. of flour,	@ $12		on 4 months;
13 chests of tea,	@ $50		on 6 months.

What is the equated time of paying the entire bill?

714. Theodore Church sold a farm for $4000, $1000 of which was to be paid down, $375 to be paid in 3 months, $2275 in 1 year, and the remainder in 2 years; he afterward agreed to take a note; for what time must the note be given?

298. To find the equated time of an account when the items are reckoned from different dates.

What is the equated time of the following bill?

NEW YORK, Jan. 1, 1863.

M. F. SIMMONS,

1863. To L. G. HAMMOND, Dr.

Aug. 3,	To	Cash,	$500.
" 19,	"	Mdse. on 3 mos.,	$375.
Sept. 11,	"	Cash,	$590.
Dec. 25,	"	Mdse. on 9 mos.,	$100.

MODEL OPERATION.

1863.		(da.)		(da.)
Due Aug. 3,	$500 ×	0 =	$1 ×	0
" Nov. 19,	$375 ×	106 =	$1 ×	39750
" Sept. 11,	$590 ×	38 =	$1 ×	22420
1864.				
" Sept. 25,	$100 ×	412 =	$1 ×	41200
	$1565			103370

da. ($.) da.
103375 ÷ 1565 = 66 +*

Aug. 3, 1863, + 66 da. = Oct. 9, 1863. } Ans.

*NOTE.—When the remainder equals or exceeds a half a day, add it. When less, reject it.

ANALYSIS.—1. For convenience we select the earliest date at which any of the items becomes due from which to calculate the terms of credits, which is Aug. 3, 1863, consequently $500 has no credit.

2. If $375 is payable 3 months from Aug. 19, it must become due Nov. 19, and the credit extends from Aug. 3, to Nov. 19, which is 106 da.; and the credit of $375 for 106 da. is equal to the credit of $1 for 39750 da.

3. If $590 is due Sept. 11, the credit extends from Aug. 3, to Sept. 11, which is 38 da.; and the credit of $590 for 38 da. is equal to the credit of $1 for 22420 da.

4. If $100 is payable 9 mo. from Dec. 25, 1863, it must become due Sept. 25, 1864; and the credit extends from Aug. 3, 1863, to Sept. 25, 1864, which is 412 da.; and the credit of $100 for 412 da. is equal to the credit of $1 for 41200 da.

5. The credit of $1565 (the sum of the items) to the times of payment is equal to the credit of $1 for 103370 da. (the sum of the credits of $1.)

6. If the credit of $1 requires 103370 da. to be worth a certain sum; to equal the same sum, $1565 will require $\frac{1}{1565}$ of 103370 da., which is 66 da., the average term of credit.

7. The 66th da. after Aug. 3, 1863, is Oct. 9, 1863.

Therefore, the equated time of payment is Oct. 9, 1863.

(*a*.) RULE.—I. *Assume the earliest date at which any of the items become due, from which to calculate the terms of credit.*

II. *Calculate the terms of credit from this date to the time when each item becomes due.*

III. *Multiply each item by its own term of credit; divide the sum of the products by the sum of the items, and the quotient will be the average term of credit.*

For convenience in finding the number of days between two dates we give the following

(*b*.) TABLE.

Jan.	Feb.	Mar.	Apr.	May.	June.	July.	Aug.	Sept.	Oct.	Nov.	Dec.
1	32	60	91	121	152	182	213	244	274	305	335
2	33	61	92	122	153	183	214	245	275	306	336
3	34	62	93	123	154	184	215	146	276	307	337
4	35	63	94	124	155	185	216	247	277	308	338
5	36	64	95	125	156	186	217	248	278	309	339
6	37	65	96	126	157	187	218	249	279	310	340
7	38	66	97	127	158	188	219	250	280	311	341
8	39	67	98	128	159	189	220	251	281	312	342
9	40	68	99	129	160	190	221	252	282	313	343
10	41	69	100	130	161	191	222	253	283	314	344
11	42	70	101	131	162	192	223	254	284	315	345
12	43	71	102	132	163	193	224	255	285	316	346
13	44	72	103	133	164	194	225	256	286	317	347
14	45	73	104	134	165	195	226	257	287	318	348
15	46	74	105	135	166	196	227	258	288	319	349
16	47	75	106	136	167	197	228	259	289	320	350
17	48	76	107	137	168	198	229	260	290	321	351
18	49	77	108	138	169	199	230	261	291	322	352
19	50	78	109	139	170	200	231	262	292	323	353
20	51	79	110	140	171	201	232	263	293	324	354
21	52	80	111	141	172	202	233	264	294	325	355
22	53	81	112	142	173	203	234	265	295	326	356
23	54	82	113	143	174	204	235	266	296	327	357
24	55	83	114	144	175	205	236	267	297	328	358
25	56	84	115	145	176	206	237	268	298	329	359
26	57	85	116	146	177	207	238	269	299	330	360
27	58	86	117	147	178	208	239	270	300	331	361
28	59	87	118	148	179	209	240	271	301	332	362
29		88	119	149	180	210	241	272	302	333	363
30		89	120	150	181	211	242	273	303	334	364
31		90		151		212	243		304		365

NOTES.—To find from the above table the number of days between two dates, is given the following

(*a.*) RULE.—I. *When the dates are in the same year, subtract the number of days in the place of the earlier date from the number of days of the later date; the result will be the number of days required.*

II. *When the dates are in consecutive years, subtract the number of days in the earlier date from 365, and add to the remainder the number of days in the place of the later date; the result will be the number of days required.*

(*b.*) *When the year is a leap year, add one day to the result.*

For convenience in finding the interest from one to $5000 from one to 2000 days, at 6 or 7 per cent., we give the following

(c.) TABLE.*

$	1da	2	3	4	5	6	7	8	9	10	20	30	40	50	60	70	80	90	100	200
1	6 per ct.		.000	.001	.001	.001	.001	.001	.001	.002	.003	.005	.007	.008	.010	.012	.013	.015	.017	.033
	7 per ct.		.001	.001	.001	.001	.001	.002	.002	.002	.004	.006	.008	.010	.012	.014	.016	.017	.019	.039
2	.000	.001	.001	.001	.002	.002	.002	.003	.003	.003	.007	.010	.013	.017	.020	.023	.027	.030	.033	.067
	.000	.001	.001	.002	.002	.002	.003	.003	.003	.004	.008	.012	.016	.019	.023	.027	.031	.035	.039	.078
3	.000	.001	.001	.002	.002	.003	.003	.004	.004	.005	.010	.015	.020	.025	.030	.035	.040	.045	.050	.100
	.001	.001	.002	.002	.003	.003	.004	.005	.005	.006	.012	.017	.023	.029	.035	.041	.047	.052	.058	.117
4	.001	.001	.002	.003	.003	.004	.005	.005	.006	.007	.013	.020	.027	.033	.040	.047	.053	.060	.067	.133
	.001	.002	.002	.003	.004	.005	.005	.006	.007	.008	.016	.023	.031	.039	.047	.054	.062	.070	.078	.156
5	.001	.002	.002	.003	.004	.005	.006	.007	.007	.008	.017	.025	.033	.042	.050	.058	.067	.075	.083	.167
	.001	.002	.003	.004	.005	.006	.007	.008	.009	.010	.019	.029	.039	.049	.058	.068	.078	.087	.097	.194
6	.001	.002	.003	.004	.005	.006	.007	.008	.009	.010	.020	.030	.040	.050	.060	.070	.080	.090	.100	.200
	.001	.002	.003	.005	.006	.007	.008	.009	.010	.012	.023	.035	.047	.058	.070	.082	.093	.105	.117	.223
7	.001	.002	.003	.005	.006	.007	.008	.009	.010	.012	.023	.035	.047	.058	.070	.082	.093	.105	.117	.223
	.001	.003	.004	.005	.007	.008	.010	.011	.012	.014	.027	.041	.054	.068	.082	.095	.109	.122	.136	.272
8	.001	.003	.004	.005	.007	.008	.009	.011	.012	.013	.027	.040	.053	.067	.080	.093	.107	.120	.133	.267
	.002	.003	.005	.006	.008	.009	.011	.012	.014	.016	.031	.047	.062	.078	.093	.109	.124	.140	.156	.311
9	.001	.003	.004	.006	.007	.009	.010	.012	.013	.015	.030	.045	.060	.075	.090	.105	.120	.135	.150	.300
	.002	.003	.005	.007	.009	.010	.012	.014	.016	.017	.035	.052	.070	.087	.105	.122	.140	.157	.175	.350
10	.002	.003	.005	.007	.008	.010	.012	.013	.015	.017	.033	.050	.067	.083	.100	.117	.133	.150	.167	.333
	.002	.004	.006	.008	.010	.012	.014	.016	.017	.019	.039	.058	.078	.097	.117	.136	.156	.175	.194	.389
20	.003	.007	.010	.013	.017	.020	.023	.027	.030	.033	.067	.100	.133	.167	.200	.233	.267	.300	.333	.667
	.004	.008	.012	.016	.019	.023	.027	.031	.035	.039	.078	.117	.156	.194	.233	.272	.311	.350	.389	.778

30	.005	.010	.015	.020	.025	.030	.035	.040	.045	.050	.100	.150	.200	.250	.300	.350	.400	.450	.500	1.000
	.006	.012	.017	.023	.029	.035	.041	.047	.052	.058	.117	.175	.233	.292	.350	.408	.467	.525	.583	1.167
40	.007	.013	.020	.027	.033	.040	.047	.053	.060	.067	.133	.200	.267	.333	.400	.467	.533	.600	.667	1.333
	.008	.016	.023	.031	.039	.047	.054	.062	.070	.078	.156	.233	.311	.389	.467	.544	.622	.700	.778	1.556
50	.008	.017	.025	.033	.042	.050	.058	.067	.075	.083	.167	.250	.333	.417	.500	.583	.667	.750	.833	1.667
	.010	.019	.029	.039	.049	.058	.068	.078	,087	.097	.194	.292	.389	.486	.583	.681	.778	.875	.972	1.944
60	.010	.020	.030	.040	.050	.060	.070	.080	.090	.100	.200	.300	.400	.500	.600	.700	.800	.900	1.000	2.000
	.012	.023	.035	.047	.058	.070	.082	.093	.105	.117	.233	.350	.467	.583	.700	.817	.933	1.050	1.167	2.333
70	.012	.023	.035	.047	.058	.070	.082	.093	.105	.117	.233	.350	.467	.583	.700	.817	.933	1.050	1.167	2.333
	.014	.027	.041	.054	.068	.082	.095	.109	.122	.136	.272	.408	.544	.681	.817	.953	1.089	1.225	1.361	2.722
80	.013	.027	.040	.053	.067	.080	.093	.107	.120	.133	.267	.400	.533	.667	.800	.933	1.067	1.200	1.333	2.667
	.016	.031	.047	.062	.078	.093	.109	.124	.140	.156	.311	.467	.622	.778	.933	1.089	1.244	1.400	1.556	3.111
90	.015	.030	.045	.060	.075	.090	.105	.120	.135	.150	.300	.450	.600	.750	.900	1.050	1.200	1.350	1.500	3.000
	.017	.035	.052	.070	.087	.105	.122	.140	.157	.175	.350	.525	.700	.875	1.050	1.225	1.400	1.575	1.750	3.500
100	.017	.033	.050	.067	.083	.100	.117	.133	.150	.167	.333	.500	.667	.833	1.000	1.167	1.333	1.500	1.667	3.333
	.019	.039	.058	.078	.097	.117	.136	.156	.175	.194	.389	.583	.778	.972	1.167	1.361	1.556	1.750	1.944	3.889
200	.033	.067	.100	.133	.167	.200	.233	.267	.300	.833	.667	1.000	1.333	1.667	2.000	2.333	2.667	3.000	3.333	6.644
	.039	.078	.117	.156	.194	.233	.272	.311	.350	.389	.778	1.167	1.556	1.944	2.333	2.722	3.111	3.500	3.889	7.777
300	.050	.100	.150	.200	.250	.300	.350	.400	.450	.500	1.000	1.500	2.000	2.500	3.000	3.500	4.000	4.500	5.000	10.000
	.058	.117	.175	.233	.292	.350	.408	.467	.525	.583	1.167	1.750	2.333	2.917	3.500	4.083	4.667	5.250	5.833	11.666
400	.067	.133	.200	.267	.333	.400	.467	.533	.600	.667	1.333	2.000	2.667	3.333	4.000	4.667	5.333	6.000	6.667	13.333
	.078	.156	.233	.311	.389	.467	.544	.622	.700	.778	1.556	2.333	3.111	3.889	4.667	5.444	6.222	7.000	7.777	15.555
500	.083	.167	.250	.333	.417	.500	.583	.667	.750	.833	1.667	2.500	3.353	4.167	5.000	5.833	6.667	7.500	8.833	16.667
	.097	.194	.292	.389	.486	.583	.686	.778	.875	.972	1.944	2.917	3.889	4.861	5.833	6.805	7.778	8.750	9.722	19.494

EXPLANATION.—To find from the above table the interest of a given amount for a given number of days, refer to the square opposite the number of dollars at the left, and the number of days at the top. For $5000 remove the decimal point one place to the right.

* NOTES.—1 On account of space the table is condensed as much as possible; but in practice any interest table may be used, so that the interest on any amount for any number of days may be seen at a glance, and thus calculations may be greatly facilitated. This table is calculated at 360 days per year.

2. Accounts may be averaged by any per cent., but the cash balance can only be found by using the legal rate per cent.

What is the interest of $4374. for 250 days?

MODEL OPERATION.

	Prin.		Int.			
By the table	$4000	give	$133.33	for	200	days.
	"	"	33.33	"	50	"
	300	"	10.00	"	200	"
	"	"	2.50	"	50	"
	70	"	2.333	"	200	"
	"	"	.583	"	50	"
	4	"	.133	"	200	"
	"	"	.033	"	50	"
	$4374	"	$182.242	"	250	"

QUESTIONS.—Who is the drawee? (273.) Who is the payee? (274.) Who is the buyer, or remitter? (275.) What is an acceptance? (276.) What is an indorsement? (277.) What is an inland bill? (278.) What is a foreign bill? (279.) What is the course of exchange? (280.) What is par of exchange? (281.) When is exchange said to be at a premium? (282.) When is it said to be at a discount? (283.) Give a rule for finding the cost of a sight draft. (284., *a.*) When the draft is on time? (284., *b.*)

LESSON XCV

EXAMPLES FOR PRACTICE.

715. SEYMOUR A. BRUNDIGE,

To JAMES NYLES, Dr.

1863.		
Jan. 1,	To Mdse.,	$347
" 13,	" " on 3 months,	463
Aug. 11,	" Cash,	300
Nov. 23,	" Mdse. on 60 days,	463

At what time is the whole bill due?

716. WILLIAM WHITE,

To SEYMOUR A. BRUNDIGE, Dr.

1864.				
Jan.	3	To 30 yds. Broadcloth	@ $5.00	
"	27	" 375 " Calico	@ .15	
July	13	" Cash,		$375
Sept.	11	" 400 yds. Carpeting	@ $0.50	
Oct.	13	" Mdse. on 30 days,		$300

At what date will the amount of the above bill become due?

717. JAMES GILBERT, JR.,

To FRANK BUSHNELL & Co., Dr.

1862.			
Apr.	13,	To Cash	$400
June	11,	" Mdse. on 3 months,	800
Aug.	13,	" " on 60 days,	500
Oct.	27,	" Cash,	875
1863.			
Feb.	23,	" Draft, 90 days,*	500

If the above bill be settled by note, from what date ought it to draw interest?

718. JOSEPH NYE,

To HALL, CHADWICK & Co., Dr.

1864.			
Feb.	3,	To 325 bu. of Wheat,	@ $1.20
Apr.	15,	" 467 " "	@ 1.16
June	11,	" 520 " "	@ 1.21
Sept.	13,	" 420 " "	@ 1.51
Nov.	29,	" 325 " "	@ 1.11½

What is the equated time of payment of the above account?

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*NOTE.—Add the customary three days of grace to the term of credit of the draft.

719. I wish to ascertain the time of payment of the following account with Theodore Lyon & Co.

1864.					
Jan. 30,	37 yds. Broadcloth,	@	$3.50	on	3 months.
Feb. 11,	25 bales Cotton cloth,	@	21.30	"	60 days.
June 13,	9 tons Iron,	@	50.	"	4 months.
Sept. 27,	14 cwt. Sugar,	@	9.30	"	90 days.

LESSON XCVI.

299. To find the equated time of the balance of an account when the items are reckoned from different dates.

What is the date at which the balance of my account with O. H. Hall & Co. will become due?

Dr. O. H. HALL & Co. Cr.

1864.				1864.			
Jan. 3	To Mdse. on 30 da.	300		Apr. 11	By Mdse. on 90 da.	500	
" 30	" " " 90 "	400		June 10	" "	300	
Apr. 5	" Cash,	350	50	Sept. 13	" Cash,	450	50
Nov. 27	" Mdse.	400		Dec. 12	" Draft 60 da.	500	

MODEL OPERATION.

Dr.

Due.		(da.)		(da.)
Feb. 2,	$300	×0	= $1×	0
Apr 30,	400	×87	= 1×	34800
" 5,	350½	×62	= 1×	21731
Nov. 27,	400	×298	= 1×	119200
	$1450.50			175731

Cr.

Due.		(da.)		
July 10.	$500	×158	$1×	79000
June 10,	300	×128	= 1×	38409
Sept 13,	450½	×223	= 1×	100462
1865				
Feb. 13,	500	×376	= 1×	188000
	$1750.50			405862
	1450 50			175731
	$300			230131

230131 ÷ 300 = 767 da

Feb 2, 1864 + 767 da = Mar. 10, 1866, Ans.

(*a.*) ANALYSIS.—1. For convenience *assume* Feb. 2, the earliest date at which any of the items become due, from which to calculate the terms of credit.

2. On the debit side of the account, the term of credit of \$1450½ (the sum of the items) to the dates at which it becomes due is equal to the credit of \$1 for 175731 days (the sum of the products.)

3. On the credit side of the account, the term of credit of \$1750.50 (the sum of the items) to the dates at which it becomes due is equal to the credit of \$1 for 405862 days (the sum of the products.)

4. On striking the balance, I find that I owe O. H. Hall & Co. \$300, but as he has had the use of \$1 for 230131 days more than myself, I ought to retain the balance of \$300 until the credit is one of equity.

5. If \$1 requires a term of credit for 230131 days to be worth a certain amount, \$300 will require $\frac{1}{300}$ of 230131 days, which is 767 days, to be worth the same amount; hence I must retain the balance for 767 days after the assumed date, which is Mar. 10, 1866.

Therefore, the balance of \$300 will become due Mar. 10, 1866.

What is the date at which the balance of my account with E. W. Brewer will become due, and what will be the cash balance Jan. 1st, 1864?

E. W. Brewer.

1863.			1863.		
Apr. 9	To Mdse.	300	June 10	By Mdse.	200
June 7	" "	500	Sept. 5	" Cash,	300
Aug. 3	" "	350	Nov. 11	" "	100
Dec. 11	" Cash,	200	Dec. 21	" "	400

MODEL OPERATION.

Dr. Due	(da.)	(da.)	Cr. Due.	(da.)	
Apr 9	$300×0	=$1×0	June 10	$200×62	– $1×12400
June 7	500×59	= 1×29500	Sept. 5	300×149	= 1×44700
Aug. 3	850×116	= 1×40600	Nov. 11	100×216	= 1×21600
Dec. 11	200×246	= 1×49200	Dec. 21	400×256	– 1×102400
	$1350	119300		$1000	181100
	$1000				119300
	$350				61800

$443 \times .000\frac{1}{6} = .073\frac{5}{6}$

291
1050
2450

25.841, interest.
850

$375.84, Cash balance, Jan. 1, 1864.

$61800 \div 350 = 177$ days.

Apr. 9, 1863—177 days equals Oct. 14, 1862.

The number of days between Oct. 14, 1862, and Jan. 1, 1864, is 443 days.

(*b.*) Analysis.—1. Find, as in the previous example, the sum of the items and also the sum of the products on each side of the account.

2. On striking the balance I find that E. W. Brewer owes me $350, but, as he has had the use of $1 for 61800 days more than myself, he ought to pay the balance of $350 soon enough to make the credit one of equity.

3. If $1 requires the term of credit of 61800 days to be worth a certain amount, $350 will require $\frac{1}{350}$ of 61800 days, which is 177 da. to be worth the same amount; hence I ought to receive the balance of $350 177 days before the *assumed date.*

4. Therefore, the balance will become due 177 days *before* Apr. 9, 1863, which is Oct. 14, 1862.

5. If the balance of $350 becomes due Oct. 14, 1862, it must have been on interest from that date to Jan. 1, 1864. The number of days between these dates is 443, and the interest of $350 for this time is $25.841, which added to $350 gives the cash balance of $375.84.

(*c.*) RULE.—I. *Find the time when each item of the account is due, and their respective amounts.*

II. ASSUME *the* EARLIEST *date at which any of the items of the account become due, and* FROM IT *calculate the terms of credit.*

III. *Multiply each item by its own term of credit, and divide the balance of the sum of the products by the balance of the sum of the items of the account; the quotient will be the interval of time, which must be reckoned from the* ASSUMED DATE FORWARD, *when the balances are on the same side of the account, but* BACKWARD *from the* ASSUMED DATE, *when the balances are on opposite sides of the account: this will be the date at which the balance becomes due.*

IV. *When the balance of the account becomes due* BEFORE *the time of settlement, calculate the interest at the given per cent. for the interval of time between the date at which the balance becomes due and the time of settlement; add it to the balance: the result will be the cash balance.*

V. *When the balance of the account becomes due* AFTER *the time of settlement, calculate the* BANK *discount on the balance at the given rate per cent., for the interval of time between the settlement and the date at which the balance becomes due; deduct it from the balance of the account: the remainder will be the cash balance.*

QUESTIONS.—Give the rule for finding the cost of a bill of exchange on England. (286., *a.*) How is English exchange quoted? (286., *a.*, 1.) Give the analysis of finding the cost of a bill of exchange on France. (290., *a.*) How is exchange on France quoted? (287., *a.*, 2.) How are bills of exchange usually drawn? (287., *a.*, 3.) What is the exchange value of the pound sterling? (286.) What is the exchange value of the franc? (287.) How are bills on England drawn? (286.) How are bills on France drawn? (287.)

LESSON XCVI.

EXAMPLES FOR PRACTICE.

720.

Dr. Samuel J. Miller & Co. **Cr.**

1864.			1864.		
Jan. 3	To Mdse. on 30da.,	400	Apr. 9	By Cash,	430
Aug. 17	" " " 90 "	300	Aug. 11	" Mdse. on 20 da.,	500
Sept. 17	" Draft " 30 "	250	Nov. 11	" Cash,	400

What is the cash balance of the above account Jan. 1, 1865?

721.

Dr. Geo. W. Howell & Co. **Cr.**

1860.			1860.		
Apr. 11	To Mdse. on 2 mo.,	300	May 1	By Cash,	500
June 13	" " " 20 da.,	400	July 27	" Draft on 90 da.,	450
Sept. 12	" Cash,	356	Dec. 11	" Cash,	375
Nov. 13	" Mdse.	490			

What is the date at which the balance of the above account becomes due?

722.

Dr. Isaac Vanness. **Cr.**

1863.			1863.			
June 7	To Mdse. on 90da.,	550	Apr. 9	By Cash,	575	50
Aug. 9	" " " 30 "	380	Oct. 11	" Draft on 20 da.,	400	
Sept. 11	" Mdse.	440	Dec. 18	" Cash,	300	
			" 9	" "	800	

What is the cash balance of the above account Dec. 25, 1863?

723.

Dr. **Charles F. Mawbey.** **Cr.**

1864.				1864.			
Jan.	8	To Cash,	200	Apr.	4	By Mdse. on 30 da.,	475
July	18	" Mdse. on 60da.,	350	July	6	" Cash,	450
Aug.	18	" ' " 90 "	420	Aug.	26	" Mdse. on 60 da.,	375

What is the cash balance of the above account Jan. 1, 1865?

Questions.—What is exchange? (270.) What is a bill of exchange? (271.) Who is the drawer or maker? (272.) The drawee? (273.) The payee? (274.) The buyer or remitter? (275.) What is an acceptance? (276.) What, an indorsement? (277.) What is the difference between an inland bill and a foreign bill? (278.) What is the course of exchange? (280.) What is par of exchange? (281.) When is exchange said to be at a premium? (282.) When at a discount? (283.)

LESSON XCVII.

300.* AVERAGING ACCOUNTS BY INTEREST.

(*a.*) What is the date at which the following bill is due?

(*b.*) What is the cash balance at 6 per cent., Dec. 12.

Jerrie L. Fordham, **To G. W. Howell, Dr.**

1864.			Days.	Interest.			
				$	cts.	$	cts.
July	3	To 500 bu. Corn, @ $1.00				500	
"	15	" 300 " " " .80	12		480	240	
Aug.	5	" 200 " Wheat, " 1.50	33	1	65	300	
"	28	" 500 " " " 1.00	56	4	667	500	
Sept.	11	" 300 " Oats, " .50	70	1	75	150	
				8	547	1690	

*Note—For the following brief, practical method of averaging, or of finding the cash balance of accounts, we are indebted to R. S. Delisser, Esq., of N. Y. city, who has kindly permitted us to use it in this work.

The advantage of this method is, that, by ruling an interest column in the ledger, and by the aid of an appropriate interest table, the amount of interest on each item, from the *assumed* date to the date of purchase, may be entered while *posting*. Upon settlement, either the average or the cash balance may be found by adding the item and the interest column, and, at a single operation, determining the equated time or cash balance, the whole work requiring but a few moments of time.

This method is now used by the principal merchants in New York, and other large cities, and a more complete exposition of the system, with tables, &c., may be found in Mr. Delisser's published work.

MODEL OPERATION.

	(a.)	(b.)
1.	\$1690÷6=\$.2816.	Int. on \$1690 for 162 da.=\$45.63.
2.	8.547÷.2816=30 da.	\$45.63—\$8.547=37.083.
3.	July 3,+30 da=Aug. 2, Ans.	\$37.083+\$1690=1727.08, Ans.

Analysis.—(*a.*) By the table (298., *b.*) find the term of credit of each item from July 3, the *assumed* date, and write it in the column of days.

By the table (298., *c.*) find the interest on each item for its term of credit, and write it in the interest column.

1. If the interest on \$6. for one day is 1 mill, the interest on \$1690 for one day will be as many mills as \$6. are contained times in \$1690, which are $281\frac{6}{10}$.

2. If 1690 require *one* day to give \$0.2816 interest, to give \$8.547 interest it will require as many days as \$0.2816 are contained times in \$8.547, which are 30.

3. 30 days added to July 3d, equals Aug. 2d, the equated time.

(*b.*) The interest on \$1690 from July 3, the *assumed* date, to Dec. 12, the time of settlement (162 da.), is \$45.63.

If the interest on the several items to the dates at which they become due is \$8.547, then \$8.547 deducted from \$45.63, the interest of \$1690 to Dec. 12th, leaves \$37.083, the required interest, which added to \$1690 amounts to 1727.08, the cash balance.

What is the date at which the balance of the following account becomes due?

Dr. CHARLES ROCKWOOD.* Cr.

1864.			Int.					Int.	
Jan.	9	To Mdse. on 30 da.		5000 00	Apr.	3	By Cash,	4 50	500 00
Apr.	11	" " " 20 "	6 834	500 00	July	12	" "	11 54	450 00
Aug	13	" Cash,	6 20	200 00	Aug	20	" Mdse. on 30 da.	27 417	500 00
			13.034	5700.00				43 457	1450 00

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*Note.—The cash balance of this account may be found as follows:—Deduct \$13.034 from the interest of \$5700 from Feb. 8 to the time of settlement, and add the remainder to \$5700 on the debit side of the account. In the same manner proceed with the credit side, and the balance of the account will be the cash balance.

MODEL OPERATION.

	Due.		da.	Int.	Due.		da.	Int.
Assumed Date.	Feb. 8,	$5000	0	$0.	Apr. 8,	$500	54	$4.50
	May 1,	500	82	6.834	July 12,	450	154	11.54
	Aug. 13,	200	186	6.20	Dec. 28,	500	323	26.917
		$5700		$13.034		$1450		$42.957
		1450						13.034
		$4250 balance of items.						$29.923 balance of Int.

1. 4250 ÷ 6 = 708.3 mills; $30.423 .7083 = 42 da.
2. Feb. 8—42 da. = Dec. 28, 1863, Ans.

Aɴᴀʟʏsɪs.—Assume Feb. 8, 1864, from which to calculate credits. Write the interest on each credit, in the interest column, and find the balance of interest and the balance of items.

This balance shows that Chas. Rockwood owes me $4250, and the value of his credit in the account amounts to $29.923 more than mine, consequently the balance of the account should become due previous to Feb. 8, 1864, for a time sufficient to produce $29.923.

1. If $4250 require *one* day to give $.7083 interest, it will require as many days to give $29.923 interest as $0.7083 is contained times in $29.923, which is 42.

2. 42 days previous to Feb. 8, 1864, is Dec. 28, 1863, the equated time.

Hence, to find the equated time of an account by interest, we have the following

Rᴜʟᴇ.—I. *Find the interest by the table on each item from the* ASSUMED *date to the time at which it becomes due.*

II. *Divide the balance of interest by the interest of the balance for* 1 *day and the quotient will be the average term of credit.*

III. *Add the average term of credit to the assumed date, if both interest and item balance are on the same side of the account; but subtract it if they are on the opposite sides of the account.*

To find the cash balance.

RULE.—I. *Subtract the sum of the interest column from the interest of the sum of the items, from the* ASSUMED *date to the time of settlement, taking each side of the account separately.*

II. *Add each remainder to the sum of its own items, and the balance of the account will be the cash balance required.*

LESSON XCVIII.

724.

Dr. EDWARD JONES. **Cr.**

Date.			Int.	Items.		Date.			Int.	Items.	
1864.						1864.					
Jan.	9	To 400 bu. Whe't @ $1.25		500	00	Jan.	25	By Cash		500	00
Mar.	11	" 300 " Oats @ .75		225	00	Apr.	18	" Draft 30 da.		300	00
Aug	9	" 500 " " @ .80		400	00	July	12	" " 20 "		200	00

What is the equated time of settlement of the above account?

725.

Dr. S. A. POTTER & Co. **Cr.**

Date.			Int.	Items.		Date.			Int.	Items.	
1864.						1864.					
Feb.	11	To 40 doz. Copy B. @ $1.20		48	00	Feb.	13	By Cash		50	00
Apr.	9	" 50 " " @ 1.30		65	00	May	29	" "		70	00
June	23	" 80 " " @ 1.10		88	00	Aug.	11	" "		100	00
"	29	" 200 B. Keep. @ .30		60	00	"	25	" Dr'ft 30 da.		75	00

What is the cash balance of the above account Nov. 1st, 1864?

726.

Dr. MARTIN R. DENNIS. **Cr.**

Date.			Int.	Items.		Date.			Int.	Items.	
1864.						1864.					
Mar.	29	To 5000 Prim. Arith. @ .15		750	00	Apr.	1	By Cash		1000	00
Apr.	9	" 2500 " Gram. @ .30		750	00	"	29	" "		500	00
June	23	" 2000 Read. No. 2 @ .30		600	00	June	1	" Dr'ft 90 da.		800	00
Sept.	20	" 1000 " No. 1 B. @ .10		100	00	Aug.	27	" " 90 "		300	00

What is the cash balance of the above account on Oct. 1, 1864?

727.

Dr. F. C. BROWNELL & Co. **Cr.**

Date			Int.	Items.		Date.			Int.	Items.	
1863.						1963.					
Jan.	8	To Sundries		180	00	Jan.	1	By Aparatus, Maps, &c.		200	00
"	27	" "		500	00	Mar.	19	" Books, Charts, &c		600	00
Apr.	19	" Cash		800	00	June	12	" Aparatus, Globes, &c.		800	00
Sept.	11	" "		600	00	Aug.	27	" " " "		800	00

The above account was settled by note. What is the amount, and when should it become due?

LESSON XCIX.

PARTNERSHIP.

301. A **Partnership** is an association of two or more persons in trade under a certain name, with an agreement to share the profits and losses of business.

302. The **Partners** are the individuals who transact business together.

303. The **Capital**, or **Stock**, is the money or property used by the firm in business.

304. A **Dividend** is the profits to be divided between the partners. A tax levied to meet losses is called an *assessment*.

305. To find each partner's share of the dividend or assessment, when the capital of each has been used for equal periods of time.

A, B, and C engage in mercantile business: A furnished $400; B, $684; and C, $2500. At the end of one year their profits amount to $1200; what is each partner's share?

MODEL OPERATION.

$400, A's capital.
684, B's "
2500, C's "

gain. gain on $1.
$1200÷3584=.33482,=33$\frac{482}{1000}$ per cent.
$400×.33482=$133.93, A's share of the dividend.
$684×.33482=$229.01, B's " " "
$2500×.33482=$837.05, C's " " "

Analysis.—1. If A furnishes $400, B $684, and C $2500, they all furnish the sum of these quantities, which is $3584.

2. If the profit on a capital of $3584 is $1200, the profits on $1 is $\frac{1}{3584}$ of $1200, which is $0.33482, or 33$\frac{482}{1000}$ per cent.

3. If a capital of $1 is entitled to a profit of $0.33482, A's capital of $400 is entitled to 400 times $0.33482, which is $133.93; B's capital of $684 is entitled to $229.01; and C's capital of $2500 is entitled to $837.05.

Therefore, according to the conditions of the question, A's share of the dividend is $133.93; B's, $229,01; and C's, $837.05.

(*a.*) Rule.—I. *Divide the amount gained by the entire capital, and the result will be the amount on* $1, *or the per cent.*

II. *Multiply each partner's capital by the amount gained on* $1, *or the per cent., and the result will be each partner's share of the dividend.*

EXAMPLES FOR PRACTICE.

728. There is a joint stock of $3500, of which A owns $370, and B, $1970; the annual dividend amounts to $1275; what is each man's share?

729. There is a ship which, with its cargo, is worth $35000; of which A owns $5000; B, $10000; and C, the

remainder; the profits of a voyage amounted to $15000; what is each man's share?

730. The stock of a railroad is $4,000,000; the annual expenses are $500000; the annual earnings amount to $1,000,000. A owns $35000 of the stock; B, $8750; and C, $50000. To what share of the profits will each be entitled?

731. A firm of three partners owns a ship and its cargo valued at $40000; A, one of the partners, owns $\frac{1}{2}$; B, $\frac{1}{3}$; and C, the remainder. The ship, in a storm, was injured to the amount of $10,384; what was each man's share of the loss?

732. Three men owned a factory: A owned $\frac{1}{3}$; B, $\frac{2}{5}$; and C, the remainder; it was injured by fire to the amount of $3750; what was each man's share of the loss?

Questions.—What is the practice in regard to days of grace? (283., *a.*) Give a rule for finding the cost of a sight draft. (284., *a.*) Give the rule when on time. (284., *b.*) What is the nominal value of 1£. in English exchange? (286.) How is the value quoted? (286.) What is the value of the franc in a bill on France? (287.) How quoted? (287.) How are foreign bills of exchange usually drawn? (287., *a.*) Give a rule for finding the cost of a bill on England. (288., *a.*)

LESSON C.

306. To find each partner's share of the dividend, when the capital of each is used for unequal times.

A and B entered into partnership; A put in $340 for 3 months; and B, $230 for 4 months; they gained $320; what was each one's share of the profits?

MODEL OPERATION.

A's cap. \$340×3=\$1020, for 1 mo.
B's " 230×4= 920, " 1 "

$\$320 \div \$1940 = \overset{\text{gain on \$1}}{\$0.16494} = 16\frac{494}{1000}$ per cent.
\$1020×.16494=\$168.238+, A's share of the profits.
\$920×.16494=\$151.744+, B's " " "

ANALYSIS.—1. The profits on A's capital of \$340 for 3 months are equal to the profits on 3 times \$340 for 1 month, or \$1020; and the profits on B's capital of \$230 for 4 months are equal to the profits on \$920 for 1 month.

2. If A puts in \$1020 for a certain time, and B puts in \$920 for the same time, the entire capital is the sum of these quantities, which is \$1940.

3. If a capital of \$1940 gains \$320 in a certain time, \$1 will gain $\frac{1}{1940}$ of \$320, which is \$0.16494+, equal to $16\frac{494}{1000}$ per cent.

4. If \$1 capital gains \$0.16494+, \$1020 capital will gain 1020 times \$0.16494, which is \$168.238, A's share of the profits; and \$920 will gain 920 times \$0.16494, which is \$151.744, B's share of the profits.

Therefore, according to the conditions of partnership, A's share of the profits is \$168.238, and B's share is \$151.744.

(*a.*) RULE.—I. *Multiply each partner's capital by the time during which it is employed, and divide the gain or loss by the sum of the products; the result will be the amount on \$1, or the gain or loss per cent.*

II. *Multiply the product of each partner's capital by the gain or loss per cent.; the result will be his share of the gain or loss.*

EXAMPLES FOR PRACTICE.

733. Three persons traded together: X put in \$300 for

8 months; Y, $430 for 9 months; and Z, $400 for 7 months: they gained $900; how much was each man's share of the profits?

734. Three men built a house: A furnished 7 workmen for 7 days; B, 9 workmen for 11 days; and C, 13 workmen for 4 days. They received $790 for the job; how much should each of the contractors receive?

735. Three men hired a pasture for $90: A pastured 8 cows 90 days; B, 13 cows 20 days; and C, 115 cows 11 days; how much should each pay?

736. Two merchants enter into partnership for two years, each investing $1000. At the end of 9 months A put in $500 more, and B took out $500; at the end of 18 months A doubled his capital, and B put in twice as much as he already had in; at the expiration of the two years they had gained $8713.50: what was each man's share of the gain?

737. Two persons engaged in business with a capital of $5000. A's share of the profits amounted to $473.13, and B's share of the profits amounted to $274.12; what amount of the capital did each furnish?

738. It took two persons 21 days to build a wall, for which they received $25; A's share of the money amounted to $6.50, and B received the remainder; how many days did each work?

739. Two men engaged in business with a capital of $5000. A's capital was in the business 5 months, and he received, as his share of the profits, $473.12; B's capital was in the business 9 months, and he received, as his share of the profits, $325.38; how much of the capital did each furnish?

QUESTIONS.—Give the rule for finding the amount of a bill of exchange that can be purchased for a given sum of U. S. currency.

(289., *a.*) Give the analysis of finding the cost of a bill of exchange on France. (290., *a.*) Give the analysis for finding the amount of a bill on France that can be purchased for a given sum of U. S. currency. (291., *a.*)

SECTION XII.

LESSON I.

ALLIGATION.

307. Alligation Medial is the process of finding the *mean price* or *quality* of a mixture, when the quantity of each ingredient and its price or quality are known.

A wine merchant mixes 12 gallons of wine worth $1.50 per gallon, and 9 gallons of brandy worth $2. per gallon, with 5 gallons of water; what is the value of the mixture?

MODEL OPERATION.

$$\begin{array}{rcrcr}
\$1.50 & \times & \overset{\text{(gals.)}}{12} & = & \$18.00. \\
2.00 & \times & 9 & = & 18.00. \\
0 & \times & 5 & = & 0. \\
\hline
 & & & 26) & 36.00 \\
\hline
 & & & & \$1.38, \text{ Ans.}
\end{array}$$

ANALYSIS.—1. Since 12 gallons of wine at $1.50 per gal. are worth $18., and 9 gallons of brandy at $2. are worth $18., and 5 gallons of water are worth nothing, the whole 26 gallons will be worth the sum of $18. and $18., which is $36.00.

2. If 26 gallons of the mixture are worth $36.00, one gallon is worth $\frac{1}{26}$ of $36., which is $1.38.

RULE.—*Divide the entire cost of the given simples by the entire quantity, and the result will be the mean price.*

1. A grocer mixed 13 gallons of water with 40 gallons of brandy worth $1.25 per gallon; will he gain, or lose, by selling the mixture at 5 cts. per gill?

2. A grocer mixed together 20 gallons of molasses worth 75 cts., 13 gallons worth 50 cts., 30 gallons worth $1., and 50 gallons worth 30 cts.; what are 15 gallons of the mixture worth?

3. A farmer mixed together 5 bushels of oats worth 40 cts., 9 bushels of rye worth 50 cts., and 30 bushels of corn worth 75 cts.; what is one bushel of the mixture worth?

QUESTIONS.—When is a bill of exchange said to be at a premium? (282.) When at a discount? (283.) What is the difference between present worth and discount? (259.) (260.) What is a bank? (261.) When is a note said to be at maturity? (266.) What are the proceeds of a note? (267.) What is per cent.? (183.) What is an agent? (188.) What is a charter? (192.) What is the market value of stocks? (200.) What are profit and loss? (206.)

LESSON II.

308. Alligation Alternate is the process of finding what quantity of simples, whose prices or qualities are given, must be taken to make a mixture of any given price or quality.

309. To find what quantity of each simple must be taken to form a mixture of a given value.

I have different qualities of sugar worth respectively 10, 11, 12, 14, and 15 cts. per lb.; what proportion of each must I take to make a mixture worth 13 cents?

MODEL OPERATION.

$$
\begin{array}{l}
\\
\text{13 cts.}
\end{array}
\left\{
\begin{array}{l|lll}
 & & & \text{Proportional quantities.} \\
\text{10 cts.} & \text{to gain 1 ct.} & \text{use} & \tfrac{1}{3}\text{ lb.} = \tfrac{1}{3}\text{ lb.} \\
\text{11 cts.} & \text{" " "} & \text{"} & \tfrac{1}{2}\text{ lb.} = \tfrac{1}{2}\text{ lb.} \\
\text{12 cts.} & \text{" " "} & \text{"} & 1\text{ lb.} = 1\text{ lb.} \\
\text{14 cts.} & \text{to lose } 1+\tfrac{1}{2}^{*}\text{ ct.} & \text{"} & 1+\tfrac{1}{2}\text{ lb.} = 1\tfrac{1}{2}\text{ lb.} \\
\text{15 cts.} & \text{" " } \tfrac{1}{2}+\tfrac{1}{2}^{*}\text{ ct.} & \text{"} & 1+\tfrac{1}{4}\text{ lb.} = \tfrac{3}{4}\text{ lb.}
\end{array}
\right\} \times 12 =
\left\{
\begin{array}{r}
\text{4 lbs.} \\
\text{6 lbs.} \\
\text{12 lbs.} \\
\text{18 lbs.} \\
\text{9 lbs.}
\end{array}
\right.
$$

*NOTE.—The difference between the total loss and the total gain is *one cent*, which added equally to the losses makes 1½ cents for each.

ANALYSIS.—1. For convenience arrange the prices of the simples in a column, in the order of their values, with the given mean at the left.

2. Since the total gain on the quantity of simples used must equal the total loss, first take that portion of each simple that will equal the gain of 1 cent upon selling at the mean price.

3. If, by using one pound of sugar at 10 cts. in a mixture worth 13 cts., the gain is 3 cts.; to gain 1 ct., use $\frac{1}{3}$ of a pound.

4. If, by using one pound of sugar at 11 cts. in a mixture worth 13 cts., the gain is 2 cts.; to gain 1 ct., use $\frac{1}{2}$ of a pound.

5. If, by using one pound of sugar at 12 cts. in a mixture worth 13 cts., the gain is 1 ct.; to gain 1 ct., use 1 pound.

6. If, by using one pound of sugar at 14 cts. in a mixture worth 13 cts., the loss is 1 ct.; to lose 1 ct., use 1 pound.

7. If, by using one pound of sugar at 15 cts. in a mixture worth 13 cts., the loss is 2 cts.; to lose 1 ct., use $\frac{1}{2}$ of a pound.

8. Since the total gain is 3 cts., and the total loss is 2 cts., add the difference, 1 ct., equally to the losses, making $1\frac{1}{2}$ cent for each.

9. If 1 pound of sugar at 14 cts. must be used, to lose 1 cent; to lose $\frac{1}{2}$ cent, $\frac{1}{2}$ pound must be used, which added to 1 pound makes $1\frac{1}{2}$ lbs., the proportional quantity.

10. If $\frac{1}{2}$ lb. of sugar at 15 cts. must be used, to lose 1 cent; to lose $\frac{1}{2}$ cent, $\frac{1}{4}$ lb. must be used, which added to $\frac{1}{2}$ lb. makes $\frac{3}{4}$ lb., the proportional quantity.

Therefore, according to the conditions of the questions, the proportional quantities are $\frac{1}{3}$ lb. at 10 cts., $\frac{1}{2}$ lb. at 11 cts., 1 lb. at 12 cts., $1\frac{1}{2}$ lb. at 14 cts., and $\frac{3}{4}$ lb. at 15 cts. Multiplying by 12*, the least common multiple of the denominators, we have 4 lbs. at 10 cts., 6 lbs. at 11 cts., 12 lbs. at 12 cts., 18 lbs. at 14 cts., and 9 lbs. at 15 cts.

RULE.—I. *Arrange the prices or qualities in a column,*

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*NOTE.—*If proportional quantities be multiplied by any number, they will still be proportional.* The most convenient method of changing two or more fractional numbers to integers without changing their ratios is to multiply them by the least common multiple of their denominators.

in the order of their values, with the given mean price at the left.

II. *Compare the price of each simple with the mean price, and take that portion of each which will gain one of the mean price.*

III. *Find the difference between the total loss, and the total gain; if it be a gain, add it equally to the losses; if a loss, add it equally to the gains, adding that portion of each quantity required by the added gain or loss to the portion already found, and the result will be the proportional quantity required.*

IV. *If the proportional quantities are fractional, multiply them by the least common multiple of the denominators.*

EXAMPLES FOR PRACTICE.

4. A farmer wishes to mix corn at 75 cts. per bushel with rye at 55 cts., oats at 50 cts., and wheat at 95 cts.; what quantity of each must he take to make a mixture worth 70 cts. per bushel?

5. I have different kinds of salt worth respectively 25, 30, 35, and 50 cts. per bushel; how much of each kind must be taken in order that the mixture may be sold without loss at 42 cts. per bushel?

6. How much tea at 40 cts., 50 cts., 60 cts., and 70 cts. per pound, must be taken, that the mixture may be worth 45 cts. per pound? 55 cents per lb.? 68 cents per lb.?

LESSON III.

310. To find what quantity of the other simples is required to form a mixture at a given price, when the quantity of one of the simples is limited.

A merchant has tea at 40 cts., 50 cts., 75 cts., 95 cts., and 100 cts. per pound; how many pounds of each must he use with 25 lbs. of that worth 40 cts. to form a mixture worth 88 cts.?

MODEL OPERATION.

$$88\text{ cts.}\left\{\begin{array}{r|llr}40\text{ cts.} & \text{to gain 1 ct.} & \text{use} & \frac{1}{48}\text{ lb.}=\frac{1}{48}\text{ lb.}\\ 50\text{ cts.} & \text{“ “ “} & \text{“} & \frac{1}{38}\text{ lb.}=\frac{1}{38}\text{ lb.}\\ 75\text{ cts.} & \text{“ “ “} & \text{“} & \frac{1}{13}\text{ lb.}=\frac{1}{13}\text{ lb.}\\ 95\text{ cts.} & \text{to lose }1\frac{1}{2}{}^{*}\text{ ct.} & \text{“} & \frac{1}{7}+\frac{1}{14}\text{ lb.}=\frac{3}{14}\text{ lb.}\\ 100\text{ cts.} & \text{“ “ }1\frac{1}{2}{}^{*}\text{ ct.} & \text{“} & \frac{1}{12}+\frac{1}{24}\text{ lb.}=\frac{1}{8}\text{ lb}\end{array}\right\}\times1200=\left\{\begin{array}{l}25\text{ lbs.}\\ 31\frac{11}{19}\text{ lbs.}\\ 92\frac{4}{13}\text{ lbs.}\\ 257\frac{1}{7}\text{ lbs.}\\ 150\text{ lbs.}\end{array}\right.$$

$25\text{ lbs.}\div\frac{1}{48}\text{ lb.}$ at 40 cts. $=1200$ times the proportional quantities.

Analysis.—1. Arrange the prices and find the proportional quantities as in § 308. They are $\frac{1}{48}$ lb., $\frac{1}{38}$ lb., $\frac{1}{13}$ lb., $\frac{3}{14}$ lb., and $\frac{1}{8}$ lb.

2. If the proportional quantity for the mixture at 40 cts. is $\frac{1}{48}$ lb., as many times the other proportional quantities will be required in the mixture containing 25 lbs. at 40 cts. as $\frac{1}{48}$ of a pound is contained times in 25 lbs., which is 1200 times.

3. 1200 times $\frac{1}{38}$ lb. $=31\frac{11}{19}$ lbs.; 1200 times $\frac{1}{13}$ lb. $=92\frac{4}{13}$ lbs.; 1200 times $\frac{3}{14}$ lb. $=257\frac{1}{7}$ lbs.; 1200 times $\frac{1}{8}$ lb. $=150$ lbs.

Therefore, according to the condition of the problem there may be used in the mixture with 25 lbs. at 40 cts., $31\frac{11}{19}$ lbs. at 50 cts., $92\frac{4}{13}$ lbs. at 75 cts., $257\frac{1}{7}$ lbs. at 95 cts., and 150 lbs. at 100 cts.

Rule.—I. *Find the proportional quantities as in* § 308.

II. *Divide the given quantity by the proportional quantity of the required price, and multiply the quotient by each of the other proportional quantities.*

EXAMPLES FOR PRACTICE.

7. I have sugar at 11 cts., 13 cts., 15 cts., and 17 cts. per pound; how much of each must I mix with 10 pounds of the 13 cent sugar to make a mixture worth 14 cts. per pound?

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*Note.—The difference between the total loss and the total gain is 1 ct. which added equally to the losses makes $1\frac{1}{2}$ cts. each.

8. A farmer wishes to mix 20 bushels of corn worth 50 cts. per bushel, with rye worth 75 cts., barley worth 30 cts., and oats worth 38 cts.; how much of each must he take to make a mixture worth 45 cents?

9. A merchant has molasses worth 40 cts., 50 cts., 60 cts., and 70 cts. per gallon; how much of each must he mix with 10 gallons of the 50 cent molasses to make a mixture worth 55 cts.? To make one worth 62 cents?

QUESTIONS.—What is alligation medial? (307.) What is alligation alternate? (308.) Give the rule for finding the mean price, when the price or quality of a number of simples is given? (309.) Give the rule for finding the proportional quantity of a number of simples, when the quantity of one is limited. (310.)

LESSON IV.

311. To find what quantity of each simple is required to form a mixture at a given price when the quantity of the mixture is limited.

I have sugar at 10 cts., 12 cts., 14 cts., and 15 cts. per pound, and wish to make a mixture of 100 pounds worth 11 cts. per pound; how much of each must I use?

MODEL OPERATION.

$$11\text{ cts.}\left\{\begin{array}{l|llrl} 10\text{ cts.} & \text{to gain } 1+2^{*}\text{ cts.} & \text{use} & 3 & \text{lbs.}\\ 12\text{ cts.} & \text{to lose } 1\text{ ct.} & \text{``} & 1 & \text{lb.}\\ 14\text{ cts.} & \text{``} \quad \text{``} \quad \text{``} & \text{``} & \frac{1}{3} & \text{lb.}\\ 15\text{ cts.} & \text{``} \quad \text{``} \quad \text{``} & \text{``} & \frac{1}{4} & \text{lb.} \end{array}\right\}\times 21\tfrac{9}{11}=\left\{\begin{array}{r} 65\tfrac{5}{11}\text{ lbs.}\\ 21\tfrac{9}{11}\text{ lbs.}\\ 7\tfrac{3}{11}\text{ lbs.}\\ 5\tfrac{5}{11}\text{ lbs.} \end{array}\right.$$

$$100\text{ lbs.}\div 4\tfrac{7}{12}\text{ lbs.}=21\tfrac{9}{11}.$$

ANALYSIS.—1. Arrange the prices, and find the proportional quantities as in § 308. They are 3 lbs., 1 lb., $\frac{1}{3}$ lb., and $\frac{1}{4}$ lb.

2. Since the sum of the proportional quantities makes but $4\frac{7}{12}$ lbs. of the required quantity of the mixture, it will take as

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*NOTE.—The difference between the loss and gain is 2 cents, which added to the gain makes 3 cents.

many times each of the proportional quantities as $4\frac{7}{12}$ lbs. are contained times in 100 lbs., which are $21\frac{9}{11}$.

3. $21\frac{9}{11}$ times 3 lbs. are $65\frac{5}{11}$ lbs.; $21\frac{9}{11}$ times 1 lb. are $21\frac{9}{11}$ lbs.; $21\frac{9}{11}$ times $\frac{1}{3}$ lb. are $7\frac{3}{11}$ lbs.; $21\frac{9}{11}$ times $\frac{1}{4}$ lb. are $5\frac{5}{11}$ lbs.

Therefore, according to the condition of the question $65\frac{5}{11}$ lbs. at 10 cts., $21\frac{9}{11}$ lbs. at 12 cts., $7\frac{3}{11}$ lbs. at 14 cts., and $5\frac{5}{11}$ lbs. at 15 cts. are required to make a mixture of 100 lbs. worth 11 cents per pound.

RULE.—I. *Find the proportional quantities as in* § 308.

II. *Divide the given quantity by the sum of the proportional quantities, and multiply each of the proportional quantities by the quotient thus obtained.*

EXAMPLES FOR PRACTICE.

10. A farmer has a box that will hold 50 bushels: he has corn worth 75 cts., oats worth 37 cts., and barley worth 25 cts.; how much must he take of each, to fill the box with a mixture worth 30 cents?

11. I have a hogshead holding 65 gallons: I wish to fill it with brandy worth $2 per gallon, alcohol worth $1.50 per gallon, and water, so that the mixture may be worth $1.25; how much of each must I take?

12. A merchant has a barrel which will hold 200 pounds of sugar, and he wishes to fill it with sugars worth respectively 10 cts., 12 cts., 14 cts., and 20 cts. per pound, so that the mixture may be worth 15 cts.; how much of each must he take?

QUESTIONS.—Give the rule for finding the proportional quantity of each simple when the entire quantity is limited. (311.) What is Addition? (31.) Subtraction? (39.) Multiplication? (46.) Division? (54.)

SECTION XIII.

LESSON I.

RATIO.

312. **Ratio** is the relation in respect to magnitude which one number has to another of the same kind.

313. The **Terms** of a ratio are *antecedent* and *consequent*, and when taken together they are called a *couplet.*

314. The **Antecedent** is the first term of the couplet, and is the *standard* by which the other number is measured.

315. The **Consequent** is the second term of the couplet, and is the quantity *measured.*

316. Ratio may be expressed in two ways:—

1st. By two dots between the terms of the couplet; as, 3:12.

2nd. In the form of a fraction; as, $\frac{12}{3}$, or 12 divided by 3.

(*a.*) NOTE.—All numbers are finally compared with *unity* as a standard, hence when we say that the ratio 4:8 is 2, it is understood that the ratio 4:8 is equal to the ratio 1:2; and the ratio 4:3 is $\frac{3}{4}$ or 4:3 is the same ratio as 1:$\frac{3}{4}$.

MENTAL EXERCISES.

What part of 12 is 4?

ILLUSTRATION.—4 is $\frac{4}{12}$ of 12; or the ratio 12:4$=\frac{4}{12}$,$=\frac{1}{3}$; that is, 12 has the same ratio to 4 that 1 has to $\frac{1}{3}$.

1. What part of 20 is 2? is 5? is 7? is 8? is 6? is 9?
2. What part of 24 is 3? is 9? is 24? is 30?
3. What part of 2 is 1? is 2? is 8? is 4?
4. What is the ratio of 6 to 7?

ILLUSTRATION.—The ratio $6:7=\frac{7}{6},=1\frac{1}{6}$; or the ratio $6:7$ is the same as the ratio $1:1\frac{1}{6}$.

5. What is the ratio of 3 to 12? of 8 to 13? of 9 to 17?

6. What is the ratio of 5 to 6? of 3 to 7? of 7 to 3?

7. What is the ratio of $\frac{3}{4}$ to $3\frac{1}{3}$?

ILLUSTRATION.—The ratio of $\frac{3}{4}$ to $3\frac{1}{3}$ is $\frac{3\frac{1}{3}}{\frac{3}{4}},=\frac{40}{9},=4\frac{4}{9}$; or the ratio $\frac{3}{4}:3\frac{1}{3}$ is the same as the ratio $1:4\frac{4}{9}$.

8. What is the ratio of $\frac{1}{2}$ to $\frac{1}{4}$? of $\frac{1}{4}$ to $\frac{1}{2}$? of $\frac{5}{8}$ to $\frac{3}{7}$? of $3\frac{1}{2}$ to $\frac{1}{4}$?

9. What is the ratio of 5 qts. to 3 pts.? of 1 gal. to 3 qts. 1 pt.?

10. What is the ratio of 7 oz. to 3 oz.? of 4 lbs. to 1 lb. 6 oz.?

11. If the consequent is 4 and the ratio is 3, what is the antecedent?

ANALYSIS.—If the ratio is 3, or $1:3$, then, as the antecedent is $\frac{1}{3}$ as large as the consequent 3, the antecedent sought must be $\frac{1}{3}$ as much as the given consequent, and $\frac{1}{3}$ of 4 is $1\frac{1}{3}$, which is the antecedent required.

12. If the consequent is 3 the ratio 9, what is the antecedent?

13. If the antecedent is 5 and the ratio 3, what is the consequent?

QUESTIONS.—What is the equation of payments? (292.) What is the term of credit? (293.) What is the equated time? (294.) What is an account? (295.) What is a balance? (296.) Give the rule for finding the equated time, when the items have the same date. (297., *a.*) Give the rule, when the items have different dates. (298., *a.*)

LESSON II.

PROPORTION.*

317. Proportion is equality of ratios; thus, the ratio 9:3 is equal to the ratio 12:4; hence 9:3 and 12:4 form a proportion; thus, 9:3=12:4.

318. A Proportion is usually expressed by four dots; (thus, 3:7::6:14, to be read 3 is to 7 as 6 is to 14.)

319. Every proportion consists of four terms; the first and fourth are called the *extremes*, and the second and third are called the *means*. The first and third are called the *antecedents*, the second and fourth are called the *consequents*.

320. PRINCIPLES.—(*a.*) If the antecedents or consequents of a proportion, or both, are multiplied or divided by the same number, the proportion will still exist.

ILLUSTRATION.—Dividing the antecedents of the proportion 4:8::10:20 by 2 we have 2:8::5:20. Dividing the consequents we have 4:4::10:10. Dividing both we have 2:4::5:10. Multiplying the consequents we have 4:16::10:40. Multiplying the antecedents we have 8:8::20:20. Multiplying both we have 8:16::20:40. Each of these is a proportion, for the ratios of each are equal.

(*b.*) In every proportion the product of the extremes is equal to the product of the means.

(*c.*) Either mean is equal to the product of the extremes divided by the other mean.

(*d.*) Either extreme is equal to the product of the means divided by the other extreme.

If 9 pounds of coffee cost $3.60, what will 11 lbs. cost?

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*NOTE.—Simple proportion only will be treated in this work.

MODEL OPERATION.

Extreme. Mean. Mean. Extreme.
9 lbs. : 11 lbs. :: $3.60 : cost = 9 lbs. : 11 lbs. :: $3.60 : $4.40
11
9)39.60
$4.40, the required extreme.

Analysis.—1. It is evident that 9 lbs. bears the same ratio to 11 lbs. that the cost of 9 lbs. bears to the cost of 11 lbs.; therefore, we have the proportion 9 lbs. : 11 lbs. :: $3.60 : cost of 11 lbs.

2. Since in every proportion the product of the extremes is equal to the product of the means, the required extreme may be found by dividing the product of the means 11 and 3.60 by the given extreme 9 (320., *d.*) which gives $4.40 for the required extreme.

Therefore, if 9 lbs. of coffee cost $3.60, 11 lbs. will cost $4.40.

Rule.—*From the conditions of the question arrange the terms so that the ratios will be equal; then* (320., *b.*, *c.*, *d.*) *find the required term.*

1. If 11 bushels of wheat cost $9, what will 17 bushels cost?

2. What will 7 gallons of molasses cost if 63 gallons cost $13.16?

3. A fox is 35 rods before a greyhound, and while the fox is running 2 rods the greyhound runs 25 rods; how far must the dog run before he can catch the fox?

LESSON III.

Any example that can be solved by proportion may be easily solved by the following simple analysis.

If 30 cords of wood cost $120, how much will 20 cords cost?

MODEL OPERATIONS.

(*a.*)

cost.

1. $\$120 \times \frac{1}{30} = \$4$, cost.
2. $\$4 \times 20 = \$80$, Ans.

(*b.*)

cost.

$$\overset{4}{\cancel{\$120}} \times \frac{1}{\cancel{30}} \times 20 = \$80, \text{ Ans.}$$

Analysis.—(*a.*) 30 cords of wood cost $120.

1. One cord will cost $\frac{1}{30}$ as much as 30 cords, which is $4.

2. 20 cords will cost 20 times as much as one cord, which is $80.

Therefore, 20 cords of wood will cost $80, if 30 cords cost $120.

(*b.*) Note.—When the pupil thoroughly understands the analysis of this operation (*a.*), the more contracted form (*b.*) may be used, the same analysis being given as before.

EXAMPLES FOR PRACTICE.

4. If 5 bushels of rye cost $4.50, how much will 35 bushels cost?

5. If 5 yards of calico cost $3.50, how much will 50 yards cost?

6. If 7 pounds of coffee cost $2.80, what will 30 pounds cost?

7. If 25 pounds of sugar cost $2.50, what will 36 pounds cost?

8. If a pole 10 ft. 8 in. high cast a shadow 4 ft. 5 in. long, what is the length of the shadow cast at the same time by a steeple 138 feet high?

9. If a pole 10 ft. 9 in. high cast a shadow 8 ft. 2 in. long, what is the height of a steeple whose shadow at the same time of day is 50 ft. 8 in. long?

10. If 15 yards of calico cost $5, how many yards can be bought for $30?

11. If 25 yards of tape cost \$5.25, how many yards can be bought for \$25?

12. If 7 barrels of flour will last a family 56 weeks, how many barrels will they require in 15 months?

LESSON IV.

If $7\frac{1}{2}$ pounds of rice cost \$1.75, how many pounds can be bought for \$112.70?

MODEL OPERATIONS.

(*a.*) (*b.*)

$$1.\ \overset{\text{lbs.}}{7\tfrac{1}{2}} = \overset{\text{lbs.}}{\frac{15}{2}};\ \overset{\text{lbs.}}{\frac{\cancel{15}}{2}} \times \frac{1}{\underset{.35}{\cancel{1.75}}} = \overset{\text{lbs.}}{\frac{3}{.70}}.\quad \overset{\text{lbs.}}{\frac{\overset{3}{\cancel{15}}}{\cancel{2}}} \times \frac{1}{\underset{.35}{\cancel{1.75}}} \times \overset{161}{\cancel{112.70}} = \overset{\text{lbs.}}{483}, \text{ Ans.}$$

$$2.\ \frac{3}{\cancel{.70}} \times \overset{161}{\cancel{112.70}} = 483 \text{ lbs., Ans.}$$

Analysis.—(*a.*) \$1.75 will purchase $\frac{15}{2}$ lbs.

1. One dollar will purchase $\frac{1}{1.75}$ as many pounds as \$1.75, which is $\frac{3}{.70}$ lbs.

2. \$112.70 will purchase 112.70 times as many pounds as one dollar, which is 483 lbs.

Therefore, 483 lbs. can be purchased for \$112.70, if $7\frac{1}{2}$ lbs. cost \$1.75.

13. How many bushels of rye can be bought for \$78.90, if 12 bushels can be bought for \$13.32?

14. If $8\frac{3}{4}$ bushels of wheat cost \$4.50, what will be the cost of $15\frac{1}{2}$ bushels at the same rate?

15. If $\frac{3}{4}$ of a barrel of cider cost \$4.78, how much will $\frac{5}{8}$ of a barrel cost?

16. A grocer sells tea by Troy weight; he receives \$50 for a box; what should he have received?

17. A grocer sells sugar, giving only 14 oz. for a pound; he received $50 for a barrel; how much does he cheat his customers?

18. John bought a knife for 25 cts. and James bought one for 35 cts.; John valued his at 48 cts.; what value should James place upon his?

19. If 6 men can do a piece of work in 10 days, how long will it take 12 men to do it?

20. If it take 44 yards of oilcloth 1.25 yds. wide to cover a floor, how many yards will it take ⅞ of a yard wide?

LESSON V.

If 20 men in 16½ days of 8 hours each, dig a ditch 88 rods long, 8 feet deep, and 3 feet wide; how many men will be required to dig a ditch 360 rods long, 12 feet deep, and 8 feet wide, in 18 days, by working 12 hours each day?

MODEL OPERATION.*

(a.)

men.

1. $\overset{10}{\cancel{20}}\times\frac{33}{\cancel{2}}=33\times10$ men.

2. $33\times10\times8=33\times10\times8$ men.

3. $\overset{3}{\cancel{33}}\times10\times\cancel{8}\times\frac{1}{\underset{\cancel{11}}{\cancel{88}}}=30$ men.

4. $\overset{15}{\cancel{30}}\times\frac{1}{\underset{4}{\cancel{8}}}=\frac{15}{4}$ men.

---

*Note.—We are indebted to L. H. Birdseye, Prof. of Math. in the Utica Advanced School, for valuable suggestions in the following operation and analysis.

5. $\frac{\overset{5}{\cancel{15}}}{4} \times \frac{1}{\cancel{3}} = \frac{5}{4}$ men.

6. $\frac{5}{\cancel{4}} \times \overset{90}{\cancel{360}} = 5 \times 90$ men.

7. $5 \times 90 \times 12 = 5 \times 90 \times 12$ men.

8. $5 \times 90 \times 12 \times 8 = 5 \times 90 \times 12 \times 8$ men.

9. $5 \times \overset{30}{\cancel{90}} \times \overset{2}{\cancel{12}} \times 8 \times \frac{1}{\underset{3}{\cancel{18}}} = 5 \times 30 \times 2 \times 8$ men.

10. $5 \times \overset{5}{\cancel{30}} \times \cancel{2} \times 8 \times \frac{1}{\underset{2}{\cancel{12}}} = 200$ men, Ans.

(*b.*)

$$\overset{\text{men.}}{\frac{\overset{10}{\cancel{20}}}{1}} \times \frac{\overset{3}{\cancel{33}}}{\cancel{2}} \times \frac{\cancel{8}}{1} \times \frac{1}{\underset{8}{\cancel{88}}} \times \frac{1}{\cancel{8}} \times \frac{1}{\cancel{3}} \times \frac{\overset{20}{\cancel{360}}}{1} \times \frac{\cancel{12}}{1} \times \frac{\cancel{8}}{1} \times \frac{1}{\cancel{18}} \times \frac{1}{\cancel{12}} = \overset{\text{men.}}{200}$$

ANALYSIS.—(*a.*) To dig a ditch in $16\frac{1}{2}$ days according to the conditions of the question 20 men are required.

1. One day will require $\frac{1}{2}$ of 33 times as many men as $16\frac{1}{2}$ days, which is 33×10 men.

2. One hour will require 8 times as many men as 8 hours, which is $33 \times 10 \times 8$ men.

3. One rod long will require $\frac{1}{88}$ as many men as 88 rods, which is 30 men.

4. One foot deep will require $\frac{1}{8}$ as many men as 8 feet, which is $\frac{15}{4}$ men.

5. One foot wide will require $\frac{1}{3}$ as many men as 3 feet, which is $\frac{5}{4}$ men.

6. 360 rods long will require 360 times as many men as one rod, which are 5×90 men.

7. 12 feet deep will require 12 times as many men as one foot, which are 5×90×12 men.

8. 8 feet wide require 8 times as many men as one foot, which are 5×90×12×8 men.

9. 18 days will require $\frac{1}{18}$ as many men as one day, which is 5×30×2×8 men.

10. 12 hours will require $\frac{1}{12}$ as many men as one hour, which is 200 men.

Therefore, according to the conditions of the questions, 200 men will be required.

LESSON VI.

21. A farmer bought $25\frac{2}{3}$ barrels of flour at $\$6\frac{6}{11}$ a barrel, and paid for it in sheep at $3 per head; how many sheep did it take?

22. $4\frac{2}{3}$ barrels of sugar cost $112.75; how much will $\frac{7}{10}$ of a barrel cost?

23. If 24 men in 16 days perform a certain piece of work, how many men will be necessary to accomplish 3 times as much work in $\frac{3}{4}$ of a day?

24. If it require 2 bushels of provender to feed 4 mules $\frac{1}{3}$ of a day, how many mules would it take to consume 144 bushels in $\frac{3}{5}$ of a day?

25. If 12 men, by working 6 hours a day, build 72 rods of fence in 8 days, how many rods can 20 men build in 25 days, by working 12 hours a day?

26. 16 men can gather 44 acres of grain in 12 days by working 10 hours a day; how many men will be required to gather 440 acres in 10 days, by working 16 hours a day?

27. 12 men can cut 36 cords of wood in 8 days by working 10 hours a day; how many cords can 50 men cut in 26 days by working 12 hours a day?

28. In a certain factory 9448 yards of cloth were made by employing 120 hands 78 days, 10 hours a day; how much more could be made by employing 300 hands 4 times as many days for 10 hours 10 minutes each day?

29. I have a marble slab which is 25 feet long, 5 feet wide, and 3½ inches thick, and weighs 825 pounds; what will be the weight of a slab whose length is 8 feet, width 3 feet, and thickness 3½ feet?

30. 8 men are paid $48 for 6 days' labor; how many men may be employed 16 days for $96?

LESSON VII.

31. If 6 men can dig a ditch 80 feet long, 6 feet wide, and 4 feet deep, in 15 days, in what time can 18 men dig one 240 feet long, 8 feet wide, and 6 feet deep?

32. If 8 men pay $136.20 for 5 weeks' board, how much must 8 men pay for 3½ weeks' board?

33. The walls of a fortification are to be raised to the height of 27 feet; a squad of 12 men raised them 9 feet in 6 days: how many men must be employed to finish them in 4 days?

34. 5 bushels of wheat are worth 12 bushels of rye; 8 bushels of rye are worth 20 bushels of oats; and 9 bushels of oats are worth $4: how many bushels of wheat will $50 buy?

35. If 8 yds. of cloth 1¾ yds. wide cost $50, what will

be the cost of 14¾ yds. of cloth of the same quality, 2¼ yds wide?

36. I have a piece of coal which is 2 feet long, one foot wide, and 6 inches thick, and weighs 55 pounds; what will be the weight of a block 110 feet long, 92 feet wide, and 24 feet thick?

37 I have a stone 4 inches wide, 8 inches long, 3 inches thick, and weighing 2½ lbs.; what will be the weight of a monument, of the same kind of stone, 30 feet long, 12 feet wide, and 12 feet thick?

38. If $500 will gain $30.12 in 4 mo. 12 days, at 9 per cent., how much will $750 gain in 2 yrs. 9 mo. 8 da., at 6 per cent.?

39. If a garrison of 900 men eat $4540 worth of bread in 6 months, when flour is worth $7.50 a barrel, how many dollars' worth of bread will 578 men eat in 5 mo. 12 da., when flour is worth $9 a barrel?

40. A squad of 24 men by working 8 hours a day, can, in 18 days, erect a wall 95 rods long, 12 feet thick, and 9 feet high; how many men working 12 hours a day for 24 days will be required for erecting a wall 380 rods long, 9 feet thick, and 6 feet high?

SECTION XIV.

INVOLUTION AND EVOLUTION.

LESSON I.

INVOLUTION.

321. Involution is the method of finding any power.

322. A Power is a quantity produced by taking a given number a certain number of times as a factor. The factor thus taken is called the *root* of the power.

323. The **Exponent** or **Index** of the power is the number denoting the power, and is indicated by a small figure placed at the right of the *root*, thus, 9^3 indicates the third power of 9 or $9\times9\times9$; $(\frac{2}{3})^4$ indicates the fourth power of $\frac{2}{3}$.

ILLUSTRATIONS.

The first power of five $=5^1=5$.
" second " " $=5\times5=5^2=25$.
" third " " $=5\times5\times5=5^3=125$.
" fourth " " $=5\times5\times5\times5=5^4=625$.
" fifth " " $=5\times5\times5\times5\times5=5^5=3125$.
" first " two-thirds $=(\frac{2}{3})^1=\dfrac{2^1}{3^1}=\frac{2}{3}$.
" second " " $=\frac{2}{3}\times\frac{2}{3}=(\frac{2}{3})^2=\dfrac{2^2}{3^2}=\frac{4}{9}$.
" third " " $=\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}=(\frac{2}{3})^3=\dfrac{2^3}{3^3}=\frac{8}{27}$.

324. **To find any required power of a number.**

What is the cube of 12?

MODEL OPERATION.

$$12^3=12\times12\times12,=1728, \text{ Ans.}$$

ANALYSIS.—The cube of 12 equals 12 taken as a factor three times, which equals 1728.

1. What is the 3rd power of 5?
2. What is the 4th power of 6?
3. What is the 7th power of 2?
4. What is the 3rd power of 7?
5. What is the cube of 9?
6. What is the cube of 8?
7. What is the square of 11?
8. What is the 4th power of 9?
9. What is the 5th of $\frac{3}{4}$?
10.* What is the 4th power of $3\frac{5}{8}$?

QUESTIONS.—What is per cent.? (183.) How is per centage expressed? (184.) What is an agent or factor? (188.) What is commission? (189.) What is brokerage? (190.) What is a corpora-

* NOTE.—Reduce the mixed numbers to improper fractions.

tion? (193.) What is a charter? (192.) What is a firm? (191.) What is capital? (194.) What is a share? (195.) What is usually the value of a share? What are stockholders? (196.) When is stock said to be at par? (197.) When is stock said to be above par? (198.)

LESSON II.

EVOLUTION.

325. Evolution is the method of finding the root of a given power or number and is the reverse of involution.

326. A **Root** of any number is one of a number of equal factors which multiplied together produces the given number; thus 2 is the cube root of 8 because $2\times2\times2=8$.

327. The **Radical Sign** is a character ($\sqrt{}$), which is placed before numbers to denote that the root is to be extracted; thus, $\sqrt{81}$. indicates that the square root of 81 is to be extracted. When other roots than the square are required, a small figure or index denoting that root is placed over the radical sign; thus, $\sqrt[3]{27}=$ the cube root of 27; $\sqrt[5]{3125}=$the fifth root of 3125

(*a.*) NOTES.—1. The root and corresponding power take the same indices or exponents; thus, the cube or 3rd power corresponds to the cube or third root; the 4th power to the 4th root.

2. The square root of any number is one of two equal factors whose product equals the number. The cube root of any number is one of three equal factors; the fourth root is one of four equal factors, &c.

3. Exact roots are called *rational quantities*, and approximate roots are called *surds*.

4. Numbers that have exact roots are called perfect powers, and numbers that have no exact roots are called imperfect powers.

SQUARE ROOT.

328. The **Square Root** is the root of the second power; and to extract the square root of any number is to find a number which being raised to the second power will produce the number.

(*a*) **Note.**—The square of the greatest digit can not occupy more than *two* places, hence every two places in the power will give one place in the root.

Illustrations.—

Roots,	1,	2,	3,	4,	5,	6,	7,	8,	9,	10.
Squares,	1,	4,	9,	16,	25,	36,	49,	64,	81,	100.

What is the square root of 3969?

Illustration.*—I have a square board containing 3969 square inches; what are its dimensions?

Fig. 1.

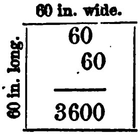


Fig. 2.

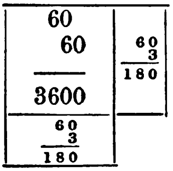
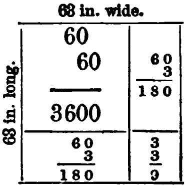


Fig. 3.



Proof.

sq. in.

$60 \times 60 = 3600$

$60 \times 3 = 180$

$60 \times 3 = 180$

$3 \times 3 = 9$

3969

MODEL OPERATION.

$$\begin{array}{rr}
 & 3969(60+3=63 \\
60\times60= & 3600 \\
\hline
60\times2=120) & 369 \\
120\times3= & 360 \\
\hline
 & 9 \\
3\times3= & 9 \\
\hline
 & 0
\end{array}$$

Analysis.—1. Since the length of the board is equal to the breadth, and the product of the length by the breadth gives the area, hence the square root of 3969 sq. in., area, will give the length of each side of the board.

2. Separate the number into periods of two figures counting from the right (228., *a*.) thus, 39,69, which shows that the root will contain two places.

3. By trial the length of the largest square is 60 in., and the

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*Note.—The extraction of the square root of an abstract number may be illustrated by the following example.

product of the length by the breadth is 3600 sq. in., (Fig. 1.) which subtracted from 39,69 sq. in. leaves a remainder of 369 sq. in.

4. In order to preserve the form of a square, the remainder must be added to two of the sides. (Fig. 2.)

5. Since the length of one of the sides is 60 in., and the length of the other side is 60 in., the entire length must be 2 times 60 in., which is 120 inches.

6. Since there are 120 sq. in. in one row of the entire addition, the addition will be as many rows wide as 120 sq. in. is contained times in 369 sq. in., which is 3; and if the length of the entire addition is 120 sq. in., and the width is 3 in., the product will be 3 times 120 sq. in., which is 360 sq. in., (Fig. 2.); 360 sq. in. subtracted from 369 sq. in. leaves 9 sq. in.

7. To complete the square (Fig. 2.), it will be necessary to add a small square 3 in. long and 3 in. wide, (Fig. 3.) containing 9 sq. in., which subtracted from 9 sq. in. leaves no remainder.

Therefore, the board is 63 in. long and 63 in. wide.

What is the square root of 391?

MODEL OPERATION.

	3,91(19.77+root.
	1
Trial divisor, 1×20=20.	291, dividend.
Complete divisor, 20+ 9=29.	261
Trial divisor, 19×20=380.	30.00, dividend.
Complete divisor, 380+ 7=387.	27.09
Trial divisor, 197×20=3940.	2.9100, dividend.
Complete divisor, 3940+ 7=3947.	2.7629
	1471, remainder.

For convenience, the following rule is adapted to the above contracted operation.

RULE.—I. *Separate the number into periods of two places each counting toward the left from units' place.*

II. *Find the root of the left hand period; subtract its square from the period; and to the remainder annex the next period for a dividend.*

III. *Multiply the root found, by* 20, *for a trial divisor, and write the quotient by the divisor as the next figure in the root; also add it to the trial divisor for a complete divisor.*

IV. *Multiply and subtract as in simple division, and to the remainder annex the next period for a new dividend.*

V. *Proceed in the same manner as before, until all the periods are brought down.*

NOTES.—1. If the dividend does not contain the complete divisor, place a cipher in the root, and then annex the next period as a new dividend.

2. If the given number contains a decimal not having an even number of decimal places, make them even by annexing ciphers before separating into periods; thus, 341.672=341.67,20.

3. When the exact root of the terms of a common fraction can not be found, change it to a decimal, and then extract the root of the decimal.

QUESTIONS.—When are stocks below par? (199.) What is the market value of stocks? (200.) What is a dividend? (201.) What are profit and loss? (206.) What is insurance? (211.) Who is the insurer? (212.) Who are the insured? (213.) What is a policy? (214.) What is the premium? (215.)

LESSON III.

EXAMPLES FOR PRACTICE.

Find the square root of each of the following numbers:—

11. $\sqrt{516961}$=what?
12. $\sqrt{182329}$=what?
13. $\sqrt{23804641}$=what?
14. $\sqrt{10673289}$=what?
15. $\sqrt{20894041}$=what?
16. $\sqrt{42025}$=what?
17. $\sqrt{1014049}$=what?
18. $\sqrt{538}$=what?
19. $\sqrt{71}$=what?
20. $\sqrt{24}$=what?

21. $\sqrt{.1024}$=what?
22. $\sqrt{.3364}$=what?
23. $\sqrt{.895}$=what?
24. $\sqrt{3.4}$=what?
25. $\sqrt{2.46}$=what?
26. $\sqrt{\frac{49}{529}}$=what?
27. $\sqrt{\frac{196}{625}}$=what?
28. $\sqrt{\frac{3721}{7569}}$=what?
29.* $\sqrt{60\frac{1}{16}}$=what?
30. $\sqrt{41\frac{3}{7}}$=what?

QUESTIONS.—What is a tax? (217.) How is a tax assessed? (218.) What is a poll tax? (219.) What is real estate? (220.) What is personal property? (221.) What is an inventory? (222.) What is the method of assessing taxes? (223.) What are duties? (224.) What is a custom house? What is a port of entry? (226.) What is tonnage? (227.)

LESSON IV.

CUBE ROOT.

329. The **Cube Root** of any number is the root of any third power or one of three equal factors whose product equals that number; and to extract the cube root of any number is to find that factor, or, in other words, a number which being raised to the third power will produce the given number.

(*a.*) NOTE.—The cube of the greatest digit can not occupy more than *three* places, hence every three places in the power will give one in the root.

ILLUSTRATION.—										
Root,	1	2	3	4	5	6	7	8	9	10.
Cube,	1	8	27	64	125	216	343	512	729	1000.

What is the cube root of 74088?

ILLUSTRATION.†—I have a cubical block containing 74088 cu. in.; what are its dimensions?

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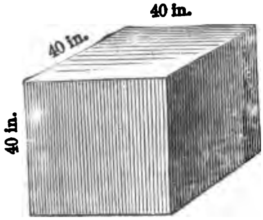
*NOTE.—Reduce the mixed numbers to improper fractions.

† NOTE.—The extraction of the cube root of an abstract number may be illustrated by the following example.

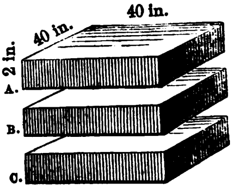
ILLUSTRATIONS.

Fig. 1.



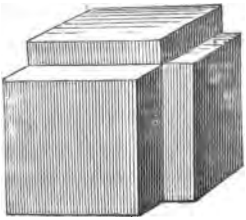
(a.)
40
40
1600
40
64000

Fig. 2.



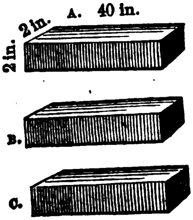
(b.)
40
40
1600, area.
3
4800 "
2
9600, cont.

Fig. 3.



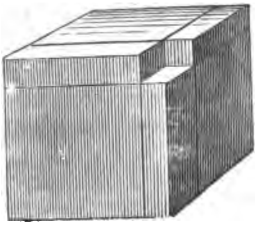
64000
9600
73600

Fig. 4.



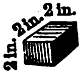
(c.)
40
2
80
2
160
3
480

Fig. 5.



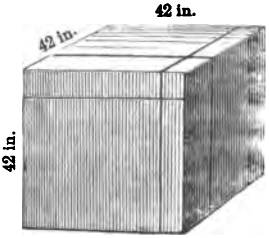
73600
480
74080

Fig. 6.



(d.)
2
2
4
2
8

Fig. 7.



(a.)
74088
64000
―――
10088
9600
―――
488
480
―――
8
8
―――
0

MODEL OPERATION.

74,088(40+2 _ 42, root.
(a) 40×40×40 = 64000
―――
40×40×3 = 4800)10088
(b.) 4800×2 = 9600
―――
488
(c.) 40×2×2×3 = 480
―――
8
(d.) 2×2×2 = 8
―――
0

ANALYSIS.—1. Since the product of the length, breadth, and thickness gives the solid contents of a cube, therefore the cube root of 74088 cu. in. will give the length of each side of the cubical block.

2. Separate the number into periods of three places, each counting from the right (329., *a.*); thus, 74'088, which shows that the root will contain two places.

3. By trial the length of the side of the largest cube is 40 inches, and the solid contents of this cube are 64000 cu. in., (40×40×40=64000; Fig. 1. *a.*) which subtracted from 74088 cu. in. leaves a remainder of 10088 cu. in. (Fig. 7. *a.*)

In order to preserve the form of a cube, the remainder must be added to three of the sides of the cube. (Fig. 3.)

4. Since the addition to each side of the cube must be 40 in. long and 40 in. wide, the addition to each side at 1 in. thick will contain 40 times 40 cu. in., which are 1600 cu. in., and the 3 sides will contain 3 times 1600 cu. in., which are 4800 cu. in. (Fig. 2., *b.*)

5. If it require 4800 cu. in. to make an addition 1 in. thick to the three sides of the cube, 1008 cu. in. will make the addition as many inches thick as 4800 cu. in. are contained times in 1008 cu. in., which are 2. If the addition at 1 in. thick contain 4800 cu. in., at 2 in. thick it will contain 2 times 4800 cu. in., which are 9600 cu. in. (Fig. 2., *b.*)

6. To farther complete the cube 3 blocks are required, each

40 in. long, 2 in. wide, and 2 in. thick, (Fig. 4.) which contain 480 cu. in. $(40\times2\times2\times3=480)$ (Fig. 4., *c.*) which subtracted from 488 cu. in. leave 8 cu. in.

7. To complete the cube is required a block (Fig. 6.) 2 in. long, 2 in. wide, and 2 in. thick, which contains 8 cu. in. $(2\times2\times2=8)$ (Fig. 6., *d.*) which subtracted from 8 cu. in. leaves no remainder. (Fig. 7., *a.*)

Therefore, the block is 42 in. long, 42 in. wide, and 42 in. thick.

QUESTIONS.—What is revenue? (228.) When are duties said to be ad valorem? (229.) When are duties said to be specific? (230.) What is an invoice? (231.) What is tare? (232.) What is leakage? (233.) What is breakage? (234.) What is gross weight? (235.) What is net weight? (236.) What is interest? (238.) What is the principal? (239.) What is rate per cent.? (240.)

LESSON V.

What is the cube root of 3874.23?

MODEL OPERATION.

	3,874.230(15.705+root.
$1^3=1.$	
$1^2\times300=300.$	2874, dividend.
$1\times5\times30=150.$	
$5\times5=25.$	
Complete divisor, 475	2375, product of the complete div., by 5.
Trial divisor, $(15)^2\times300=67500.$	499.230, new dividend.
$15\times7\times30=3150.$	
$7\times7=49.$	[divisor by 7.
Complete divisor, 70699	494.893, product of the complete
Trial divisor, $(157)^2\times300=7394700.$	4.337000000, new dividend.
Trial div. $(1570)^2\times300=739470000.$	
$1570\times5\times30=235500.$	
$5\times5=25.$	[plete div. by 5.
Complete divisor, 739705525	3.698527625, prod't of the com-
	.638472375, remainder.

For convenience in practice, the following rule is adapted to the above contracted operation:

RULE.—I. *Separate the numbers into periods of three places each, counting toward the left from units' place.*

II. *Find the root of the left hand period, subtract its cube from the period, and to the remainder annex the next period for a dividend.*

III. *Multiply the square of the root found by* 300 *for a trial divisor, and write the quotient by the divisor as the second figure in the root.*

IV. *Multiply the product of the root found, and the last figure by* 30, *and the result, plus the square of the last figure, added to the trial divisor, will form the complete divisor.*

V. *Multiply and subtract, as in simple division, and, to the remainder, annex the next period for a new dividend.*

VI. *Proceed in the same manner as before, until all the periods are brought down.*

NOTES.—1. If the dividend does not contain the complete divisor, write a cipher in the root, and annex the next period for a new dividend, with which proceed as before.

2. If the given number contains a decimal whose number of decimal places is not divisible by *three*, make it divisible by annexing ciphers, before separating into periods; thus, 4,473.2213 = 4,473.221,300.

QUESTIONS.—What is the amount? (241.) What is simple interest? (242.) What is compound interest? (243.) What is legal interest? (244.) What are partial payments? (248.) What is an indorsement? (249.) What is the rule of the Supreme Court of the United States? (250.)

LESSON VI.

EXAMPLES FOR PRACTICE.

Find the cube root of each of the following numbers;

31. 185193.

32. 80621568.

33. 17655848.

34. 257259456.

35. 1860867.
36. 1879080904.
37. 41673648.563.
38. 75686967.

QUESTIONS.—Write a rule for problem first in interest. (252.) For problems second, third, fourth, and fifth. (253.) (254.) (255.) (256.) What is discount? (259.) What is present worth? (260.) What is a bank? (261.) What is bank discount? (262.)

LESSON VII.

39. $\sqrt[3]{8.144865728}$=what?
40. $\sqrt[3]{4176.3841}$=what?
41. $\sqrt[3]{46}$=what?
42. $\sqrt[3]{9}$=what?
43. $\sqrt[3]{81\frac{5}{11}}$=what?
44. $\sqrt[3]{\frac{729}{4096}}$=what?
45. $\sqrt[3]{166\frac{3}{8}}$=what?
46. $\sqrt[3]{\frac{3}{8}}$=what?

SECTION XV.

MENSURATION.

LESSON I.

DEFINITIONS.

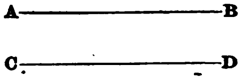
330. A **Point** is position without magnitude.

331. A **Line** has but one dimension, length; as, A B.

Fig. 1.

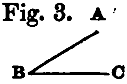
332. Parallel Lines are such as are everywhere equally distant; as, A B and C D.

Fig. 2.

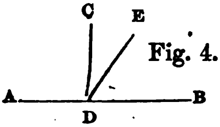


333. An **Angle** is the difference of direction between two lines which meet; as, A B C. The point of meeting is called the vertex of the angle; and when the angle is named the letter at the vertex is placed second; as, A B C.

Fig. 3.



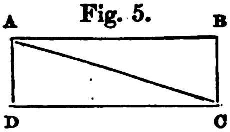
334. A **Right Angle** is one formed by a straight line meeting another perpendicularly, and the two angles thus formed are therefore equal; as, A D C and C D B.



335. An **Acute Angle** is one less than a right angle; as, E D B, Fig. 4.

336. An **Obtuse Angle*** is one greater than a right angle; as, A D E, Fig. 4.

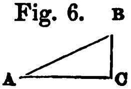
337. A **Surface** has two dimensions, length and breadth; as, A B C D.



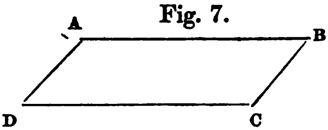
338. A **Triangle** is a figure having three sides (and three angles); as, A B C, Fig. 5.

339. The **Altitude** of a triangle is the distance of the vertex from the base or from the base produced; as, B C is the altitude of the triangle A B C, Fig. 6.

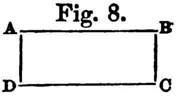
340. A **Right Angled Triangle** is a triangle having one right angle; as, A C B, Fig. 6. The line opposite the right angle is called the hypothenuse; as, A B.



341. A **Parallelogram** is a four-sided figure, whose opposite sides are parallel; as, A B C D, Fig. 7.



342. A **Rectangle** is a parallelogram all of whose angles are right angles; as, A B C D, Fig. 8.



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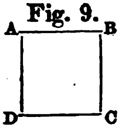
*As the right angle contains 90° it follows that the acute angle contains less and the obtuse angle more than 90°.

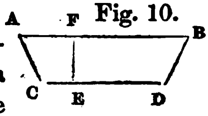
343 A **Square** is a rectangle whose sides are equal; as, A B C D, Fig. 9.

Fig. 9.



344. A **Trapezoid** is a four-sided figure having but two of its sides parallel; as, A B C D, Fig. 10.

Fig. 10.

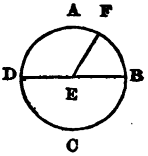


345. The **Altitude** of a Parallelogram, a Rectangle, a Square, or a Trapezoid, is the distance between the base and the parallel side opposite the base, or that side produced; as, E F, Fig. 10.

346. A **Circle** is a plane surface bounded by a curved line, every point of which is equally distant from a point within called the centre; as, A B C D.

347. The **Circumference** of a circle is the curved line by which it is bounded; as, A B C D.

Fig. 11.

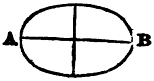


348. The **Diameter** of a circle is a straight line passing through the centre and terminating in the circumference; as, D E B.

349. The **Radius** of a circle is the distance from the centre to any point in the circumference; as, E F.

350. An **Ellipse** is a plane curve such that the sums of the distances of any points in the bounding line from two points within called the foci are always equal.

Fig. 12.



The line A B passing through the foci is called the major diameter; and the diameter perpendicular to A B at its centre is called the minor diameter.

351. A **Solid** has three dimensions, —length, breadth, and thickness; as, A B C, Fig. 13.

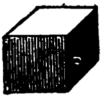


Fig. 13.

352. A **Prism** is a solid whose sides are parallelograms, and whose ends are equal and similar.

(*a.*) When the ends of a prism are triangular, it is called a *triangular prism;* as, Fig. 14.

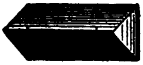


Fig. 14.

(*b.*) When the ends of a prism are square, it is called a *square prism;* as, Fig. 15.

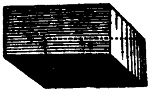


Fig. 15.

(*c.*) When the ends of a prism are hexagonal, it is called an *hexagonal prism;* as, Fig. 16.

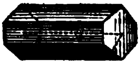


Fig. 16.

(*d.*) When the ends of a prism are circular, it is called a *cylinder;* as, Fig. 17.

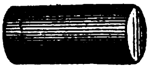


Fig. 17.

(*e.*) When all the sides of a rectangular prism are square it is called a *cube;* as, Fig. 18.

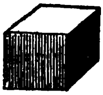


Fig. 18.

353. A **Pyramid** is a solid the base of which is a plain rectilinear figure, and having sides which are triangles whose vertices meet at a point at the top called the vertex of the pyramid. Fig. 19.

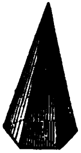


Fig. 19.

(*a.*) The **Altitude** of a pyramid or a cone is measured by a straight line let fall from the vertex perpendicular to the base; as, Fig. 20.

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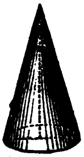
*A cylinder is a regular polygon or prism with an infinite number of sides.

Fig. 20.



354. A **Cone** is a solid, the base of which is a circle, and which tapers uniformly to a point at the top called the vertex of the cone. Fig. 20

355. A **Frustum** of a pyramid or a cone is the part that remains after cutting off the top by a plane parallel to the base.

Fig. 21.

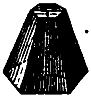


Fig. 22.

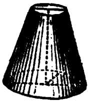
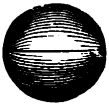


Fig. 21 represents the frustum of a pyramid.

Fig. 22 represents the frustum of a cone.

356. A **Sphere** is a solid, bounded by a convex surface, every point of which is equally distant from a point within called the centre; as, Fig. 23.

Fig. 23.



357. A **Spheroid** is a solid, generated by the revolution of an ellipse about one of its diameters. If the ellipse revolves about its major diameter the spheroid is called *prolate.* If about its minor diameter the spheroid is called *oblate.*

Fig. 24.

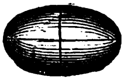


Fig. 25.

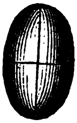


Fig. 24 represents a prolate spheroid.

Fig. 25 represents an oblate spheroid.

LESSON II.

PROBLEMS IN MENSURATION.

358. The **Area** of a plain figure, or of the surface of a solid, is the superficial contents in square measure.

359. The **Solidity** of a body is its contents in cubic measure.

PROBLEM I.—The base and altitude of a triangle being given, to find its area.

RULE.—*Multiply the base by half the altitude.*

ILLUSTRATION.—The base of a triangle is 8 ft., and the altitude is 4 feet; what is the area?

MODEL OPERATION.

base.		half alt.		area.
8 ft.	×	2	=	16 sq. ft.

PROB. II.—The sides of a triangle being given to find the area.

RULE.—*From half the sum of the three sides subtract each side separately, then multiply the continued product of these remainders by half the sum of the sides, and extract the square root of the product, and the result will be the area.*

ILLUSTRATION.—The sides of a triangle are respectively, 12, 18, and 24 feet; what is the area?

MODEL OPERATION.

12+18+24=54, sum of the sides.
54÷ 2=27 half, sum of the sides.
27—12=15, 1st remainder.
27—18= 9, 2nd remainder.
27—24= 3, 3rd remainder.
15×9×3×27=10935.
$\sqrt{10935}$=104.57, sq. ft. area.

PROB. III.—The sides of a rectangle or square being given, to find the area.

RULE.—*Multiply the length by the breadth.*

ILLUSTRATION.—The length of a rectangular figure is 12 ft. and the width is 9 feet; what is the area?

MODEL OPERATION.

$12 \times 9 = 108$, area.

PROB. IV.—The base and altitude of a parallelogram being given, to find the area.

RULE.—*Multiply the base by the altitude.*

ILLUSTRATION.—There is a parallelogram whose length is 12 feet, and altitude 8 feet; what is the area?

MODEL OPERATION.

$12 \times 8 = 96$, area.

QUESTIONS.—What is involution? (321.) What is a power? (322.) What is an exponent? (323.) What is evolution? (325.) What is the root of a number? (326.) What is a radical sign? (327.) What is the square root? (328.) What is the cube root? (329.)

LESSON III.

PROB. V.—The altitude and the parallel bases of a trapezoid being given, to find the area.

RULE.—*Multiply the altitude by half the sum of the parallel sides.*

ILLUSTRATION.—What is the area of a trapezoid the parallel sides of which are 12 and 18 feet respectively, and the altitude 8 feet?

MODEL OPERATION.

$12 + 18 = 30$; $30 \div 2 = 15$; $15 \times 8 = 120$, area.

PROB. VI.—The diameter of a circle being given, to find the circumference.

RULE.—*Multiply the diameter by* 3.1416.

ILLUSTRATION.—The diameter of a circle is 8 feet; what is the circumference?

MODEL OPERATION.

$$3.1416 \times 8 = 25.1328 \text{ ft., circumference.}$$

PROB. VII.—The circumference of a circle being given, to find its diameter. Fig. 11.

RULE.—*Divide the circumference by* 3.1416.

ILLUSTRATION.—The circumference of a circle is 25.1328 feet; what is the diameter?

MODEL OPERATION.

$$25.1328 \div 3.1416 = 8 \text{ ft., diameter.}$$

PROB. VIII.—The diameter of a circle being given, to find the area. Fig. 11.

RULE.—*Multiply the square of the diameter by* .7854.

ILLUSTRATION.—The diameter of a circle is 9 inches; what is the area?

MODEL OPERATION.

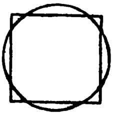
$$9^2 = 81;\ 81 \times .7854 = 63.6174, \text{ area.}$$

QUESTIONS.—What is a point? (330.) What is a line? (331.) What are parallel lines? (332.) What is a right angle? (334.) What is an acute angle? (335.) What is an obtuse angle? (336.) What dimensions has a surface? (337.) What dimensions has a solid? (351.) What is a triangle? (338.) What is the altitude of a triangle? (339.)

LESSON IV.

PROB. IX.—The diameter of a circle being given, to find the side of the square having an equal area.

Fig. 26.



RULE.—*Multiply the diameter by* .8862.

ILLUSTRATION.—The diameter of a circle is 9 inches; what is the side of the square having an equal area? Fig. 26.

MODEL OPERATION.

$.8862 \times 9 = 7.9758$, side of square.

Prob. X.—The circumference of a circle being given, to find the side of a square having an equal area. Fig. 26.

Rule.—*Multiply the circumference by* .28209.

Illustration.—The circumference of a circle is 12 inches; what is the side of a square having an equal area?

MODEL OPERATION.

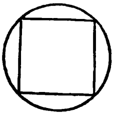
$.28209 \times 12 = 3.38508$, side of square.

Prob. XI.—The diameter of a circle being given, to find the side of the inscribed square. Fig. 27.

Rule.—*Multiply the diameter by* .7071.

Illustration.—What is the thickness of a square stick of timber that can be sawed from a log 20 inches in diameter?

Fig. 27.



MODEL OPERATION.

$.7071 \times 20 = 14.1420$, the thickness in inches.

Prob. XII.—The circumference being given, to find the side of the inscribed square.

Rule.—*Multiply the circumference by* .22508.

Illustration.—What is the thickness of the square stick of timber that can be hewn from a log 100 inches in circumference?

MODEL OPERATION.

$.22508 \times 100 = 22.50800$, the thickness in inches.

Prob. XIII.—The diameters of an ellipse being given, to find the area. Fig. 13.

Rule.—*Multiply the product of the two diameters by* .7854.

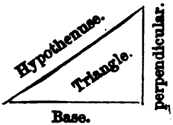
ILLUSTRATION.—What is the area of an ellipse whose major diameter is 12 inches, and whose minor diameter is 10 inches?

MODEL OPERATION.

$12 \times 10 = 120$; $120 \times .7854 = 94.248$ inches, the area of the ellipse.

LESSON V.

Fig. 28.



PROB. XIV.—The base and perpendicular of a right-angled triangle being given, to find the *hypothenuse.* Fig. 28.

RULE.—*Add the squares of the* BASE *and* PERPENDICULAR, *and extract the square root of their sums.*

(*a.*) To find the perpendicular, when the hypothenuse and base are given.

RULE.—*From the square of the hypothenuse, subtract the square of the base, and extract the square root of the remainder.*

(*b.*) To find the base, when the hypothenuse and perpendicular are given.

RULE.—*From the square of the hypothenuse, subtract the square of the perpendicular, and extract the square root of the remainder.*

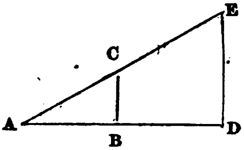
ILLUSTRATION.—The base of a right-angled triangle is 12 inches, and the perpendicular is 9 inches; what is the hypothenuse?

MODEL OPERATION.

$$\begin{array}{rl} (12)^2 = & 144 \\ 9^2 = & 81 \\ \hline & \sqrt{225} = 15 \text{ in., Ans.} \end{array}$$

Prob. XV.—The three sides of a right-angled triangle, and one side of a similar triangle being given, to find the other sides.

Fig. 29.



Rule.—*Construct two similar right-angled triangles; as,* A B C; A D E; *then* A B:A D::B C:D E *and* A B:A D::A C:A E.

Illustration.—The base of a right-angled triangle is 9 feet, the perpendicular is 9 feet, and the base of a similar triangle is 100 feet; what is the perpendicular?

MODEL OPERATION.

AB:AD::BC:DE; as, AB=9ft., AD=100ft., and BC=12.
Therefore 9:100 :: 12 :DE
12

9)1200

133⅓=DE, Ans.

Prob. XVI.—The length, and the perimeter of the end of a prism being given, to find the surface. Fig. 15.

Rule.—*Multiply the length by the perimeter, and to the product add the areas of the ends.*

Illustration.—The length of a triangular prism is 9 feet; and the sides are respectively 2, 3, and 4 feet wide; what is the surface?

MODEL OPERATION.

9×9=81. area of sides.
5.808 area of ends.*

86.808 sq. ft., Ans.

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*Note.—To find the area of the triangles forming the ends see Prob. II.

Prob. XVII.—The length and perimeter of a prism being given, to find its solidity. Fig. 17.

Rule.—*Multiply the area of one end by the length.*

Illustration.—What is the solidity of a cylindrical prism, the length being 12 feet, and the diameter 3 feet?

MODEL OPERATION.

$3^2 \times .7854 = 7.0686$, area of the end.
$7.0686 \times 12 = 84.8232$ cu. ft., Ans.

Prob. XVIII.—The side of a cube being given, to find the area of its surface. Fig. 18.

Rule.—*Multiply the area of one of its sides by* 6.

Illustration.—I wish to make a cubical box the length of whose side is 9 feet; how many square feet of boards do I require, taking no account of their thickness?

MODEL OPERATION.

$9 \times 9 = 81$
$81 \times 6 = 486$ sq. ft., Ans.

Questions.—What is a square? (343.) What is a rectangle? (342.) What is a parallelogram? (341.) What is a trapezoid? (344.) What is a circle? (346.) What is a circumference? (347.) What is a diameter? (348.) What is a radius? (349.) What is an ellipse? (350.)

LESSON VI.

Prob. XIX.—The side of a cube being given, to find its solidity. Fig. 18.

Rule.—*Multiply the product of the length and breadth by the thickness.*

Illustration.—What are the solid contents of a cubical block of marble the side of which is 9 feet?

MODEL OPERATION.

$9\times9\times9=729$ cu. ft., Ans.

Prob. XX.—The slant height and the side of the base of a pyramid being given, to find its area. Fig. 19.

Rule.—*To the areas of the triangles which form its sides, add the area of its base.*

Illustration.—What is the area of a square pyramid, the slant height being 15 feet, and the side of the base 3 feet?

MODEL OPERATION.

$22.6\times4=90.4$, area of sides.*
$3\times3=9$, area of base.
$90.4+9=99.4$ sq. ft., Ans.

Prob. XXI.—The altitude and base of a pyramid being given, to find its solidity. Fig. 19.

Rule.—*Multiply the area of its base by one-third of its altitude.*

Illustration.—How many cubic feet in a triangular pyramid, the altitude of which is 12 feet, and the length of each side of whose base is 4 feet?

MODEL OPERATION.

$\overset{\text{area of base.}}{6.92}\times\frac{1}{3}$ of $12=27.68$ cu. ft., Ans.

Prob. XXII.—The slant height and diameter of the base of a cone being given, to find its surface. Fig. 20.

Rule.—*Multiply the circumference of the base by half the sum of the slant height and the radius of the base.*

Illustration.—What is the surface of a cone whose slant height is 20 inches, and whose diameter is 8 inches?

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*Note.—To find the area of the triangles which form its sides, see Prob. II.

MODEL OPERATION.

$20+4=24$, sum of slant height and radius.
$8\times 3.1416=25.1328$, circumference.
$25.1328\times\frac{1}{2}$ of $24=301.5936$ sq. ft., Ans.

QUESTIONS.—What is a prism? (352.) What is a circle? (346., *d.*) What is a pyramid? (353.) What is a cone? (354.) What is the frustum of a cone or pyramid? (355.) What is a sphere? (356.) What is a spheroid? (357.) What is an area? (358.) What is the solidity of a body? (359.)

LESSON VII.

PROB. XXIII.—The altitude and the diameter of the base of a cone being given, to find its solidity. Fig. 20.

RULE.—*Multiply the area of the base by one-third of the altitude.*

ILLUSTRATION.—What is the solidity of a cone whose altitude is 12 inches, and whose diameter is 3 inches?

MODEL OPERATION.

$3^2\times.7854=7.0686$, area of base.
$7.0686\times\frac{1}{3}$ of $12=28.2744$ cu. in., Ans.

PROB. XXIV.—The perimeter of the ends and the slant height of a frustum of a cone being given, to find the area. Fig. 22.

RULE.—*Multiply the sum of the perimeter of the two ends by one-half the slant height, and to the product add the areas of the ends.*

ILLUSTRATION.—What is the surface of the frustum of a cone, the slant height being 30 inches, the diameter of one end 12 inches, and that of the other 4 inches?

MODEL OPERATION.

$$\left.\begin{array}{r}12\times3.1416=\ 37.6992\\ 4\times3.1416=\ 12.5664\end{array}\right\}\text{perimeters.}$$

$$\overline{50.2656}\times\tfrac{1}{3}\text{ of }30\qquad=753.984\text{, convex surface.}$$

$$\left.\begin{array}{r}4^2\times.7854=\ 12.5664\\ (12)^2\times.7854=113.0976\end{array}\right\}\text{area of ends. }=\underline{125.664}$$

$$879.648\text{ sq. in., Ans.}$$

PROB. XXV.—The perimeter of the ends and the altitude of a frustum of a pyramid or cone being given, to find the solidity. Figs. 22, 23.

RULE.—*To the square root of the product of the areas of the ends, add the areas of the ends, and multiply the sum by one-third the altitude.*

APPLICATION.—How many cubic inches in a circular block of marble 20 inches long, 4 inches in diameter at one end, and 8 inches at the other?

MODEL OPERATION.

$$\left.\begin{array}{r}4^2\times.7854=\ 16\times.7854\\ 8^2\times.7854=\ 64\times.7854\end{array}\right\}\text{areas of ends.}$$

$$\sqrt{4^2\times8^2\times.7854^2}=\ 32\times.7854\text{ sq. root of prod. of areas.}$$

$$\overline{112}\times.7854\times\tfrac{1}{3}\text{ of }20=586.432\text{ cu. in., Ans.}$$

PROB. XXVI.—The diameter of a sphere being given, to find its area. Fig. 24.

RULE.—*Multiply the circumference by its diameter.*

ILLUSTRATION.—How much leather will it take to cover a ball 5 inches in diameter?

MODEL OPERATION.

$5\times3.1416=15.708$, circumference.

$15.708\times5=78.54$ sq. in., Ans.

LESSON VIII.

Prob. XXVII.—The diameter of a sphere being given, to find the solidity. Fig. 23.

Rule.—*Multiply the cube of the diameter by* .5236.

Illustration.—How many cubic inches in a cannon ball 6 inches in diameter?

MODEL OPERATION.

$6^3 \times .5236 = 113.0976$ cu. in., Ans.

Prob. XXVIII.—The major and the minor diameter of a prolate spheroid being given, to find its solidity. Fig. 24.

Rule.—*Multiply the product of the square oj the minor into the major diameter, by* .5236.

Illustration.—What is the solidity of a prolate spheroid whose major diameter is 30 inches and whose minor diameter is 20 inches?

MODEL OPERATION.

$(20)^2 \times 30 = 12000$, product.
$12000 \times .5236 = 6283.2$ cu. in., Ans.

Prob. XXIX.—The major and minor diameters of an oblate spheroid being given, to find its solidity. Fig. 25.

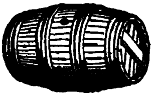
Rule.—*Multiply the product of the square of the major into the minor diameter, by* .5236.

Illustration.—What is the solidity of an oblate spheroid whose diameters are respeetively 30 and 10 inches?

MODEL OPERATION.

$(30)^2 \times 10 = 9000$, prod.
$9000 \times .5236 = 4712.4$ cu. in., Ans.

Fig. 30.



PROB. XXX.—The length, the bung, and the head diameters of a cask being given, to find the contents in gallons. Fig. 30.

RULE.—I. *Add to the head diameter two-thirds* the difference between the head and bung diameters, and the result will be the mean diameter.*

II. *Multiply the product of the square of the mean diameter into the length, by* .0034.

ILLUSTRATION.—How many gallons in a cask whose bung diameter is 18 inches, head diameter 12 inches, and length 25 inches?

MODEL OPERATION.

$12+\frac{2}{3}$ of $6=16$, mean diameter.

$(16)^2\times25\times.0034=21.76$ gal., Ans

LESSON IX.

PRACTICAL EXAMPLES IN MENSURATION.

1. There is a triangular field, the length of one side of which is 13 ch. 25 l., and the distance between this side and the vertex of the opposite angle is 9 ch. 13 l.; how many acres does the piece of land contain? (See Prob. I.)

2. There is a triangular piece of land; the length of each side is 14 ch. 29 l.; how many acres does the piece contain? (See Prob. II.)

3. I bought a square piece of land 130 rds. long, and 80 rds. wide; how much did it cost me, at $75. per acre? (See Prob. III.)

4. Of a diamond shaped piece of land 30 rds. long, and 20 rds. wide, I sold 2¾ acres; how much remained? (See Prob. IV.)

5. I have a board 12 feet in length, 5 inches wide at one

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*NOTE.—If the staves are not much curved add six-tenths.

end, and 9 inches at the other; how much is it worth at 3 cts. per sq. ft.? (See Prob. V.)

6. What is the length of a piece of wagon tire that will just fit a wheel 2 ft. 6 in. in diameter? (See Prob. VI.)

7. What is the diameter of a wheel that will revolve 236 times in going 198 rds.? (See Prob. VII.)

8. How many acres in a circular pond, the diameter of which is $\frac{5}{8}$ of a mile? (See Prob. VIII.)

9. I have a circular band of iron, the inner circle of which is 15 inches in diameter; what is the width of one side of a square stick of timber that the band will just fit? (See Prob. IX.)

10. A mechanic has a band of iron 27 inches long; what must be the width of one side of a square stick of timber which the band will just compass? (See Prob. X.)

11. How large will be a square stick of timber that can be sawed from a log 27 inches in diameter? (See Prob. XI.)

12. The circumference of a log is 5 feet 2 inches; what is the thickness of the largest square stick of timber that can be hewn from it? (See Prob. XII.)

13. How many square feet in the ceiling of an elliptical room, the length of which is 13 feet, and the width 8 feet? (See Prob. XIII.)

QUESTIONS.—Give the rule for Prob. I. (359.) For Prob. II. For Prob. III. For Prob. IV. For Prob. V. For Prob. VI. For Prob. VII.

LESSON X.

14. There is a wall 33 feet high. The foot of a ladder is nine feet from the wall; how long must the ladder be to reach the top of the wall? (See Prob. XIV.)

15. How much will it cost to plaster the walls and ceiling

of a triangular room 11 feet high, each side being 8 feet 3 in. long; at 50 cts. per square yard? (See Prob. XVI.)

16. How many cubic feet of timber in a pentagonal shaft 25 feet long, the area of the end being 320 sq. inches? (See Prob. XVII.)

17. How many feet of boards 1 inch thick will it take to make a cubical box, the side of which is 8 feet 7 inches; and how much will it cost at 3 cts. per square foot? (See Prob. XVIII.)

18. How many gallons of water can be poured into a cistern 8 ft. 8 in. long, 7 ft. 3 in. wide, and 6 ft. 9 in. deep? (See Prob. XIX.)

19.* How much will it cost to paint the pyramidal spire of a church, each of the four sides being 12 feet at the base, and having a slant height of 45 feet, at 50 cts. per square foot? (See Prob. XX.)

20. How many cubic feet in a pentagonal pyramid, the length of which is 25 feet, and the area of whose base is 12 sq. feet? (See Prob. XXI.)

21.* How much will it cost to paint the circular spire of a church, the slant height of which is 45 feet, and the diameter of whose base is 9 feet, at 25 cts. per square foot? (See Prob. XXII.)

22. How many tons of hay in a conical hay-stack 20 feet high and 40 feet in circumference, if 1 cubic foot weigh $\frac{3}{4}$ of a pound? (See Prob. XXIII.)

23. How many feet of inch rope will it take to wind a circular stick of timber 8 inches in diameter at one end, and 12 inches in diameter at the other? (See Prob. XXIV.)

24. At 3 cts. per cu. ft. what will be the cost of a round

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* NOTE.—The area of the base in these examples is to be deducted.

stick of timber 15 feet long, 18 inches in diameter at one end, and 11 inches at the other? (See Prob. XXV.)

LESSON XI.

25. How many square inches of leather will it require to cover a ball 6 inches in diameter? (See Prob. XXVI.)

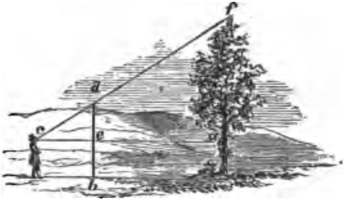
26. How much will a cannon ball weigh whose diameter is 8 inches, if 27 cubic inches weigh 12 pounds? (See Prob. XXVII.)

27. What is the solidity of a prolate spheroid 9 inches long, and 4 inches thick? (See Prob. XXVIII.)

28. What is the solidity of an oblate spheroid 9 inches long, and 4 inches thick? (See Prob. XXIX.)

29. How many gallons in a cask 46 inches long, whose bung diameter is 30, and whose head diameter is 25 inches? (See Prob. XXX.)

Fig. 31.

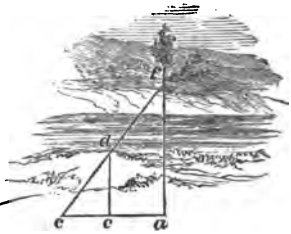


30. A man desirous of finding the height of a certain tree placed a ten foot pole perpendicularly 100 feet from the foot of the tree at B; he then stepped backward until he could just see the top of the tree over the end of the pole. On measurement he found the place on which he stood at C, was 12 feet from the pole. How high was the tree, the eye of the man being 4 feet 9 inches above the ground? (See Prob. XV.)

31. A man desirous of finding the distance of a certain tree on the opposite side of a river proceeded to construct

the right-angled triangles C A F, (Fig. 32.) making C A 10 rods. From C toward A he measured C E ½ a rod, and drew E D parallel to A F. By measurement he found E D 15 rods; what was the distance to the tree? (See Prob. XV.)

Fig. 32.



LESSON XII.

LUMBER.

360. Lumber and timber are usually computed by board measure. 1 foot by this measure has the dimensions, length, one foot, width, one foot, thickness, one inch. Round timber is sometimes computed by board measure, and sometimes by cubic measure.

Hence, to find the contents of boards, planks, joists, beams, &c., is given the following

(*a.*) RULE.—I. *Multiply the length by the breadth, when the lumber is not more than* 1 *inch thick.*

II. *When more than one inch thick, multiply together the length, the width, and the thickness taken in inches.*

EXAMPLES FOR PRACTICE.

32. What are the contents of a board 14 feet long, and 8 inches wide?

33. What are the contents of a plank 12 feet long, 9 inches wide, and 2 inches thick?

34. What are the contents of a joist 15 feet long, 4 inches wide, and 3 inches thick?

35. What will be the cost of a stick of timber 25 feet long, 8 inches wide, and 6 inches thick, at $2\frac{1}{2}$ cts. per foot?

36. How much will the following bill of lumber cost at $2\frac{2}{3}$ cts. per foot?

4 sticks 24 feet long, 6 by 8 inches.
4 " 36 " " 6 by 8 inches.
125 joists 12 feet long, 2 by 3 inches..
150 " 12 " " 2 by 4 inches.
5,000 feet of siding, at $20 per thousand.
11 bunches shingles, at $5 per bunch.
7,000 feet of boards, at $18 per thousand.
10,000 lath, at $4 per thousand.

Questions.—How is lumber usually computed? (360.) Give the rule. (360., *a.*)

SECTION XVI.

LESSON I.

MISCELLANEOUS EXAMPLES.

1. A man bought a carriage for $\$136\frac{3}{4}$, a horse for $\$380\frac{2}{7}$ more than he paid for the carriage, a house and lot for $\$9872\frac{2}{3}$ more than he paid for both the horse and carriage; how much did all cost him?

2. A merchant bought $37\frac{2}{3}$ pieces of cloth for \$9335, and sold them for $\$10610\frac{1}{7}$; how much did he gain per cent.?

3. A merchant bought $37\frac{2}{3}$ pieces of cloth for \$9335, and sold them for $\$10610\frac{1}{7}$; how much did he gain per piece?

4. A merchant bought 52 hogsheads of sugar for $\$2184\frac{2}{3}$; he sold half of them for \$1170.273; how much did he gain per hogshead?

5. A, B, and C enter into a partnership with a capital of \$669.50; A furnishes \$152; B, \$205.37; and C, the remain-

der in 21 installments; what is the amount of each installment?

6. The company invest \$208.64 in merchandise, \$247.29 in stocks, and the remainder in land at \$$31\frac{3}{8}$ per acre; after keeping the land 8 months they sold it for 8 per cent. advance; how many acres did they buy?

7. James had a certain number of marbles, when he commenced playing with Henry; in the first game he lost 36 marbles; in the second, 28; in the third game he won $\frac{5}{8}$ as many as he lost in the first and second games; he had 15 left; how many had he at first?

8. A, B, and C draw a prize of \$15236; A's share is 19 per cent., B's share is 36 per cent. more than A's share. C buys with his share a farm, for which he gives \$$48\frac{2}{7}$ per acre; how much does it contain?

LESSON II.

9. How many yards of linen $\frac{3}{4}$ of a yard wide will line $20\frac{1}{2}$ yards of broadcloth $1\frac{1}{4}$ yards wide, allowing 10 per cent. shrinkage?

10. If the dividend be $\frac{3}{8}$ and the quotient $\frac{4}{36}$, what is the divisor?

11. The sum of two fractions is $1\frac{1}{2}$, the difference is $\frac{1}{4}$; what are the fractions?

12. If 11 be added to both terms of the fraction $\frac{5}{9}$, will the value of the fraction be increased or diminished, and how much?

13. If 11 be subtracted from both terms of the fraction $\frac{10}{18}$, will the value of the fraction be increased or diminished, and how much?

14. A merchant purchased 344 yards of cloth at $41\frac{2}{3}$ cts.

a yard; had the cloth been 5$\frac{3}{7}$ cts. cheaper, how many more yards could have been purchased for the same money?

15. A, B, and C draw a prize of $15286; A's share is $4720; B's share is $\frac{2}{3}$ more than A's; C buys with his share a farm worth 48\frac{2}{7}$ per acre; how many acres does it contain?

16. How many bottles of 4$\frac{3}{7}$ pts. each, can be filled from 2$\frac{3}{4}$ hogsheads of wine?

17. The first of three numbers is 894868, the second is $\frac{3}{8}$ more than the first; the third equals the sum of the first and second; what is the sum of all the numbers?

18. How many steps of 2 feet 5 inches each would a man take in walking from Newark to New York, the distance being 9 miles?

QUESTIONS.—What is a unit? (1.) What is a number? (2.) What is the difference between an abstract and a concrete number? (3.) (4.) Of what does Arithmetic treat? (5.) What is quantity? (6.) What is a problem? (7.) What is the difference between a simple and a complex problem? (8.) (9.) Repeat an Arithmetical formula. (11.*) What is a rule? (12.) What is a sign? (13.) What are the principal signs used in Arithmetic? (32.) (33.) (34.) (43.) (51.) (62.) (116.) Repeat a complex problem. (9.) What is an axiom? (14.) Illustrate on the blackboard each of the axioms. (16.) What is the difference between the Roman and the Arabic methods of notation? (18.) (19.) What letters are used in the Roman? (18.) What figures in the Arabic? (19.) Give a complete description of the method of expressing numbers by the Roman notation. (18.) By the Arabic notation. (19.)

LESSON III.

19. A drover bought 32 head of cattle for $2400.83; 21 head for 1386\frac{1}{4}$; 25 head for $1984; and 13 head for $1040; he sold them all for 7371\frac{1}{7}$; what did he receive per head?

20. How many spoons, each weighing 3 oz. 4½ pwt. can be made from 32 lbs. 9¼ oz. 4 pwt. of silver?

21. A, B, and C can do a piece of work in 8½ days; B and C can do it in 9¾; in what time can A do it?

22. An apothecary bought 3 lb. 10 oz. of aloes by avoirdupois weight, at 35 cts. an ounce, and sold it for 3 cts. per grain apothecaries' weight; did he make or lose, and how much?

23. A druggist bought 25 lbs. opium by avoirdupois weight, at $5.25 per pound, and sold it at the same price apothecaries' weight; did he make or lose, and how much?

24. A man has four farms and two houses; his farms were worth $3896 each; his houses $9376 each; to his son he gave ¾ of three farms and one house, and to his daughter the remainder of his property; how much did each receive?

25. A train of cars runs 241880 feet in one hour and 32 seconds; how far will it run in one hour and one minute?

26. The earth in its orbit moves 802192000 miles in one year and 132 days; how far does it move in one year?

27. Three men, A, B, and C, own a farm valued at $44140; A's share is valued at $13256; B's, at $9284; C's share is worth 25\frac{3}{7}$ per acre; how many acres does he own?

QUESTIONS.—What is the difference between the simple and the local value of a figure? (20.) (21.) What is the use of a cipher? (22.) Repeat the numeration table from right to left. (25.) From left to right. (25.) What is the sum of two or more numbers? (30.) What is addition? (31.) What is the sign of equality? (33.) What is the dollar sign? (34.) How may addition be proved? (36., *f.*) Repeat the arithmetical formula for addition. (See page 15, Anal.) What are the terms of subtraction? (40.) (41.) (42.) Define each. What is the sign of subtraction? (43.) Repeat the arithmetical formula for subtraction. (See page 20, Anal.) Give the definition of each of the terms in multiplication. (46.) What is the sign of multiplication? (51.) Repeat the arithmetical formula for multiplication. (See page 27, Anal.) Define each of the terms of division. (54.) What are the signs of division? (62.) (62., *a.*) What is the first arithmetical formula in division? Ans.—*If* 6 *pounds of sugar cost* 72 *cents, one pound will cost one* SIXTH *of* 72 *cents, which is* 12 *cents.* What is the second? Ans.—*If one pound of sugar costs* 12 *cents, as many pounds may be bought for* 72 *cents as* 12 *cents are contained times in* 72 *cents, which are* 6 *times.*

LESSON IV.

28. A certain field enclosed by a board fence, is 17 ch. 30 l. long, 13 ch. 30 l. wide; how many feet long is the fence which encloses it?

29. How many posts will it take to build the fence, if placed 5 feet apart, and what will be the length of the last pannel?

30. One cubic foot of limestone weighs 175 pounds; what will be the weight of a block 13 feet 9 inches long, 14 feet 8 inches wide, and 6 feet 3 inches thick?

31. What will it cost to build the walls of a cellar 28 feet 9 inches long, 18 feet wide, and 8 feet 3 inches deep, the walls being 2 feet 3 inches thick, at $1.25 a perch?

32. A man bought 2½ bushels of chestnuts by dry measure, and sold them by wine measure; how much did he gain?

33. A milkman bought 8 gallons of milk by beer measure, and sold it by wine measure; how many quarts did he gain?

34. A man bought 3 bushels of walnuts for $6. and retailed them at 5 cts. per gill, wine measure; did he make or lose by the operation, and how much?

35. A man bought 3746 barrels of potatoes for $9460,

but finding 12½ per cent. of them spoiled he was content to sell the remainder at cost; how much did he receive a barrel?

36. A man had 300 barrels of apples, and sold 23⅔ barrels; what per cent. had he remaining?

Questions.—Illustrate each of the principles in the fundamental rules. (66.) Illustrate each of the contractions in the fundamental rules. (67.) Give a description of the United States currency, coins, &c. (68.) What is a bill of parcels? (76.) What is the difference between a simple number and a compound denominate number? (77.) (78.) What is the difference between reduction descending and reduction ascending? (80.) (81.) Give a description of English money, coins, &c. (82.) What are the uses of each of the weights? (83.) Repeat the table of each. (84., *a.*) (85., *a*) (86.) (87.) (88., *a.*) (88., *b.*) What are the dimensions of extension? (89.) Of a line? (89., *a.*) Of a surface? (89., *b.*) Of a solid? (89., *c.*) Repeat the table of Long measure. (90., *a.*) What is a knot? (90., *c.*) What is the difference in the lengths of an English and of a geographical mile? (90., *c.*, 2.) How do artificers estimate work? (92., *d.*) Repeat the table of square measure. (92., *f.*) Of surveyor's square measure? (93., *a.*) Repeat the table of Cubic measure. (94., *e.*)

LESSON V.

37. A merchant pays $80 a month for clerk hire, which is 13 per cent. of his entire income; his other expenses are $1000; what are his net profits?

38. How many times will the driving wheels of a locomotive revolve in going from Jersey City to Rahway, the distance being 18 mi. 6 fur., supposing the wheels to be 12 feet 3½ in. in circumference?

39. In 57£. 15s. how many half crowns:

40. If it take 5 yds. 3½ qr. to make a suit of clothes, how many suits can be made from 250 yds. 3 qr., and how much will remain?

41. A commission merchant in Chicago sells 370 barrels of flour at $5.30 per barrel, and 53 firkins of butter, each containing 103 lbs., at 19 cts. per pound; what was his commission, at 5 per cent.?

42. I sent my agent in New York $3874.47, with which to buy sugar; I pay him 2½ per cent. commission, and ½ per cent. insurance; what amount should he expend for sugar?

43. A man bought 35 shares of bank stock at 8½ per cent. discount, and sold them at 3 per cent. premium; how much did he gain?

44. What will a bill of exchange on London of 96£. 18s. 4d. cost, in U. S. currency, at 64 per cent. premium?

45. I wish to send $4872.30 to a friend in Liverpool; what will be the amount of the bill which I can purchase if exchange is at 57 per cent. premium?

QUESTIONS.—What are measures of capacity? (95.) What are the steps in changing gills to hogsheads? (95., *b.*) Hogsheads to gills? (95., *b.*) What is the use of Dry measure? (96.) Repeat the table. (96., *a.*) What are the steps in changing Wine measure to Beer measure? (96., *c.*) Beer to Dry measure? (96., *c.*) What is time? (97.) Repeat the table. (97., *a.*) What is the use of circular measure? (98.) Repeat the table. (98., *a.*) What is an integer? (106.) What is the difference between an odd and an even number? (107.) (108.) What is the difference between a prime and a composite number? (109.) (110.) Of what is every number composed? (112., *a.*) What is the difference between a prime and a composite factor? (112.) (113.) What is the rule for finding the prime factors of a number? (114., *d.*)

LESSON VI.

46. I have 30 shares of City R. R. stock worth 3½ per cent. premium; how many shares of Erie R. R. stock can I purchase at 12½ per cent. discount?

47. What will be the amount of the broker's commission, if

he charges me 2 per cent. for selling, and 1½ per cen.. buying?

48. I purchased the following bill of goods; what must I mark them to gain 25 per cent.?

340 yds. calico,	@ $0.13.
172 “ “	@ .14½.
34 dress shawls,	@ 4.36.
12 fine cloaks,	@ 9.00.
437 yds. fine French broadcloth,	@ 3.25.

49. Give the marking prices of the following bill at an advance of 30 per cent.?

12 doz. fine cotton thread,	@ $1.20.
3 gross pearl buttons, at	10 cts. per doz.
2 “ extra needles, at	$1.20 “
5 “ “ pins, at	.60 “
12 doz. covered buttons, large,	@ 10 cts.
12 “ “ “ medium,	@ 8 “
12 “ “ “ vest,	@ 8½ “
12 “ “ “ small,	@ 5½ “

50. How much would the above bill of goods amount to if sold at 9½ per cent. loss?

51. I sold the following articles at auction, at the following prices: what per cent. did I gain?

14 yds. calico,	which cost 11 cts.,	at 13 cts.
12 “ “	“ “ 8 “	at 10 “
9 “ “	“ “ 15 “	at 10 “
14 handkerchiefs,	“ “ 18 “	at 20 “
3 cravats,	“ “ 25 “	at 12 “
14 “	“ “ 12 “	at 15 “
25 prs. Gents' gloves	“ 50 “	at 75 “

QUESTIONS.—What does a parenthesis denote? (116.) A vinculum? (116.) What is cancellation? (117.) Upon what axiom does cancellation depend? (16., o) What is the greatest common divisor of two or more numbers? (119) What is the least common multiple of two or more numbers? (122.) What is the difference between the g c. d. and the l. c. m. of two or more numbers? (119.) (122.) What is a fraction? (123) What is the difference between a common and a decimal fraction? (124. (125.)

LESSON VII.

52. I sold the following bill of goods at auction: did I make or lose, and what per cent.?

3 vest patterns,	which	cost	$3.	for	$2.50.
4 vests, ready made,	“	“	4.	“	5.50.
5 prs. pants,	“	“	4.50	“	5.23.
3 “ “	“	“	3.00	“	2.75.
9 yds. cassimere,	“	“	.83	“	.70.
11 cravats,	“	“	.12	“	.20.
13 coats, extra,	“	“	15.00	“	20.00.

53. A man entered business Jan. 1, 1864, with a capital of $4437.68; on Feb. 7, he added $500 to his capital, and on June 10, he added $850 more. He sold his goods at an average profit of 25 per cent., and at an average credit of 6 months; what was his net profit after deducting interest on capital, and allowing 5 per cent. for bad debts?

54. A man bought a cargo of molasses, consisting of 500 hogsheads; it cost him 30 cts. per gallon, and he sold it for 40 cts.; his storage cost him 2 per cent., insurance 3 per cent., leakage 4 per cent., other expenses 5 per cent.; how much did he make by the operation?

55. An importer in New York bought 3000 yards of English broadcloth, at 10 shillings per yard; for what must he sell it to make 15 per cent. profit, exchange being 67 per cent. premium?

56. I have a bill of 500£. to pay in London; how much

can I afford to pay for butter in New York, which will sell for 15d. per pound in Liverpool, allowing 3d. per pound for exportation, exchange being 67 per cent. premium, and how many pounds will it take to pay the bill?

QUESTIONS.—What is the difference between a proper and an improper fraction? (129.) (130.) What is the difference in the uses of the numerator and the denominator of a fraction? (128.) (127.) Give an example of a proper fraction. An improper fraction. A mixed number. A simple fraction. A complex fraction. A compound fraction. Of what may fractions be considered as indications? (135.) To what does the numerator correspond? (136.) To what the denominator? How is the value of a fraction affected by multiplying the numerator? (137., *e.*) By multiplying the denominator? (137., *c.*) When a change is produced upon the numerator, what change is produced upon the value of the fraction? ANS.—*A like change.* When a change is produced upon the denominator? ANS.—*An opposite change.* When the same change is produced upon both? ANS.—*No change*

LESSON VIII.

57. I bought 4 barrels of sugar at \$30 each; I sold $\frac{1}{2}$ of it at an advance of 25 per cent., but the remainder being damaged, I was content to lose $18\frac{3}{4}$ per cent.; did I make or lose by the operation, and how much?

58. I purchased a cargo of wheat consisting of 4500 bushels, worth \$1.30 per bushel; I paid for insurance 2 per cent. on $\frac{3}{4}$ of its value; for storage $\frac{1}{2}$ per cent.; for how much must I sell it per bushel to gain 10 per cent., after deducting loss of interest on the cost for 6 months?

59. I have $\frac{3}{4}$ interest in a vessel and cargo to San Francisco, worth \$89670; the insurance, war risk, was 8 per cent., the sailing expenses were \$4000; in a storm $\frac{1}{8}$ of the cargo was thrown overboard, and the remainder was sold at a profit of 80 per cent.; did I make or lose by the venture, and how much?

60. A teacher in the district in which I li received a salary of $800, the other expenses were $275; $125.37 are received from the public fund, and the aggregate attendance was 39467 days; what was my bill for 3 pupils of 89 days each?

61. What is the amount of duty on the following invoice, at 8 per cent. *ad valorem*?

13	bales of	unbleached muslin,	@	$300.
14	"	bleached "	@	350.
3	"	cassimere,	@	600.
4	"	" extra,	@	750.
4	"	English broadcloth,	@	800.

As duties are to be paid in gold, what amount in currency will be required to pay the above bill when gold is at a premium of $58\frac{1}{4}$ per cent.?

QUESTIONS.—In how many ways may fractions be reduced? (145.) (146.) (147.) How does the reduction of fractions differ from the reduction of integral numbers? (79.) (141.) What condition is essential to the addition or the subtraction of fractions? (150.) Can integral numbers and fractions be added? ANS.—*Only by changing them to the same denominations.* How many ways may fractions be multiplied? (154.) (157.) (158.) In how many ways can fractions be divided? (155.) (156.) (159.) (160.) What is the meaning of *of* in the expression $\frac{1}{7}$ *of* 27? (155., *b.*†) What is the difference between $\frac{1}{7}$ of 5 and $\frac{1}{5}$ of 7? What is the comparison of numbers? (166.) What is essential to the comparison of numbers? (167) What are the rules for adding and subtracting fractional compound numbers? (168., *a.*) Of what does the complete decimal scale consist? (169.) How is units' place designated? (171.)

LESSON IX.

62. An importor in New York, purchased 200 chests of fine tea, each chest weighing 87 lbs., at 3s. per pound; what will be the amount of the duty in American paper currency,

the exchange being 70 per cent. premium, and ½ per cent. being allowed for tare?

63. A man sold a factory for $27000, for which he was to receive a farm worth $9000, possession to be had in 6 months; $12000 to be paid down in cash, and the remainder to be paid by note at 1 yr. and 8 months; what should have been the price, if he had sold for cash?

64. I bought, Jan. 1, 1864, a house and lot for $5700; Apr. 9, I paid $37.50 for painting; Sept. 12, $375.68 for repairs; what had my house cost me Jan. 1, 1865, at 7 per cent. interest?

65. Samuel Wright owns a house in St. Felix st., Brooklyn, for which he paid $5000; Jan. 1, 1863, he rented it for $575, to be paid quarterly; Apr. 12, he paid $23.74 for repairs; Sept. 26, paid $43.27 for other repairs; Jan. 1, 1864, he considers the property worth $500 more than when he rented it; what per cent. does his property pay, allowing 7 per cent. interest?

66. Would he have gained or lost, had he invested his money in U. S. Five-Twenty 6 per cent. bonds, payable semi-annually, gold being at 60 per cent. premium?

67. What is the amount of $374.86 for 8 yrs. 3 mo. 18 da., at 8½ per cent. interest?

68. I purchased $8000 of U. S. Five-Twenty 6 per cent. bonds, payable in gold semi-annually; would I have lost or gained, by investing the same amount on bond and mortgage at 7 per cent., payable annually, if the average premium of gold is 50 per cent.?

QUESTIONS.—What is the general law in the multiplication of decimals? (177., *e.*) In the division of decimals? (179., *e.*) Repeat the rule for the multiplication of decimals. (177., *f.*) For the division of decimals. (179., *f.*) What is the signification of per cent.? (183.) 100

per cent. is what part of a number? How are transactions in commission, brokerage, and stocks, usually estimated? ANS.—*By the per cent.* How many cases in commission, brokerage, and stocks? (202.) (203.) (204.) (205.) How many cases in profit and loss? (207.) (208) (209.) (210.) What is insurance? (211.) How many cases in insurance? (216.) What is the rule for constructing a tax table? (223., Anal.) Give a rule for finding the amount of duty on an invoice of merchandise. (287.)

LESSON X.

69. $\$864\frac{38}{100}$. BROOKLYN, Jan. 3, 1861.

Three years after date I promise to pay Lyman A. Burr, or order, eight hundred sixty-four $\frac{38}{100}$ dollars, value received.

CHARLES BISHOP.

On the above note were the following indorsements:

Aug. 11, 1861, $150.
Nov. 12, 1862, $200.
Apr. 27, 1863, $175.
June 29, 1864, $87.

What will be the amount due Sept. 27, 1865[*]?

70. My interest balance for 3 years 2 mo., at 7 per cent., is $908.67; what is my net investment?

71. I wish to place at interest, at 7 per cent., a sum that will amount to $347 at the end of 2 yrs. 6 mo.; what is the sum?

72. The heir of an estate is entitled to $8000 Apr. 17, 1866, at which date he becomes of age; on June 11, 1863, what portion of the estate must I put at interest to amount to that sum?

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[*] It must be remembered that as no mention of interest is made in the note, it can not draw interest until due; consequently, whatever payments were made before the note is due draw interest until the end of the three years, after which they are treated according to the general rule of partial payments.

73. I purchased three notes, the first of $800, for 1 yr. 8 mo. 17 da., at 3 per cent. discount; the second of $780, for 9 mo. 11 da., at 5 per cent. discount; the third of $860, for 11 mo. 8 da., at 8 per cent. discount; if I collect the face and interest of the notes, what per cent. shall I make by the operation?

74. How long will it take $340 to double itself at 6 per cent.?

75. How long will it take $80 to double itself at 7 per cent.?

76. What will be the amount of the rent of a house in 2 years 3 mo., worth $3874, renting at 10 per cent., payable quarterly? Payable quarterly in advance?

Questions.—Give the first method of calculating interest. (245.) Give the rules for calculating by thepe6 r cent. method. (247.) (247., note 2.) (247., note 2.*) What is a partial payment? (248.) What is the difference between a *joint* note and a *joint* and *several* note? (pag. 269., note 1., 2.) What is a negotiable note? How may a note payable to order be transferred? Who is an indorser? Repeat the Supreme Court rule for calculating interest on notes, when partial payments have been made. (250.) How many and what are the problems in interest? (252.) (253.) (254.) (255.) (256.) What is the difference between simple, and compound interest? (242.) (258.)

LESSON XI.

97. The cost of a certain house and barn, together with the insurance, was $2010; the premium was ½ per cent. of their value, and the house was worth 3 times as much as the barn; what was the premium of insurance and the value of each?

98. For what sum must a note be drawn so that when discounted at a bank for 3 yrs. 5 mo. 27 da., at 6 per cent., the proceeds will amount to $591.71?

99. What is the compound interest of \$375.52 for 3 yrs. 4 mo. 12 da., at 5 per cent. per annum?

100. A drover sold 168 mules for \$12127.50, and, by so doing, gained 3⅞ per cent. on the purchase price; what was the average cost per head?

101. A person purchased a coat, vest, pants, and a pair of boots, for \$28.50; he paid 1½ times as much for the boots as for the vest, 5/3 as much for the coat as for the boots; and ¾ as much for the vest as for the pants; what is the cost of each?

102. What is the discount of \$825.50 due in 2 yrs. 9 mo. 12 da., at 5½ per cent.?

103. What is the bank discount of \$825.50, due in 2 yrs. 9 mo. 12 da., at 5½ per cent.?

104. What principal will give \$114.4726 in 4 years, at 5½ per cent.?

105. The amount of a certain sum for 2⅓ years, at 6 per cent., is \$370.785; what is the interest?

106. What principal will give \$112.50 interest in 2½ years, at 7½ per cent.?

107. In what time will \$540 amount to \$669.60, at 4½ per cent.?

108. What is the amount of 37£. 19s. 9½d. on interest for 4 yrs. 5 mo., at 9¾ per cent.?

QUESTIONS.—What is the difference between discount and present worth? (259.) (260.) What is a bank? (261.) What is bank discount? (262.) What is the difference between simple discount and bank discount? (259.) (262.) What is the difference between promissory notes and bank notes? (263.) (264.)

LESSON XII.

109. A merchant sold 450 casks of wine, each containing 50 gal., for $12375, and by so doing gained 10 per cent. on the cost; what did it cost per gallon?

110. What is the interest of 37£. 19s. 9½d. for 5 yrs. 6 mo. 19 da., at 11 per cent.?

111. A and B traded in partnership for 16 mo.; during the first 10 months A's stock was $750, and B's was $800; A then increased his stock to $1200, and B diminished his stock $300; they gained $1315; what was the share of each?

112. If 40 barrels of flour at $10 per barrel, and 5 bushels of potatoes at $1⅓ per bushel, will pay for the labor of 8 men for 26 days of 9 hours each; for the labor of how many men for 6 days of 12 hours each, will 16 bbls. of flour at $9 per barrel, and 112½ bushels of potatoes at 80 cts. per bushel, pay?

113. The sum of $850 was loaned May 10th, 1851, at 6 per cent.; what will be the amount Nov. 15, 1857?

114. A man sold a quantity of goods for $158.45, and by so doing lost 3 per cent.; what was the cost?

115. Sold 100 hhds. of molasses, each containing 63 gal., for $4394.25, and, by so doing, gained 12½ per cent.; what was the cost per gallon?

116. The interest of $825 for 4 yrs. 3 mo. 19 da. is $181.50; what was the rate?

117. The amount of $825 for a certain time, at 5½ per cent., is $1006.50; what was the time?

118. I loaned my friend $400 for 5½ months, when interest was 6 per cent.; he afterward loaned me $600, when

interest was 7 per cent.; how long should I keep it, to balance the favor?

119. A merchant had 1360 yds. of calico; he sold $14\frac{2}{3}$ per cent. of it to one man, $18\frac{3}{4}$ per cent. of it to another; how many yards did he have remaining?

QUESTIONS.—What are days of grace? (265.) What is the maturity of a note? (266.) What the proceeds? (267.) Give the rule for finding the proceeds of a note discounted at a bank. (268., *a.*) What is the difference between the real and nominal values of a pound sterling? (287.) What is the equation of payments? (292.) Repeat the rule for finding the equated time of an account. (298., *a.*)

LESSON XIII.

120. A man bought two farms for $4372 each; he sold one at a gain of 11 per cent., and the other at a loss of 11 per cent.; did he gain or lose, and how much?

121. A speculator bought two horses for $136 each; he sold one at a profit of 8 per cent., and the other at a loss of 8 per cent.; what was his gain or loss per cent.?

122. Three men, A, B, and C, bought a cargo of wheat for $10000; A paid $\frac{1}{3}$, B $\frac{1}{4}$, and C the remainder; the wheat proved damaged, so that they were content to lose 25 per cent. on cost; what was the loss of each, allowing the transaction to occupy three months, when money was worth 7 per cent.?

123. I bought a building lot in Liverpool for 86£. 17s., and sold it at an advance of 16 per cent.; what was my gain, after deducting taxes to the amount of 7£. 8s. 6d.?

124. A speculator bought two houses Jan. 3, 1863, at $8000 each; he had them insured at $\frac{5}{8}$ per cent. on $\frac{3}{4}$ their value; Apr. 12, 1863, he paid $347.80 taxes, and, on Sept. 11, paid $425 for repairs; Nov. 11, he sold one at an advance of 25 per cent., and Dec. 12, he sold the other at a

discount of 2½ per cent. What did he make or lose by the operation? What per cent.?

125. I bought two horses; for one I gave $75, and for the other I gave $125; after keeping them 9 months at a cost of $12 per month board, I sold the first at an advance of 50 per cent., and the second at a loss of 4 per cent.; did I make or lose by the operation, and what per cent., allowing money to be worth 7 per cent.?

126. What is 375 per cent. of 75?

127. What is 75 per cent. of 375?

128. 375 is what per cent. of 75?

129. 75 is what per cent. of 375?

130. 75 is 375 per cent. of what number?

131. 375 is 75 per cent. of what number?

LESSON XIV.

132. The nickle cent contains 22 parts copper and 3 parts nickle; what per cent. of each does it contain?

133. 40 is 50 per cent. more than what number?

134. 50 is 40 per cent. more than what number?

135. A man sold flour at $5.40, at $6.60, and at $7.30 a barrel, and thereby made a profit of 18 per cent.; what did each cost him per barrel?

136. By selling coffee at 20 cents per pound I make a profit of 23 per cent.; for what must I sell it to make 12½ per cent.?

137. What must be the asking price of sugar costing 12 cts. that I may fall 5 per cent. and still gain 10 per cent. on the cost, allowing 12 per cent. of the sales to be bad debts?

138. A house valued at $2000 was insured for ¾ of its value for 5 years, at 1½ per cent. per annum. At the end

of the fourth year it was destroyed by fire; what was the owner's *actual* loss, no allowance being made for interest?

NOTE.—Since the owner has paid the premium for four years, he will actually receive only the amount of insurance minus the premium and its interest.

139. A merchant ships a cargo of wheat worth $46384; for what must he insure it at 3 per cent. to cover both the value of the wheat and the premium?

140. The owners of the Isaac Newton have paid 5 per cent. per annum for her insurance for the past 21 years; it was burned last December; did they make or lose by having her insured, and what per cent.?

LESSON XV.

141. What will be the duty on the following invoice of goods:

380	yds.	Broadcloth,	@	13s.
467	"	Silk lace,	@	3s.
3874	"	Worsted lace,	@	9d.
3794	"	Brussels carpeting,	@	5s. 6d.
1413	"	Ingrain "	@	2s. 3d.

The duty on the broadcloth is 43 per cent., on lace 25 per cent., on carpeting 30 per cent.; what will be the amount of the duty in U. S. currency?

142 What will be the duty at 35 per cent. on the following invoice of goods, allowing the customary breakage and leakage:

150	baskets	champagne,	@	$15.
30	casks of	malaga,	@	$50.
25	"	claret,	@	$56.
30	"	sherry,	@	$40.

143. For what must the above bill be sold to make a net profit of 30 per cent.?

144. The amount of a note is $650; the rate is 6 per cent.; the time is 5 years; what is the interest?

145. The interest is $75; the time is 2 yrs. 6 mo.; the rate is 4½ per cent.; what is the amount?

146. The amount is $375; the time is 2 yrs. 6 mo. 18 da.; the rate is 6 per cent.; the interest is $48.913: what is the principal?

147. A vessel laden with an assorted cargo sailed for Rió Janeiro; while passing the West Indies she was caught in a gale and was obliged to throw overboard a portion of her cargo to the value of $3400; what was the general average, supposing the vessel to be valued at $20000, and the cargo to be valued at $40000?

LESSON XVI.

148. A merchant bought 350 yards of cloth at $3 per yard, on 3 months credit; he immediately sold it at $4.25, giving 6 months credit; how much did he gain by the transaction, allowing 6 per cent. interest?

149. A merchant bought goods to the value of $3460, on 6 months credit; at the end of three months he sold them at an advance of 25 per cent.; he immediately purchased his own paper at a discount of 10 per cent. per annum; what did he gain by the transaction?

150. A man bought a span of horses for $350, on 6 months credit; he kept them 2 months at an expense of $30 per month; he then sold them at an advance of 18 per cent., and took a note payable in 8 months, which he discounted at a bank to meet his own note, when it became due; did he make or lose by the transaction, and how much?

151. A man bought a house for $4000, on 6 months credit; after keeping it 3 months he sold it for $4500, and took

a note payable in 9 months, which he discounted at a bank to meet his own note, when it became due; what did he gain by the transaction?

152. How much more would he have gained had he borrowed money to meet his own note until the maturity of the one he had received?

153. A man borrows $500 at 10 per cent. to purchase a note of $750, due in 6 months; it was not paid until 12 months after maturity, drawing 6 per cent. interest; will he make or lose by the transaction, and how much?

LESSON XVII.

154. I have $2000 with which to purchase butter; I pay 2 per cent. for transportation, and sell at 12½ per cent. advance on cost, giving 9 months credit. If I have this paper discounted at a bank at 6 per cent., and repeat it every month, investing all the proceeds each time, what shall I gain in 3 months?

155. What are the simple interest, and the amount; the compound interest and compound amount; the present worth, and true discount; the bank discount and proceeds of $750.80 for 2 yrs. 5 mo. 12 da., at 6½ per cent.?

156. What is the simple interest and the amount; the compound interest and compound amount; the present value and the true discount; the bank discount and the proceeds of $960, for 2 yrs. 2 mo. 2 da., at 4½ per cent.; also the face of a note, which, when discounted at a bank for the same time, and at the same rate, will produce the same amount?

157. A, B, and C, enter into copartnership, each investing $5000. A is worth to the business $1500 per year, B $1200, and C $1000; at the end of the year the net profit

is $3500; what portion of this gain belongs to each partner?

158. The same partners continue business with the same capital, each investing an equal amount; but at the end of 5 months A withdraws $3000 from, and B adds $4000 to his capital; at the end of this year the net profit is $4000; what should be the share of each?

159. The next year the same partners, with the same capital, continue business, each with equal interest; A is to conduct the business for the firm and receive 25 per cent. of the net profits, as his compensation; A and B draw from and add to the capital as in the second year; at the end of the third year, the profits are $3500; what is each partner's portion?

LESSON XVIII.

160. A, B, and C enter into a copartnership, each with a capital of $1000. A is worth to the business $1000, B $800, and C $700; they gain $2000; how should the gain be divided?

161. The same partners continue business for another year, under the same conditions as above, except that A withdraws $300 at the end of the second month, B withdraws $300 at the end of the seventh month, and C withdraws $300 at the end of the ninth month; the gain is $1500; how shall it be divided, money being worth 6 per cent. for the first 6 months, and 10 per cent. for the remainder of the year?

162. A man bought a house for $12000, for which he gave his notes payable in 3 years, with annual interest at 6 per cent.; he rents his house at 10 per cent., payable quarterly. The repairs cost him 2 per cent. annually; at the

end of the three years he sold the house at ½ per cent. discount, and took up his notes; did he make or lose by the transaction, and how much?

163. A man has $3000 invested in a certain railroad which pays 10 per cent. per annum; he owns 26 shares of mining stock of $100 each, which he purchased at 47 per cent., and which pays a semi-annual dividend of 2 per cent.; he owns real estate which yields an annual income of $3550 at 7 per cent.; what is his yearly income, and what is the amount of his investments? Would he have gained or lost if he had invested his money in U. S. 6 per cent. bonds, the interest of which is payable semi-annually in gold, if gold could be sold at $1.75?

164. A man borrowed $15000, at 7 per cent., with which to purchase stocks. He bought 37 shares of Erie, at 90 per cent.; 16 shares of Michigan Central, at 70 per cent.; and with the remainder he bought N. Y. Central, at 95 per cent. The Erie paid a semi-annual dividend of 3 per cent., the Michigan Central 2½, and N. Y. Central 4 per cent.; at the end of 18 months he sells his stock at an advance of 2¾ per cent., paying a broker ¾ per cent. for selling, and takes up his notes; does he make or lose by the transaction, and how much?

165. A and B enter a copartnership; A puts in $3000, and B $5000; but A is worth $1800 to the business, while B is worth only $700; the profit for a year is $2500; how shall it be divided?

166. A and B enter a copartnership; A puts in $3000, and withdraws $1200 at the end of 5 months; B puts in $7000, and withdraws 25 per cent. of his capital at the end of 8 months; A is worth to the business $1200, and B is worth $900; at the end of 4 months B withdraws his serv-

ices, and his place is supplied by a clerk at a salary of $600 per annum; the profits for the year are $2500; how shall it be divided?

167. The ship Western was loaded in New York with flour to the value of $12000 by A, corn to the value of $8000 by B, and light freight to the value of $15000 by C; the vessel was valued at $19000, and owned by D; the vessel and cargo was insured at $\frac{3}{4}$ of its value at 5 per cent.; on passing the West Indies she was lost, and goods to the value of only $5000 were saved; what was the entire loss to the owners, and what was the *actual* loss of each? What was the actual loss to the insurance company?

FINIS.